A canonical discrete-time nonlinear form for reduced order observers design

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Abstract: This paper deals with a normal form of a class of nonlinear dynamical systems. This form enables us to design a reduced-order observer for a class of nonlinear dynamical system. Necessary and sufficient geometrical conditions for the existence of a coordinate change to transform a given singular dynamical system into such normal form are also given.

Keywords: Nonlinear singular systems, Observability normal form, Reduced-order observer.

1. INTRODUCTION

The problem of constructing observers for nonlinear systems has attracted significant attention during the last decades. From a practical point of view the observers design for nonlinear systems is often approached by linearizing the system. One technique for constructing nonlinear observers is to linearize the error dynamics by using a change of coordinates and an output injection (see Krener and Isidori [1983], Krener and Respondek [1985], Xiao and Gao [1988], Boutat et al. [2009]). However, it turned out that the conditions for the linearizability of the error dynamics are quite stringent and hard to verify in practice. Recently, a new approach for the design of nonlinear observers which can be applied to a wider class of systems was presented (see Krener and Xiao [2002], Kazantzis and Kravaris [1998]). In (Lee and Nam [1990]; W. Lee and K. Nam, gives a necessary and sufficient condition for a discrete-time nonlinear system to be equivalent to a nonlinear full observer form. On the other hand reduced-order observers design has involving various researchers in the control. The first results on the reduced order observers for linear systems were presented by Luenberger (Luenberger [1964]).

In (Sundarapandian [2004] and Sundarapandian [2006]), the reduced order observer design for discrete-time nonlinear systems is presented. This work is a generalization of the construction of reduced order observers for linear systems developed by Luenberger (Luenberger [1964]).

However, normal nonlinear forms which support a reduced nonlinear observer, with a linear error, remains unknown. The main contribution of this paper is to gives a class of these normal forms and then to characterize the family of nonlinear dynamical systems which can be transformed by mean of a change of coordinate to these class.

The paper is organized as follows. Section 2 contains notations and a definition. Section 3 presents a nonlinear normal form and the associated reduced-order observer. Section 4 deals with geometrical conditions under which a nonlinear discrete-time dynamic system can be transformed into such normal form. Section 5 presents an example to illustrate the previous results, and Section 6 concludes the paper.

2. NOTATIONS AND DEFINITION

Let us consider the following discrete-time nonlinear dynamical system:

\[ x_{k+1} = F(x_k) \]  \quad (1a)
\[ y_k = h(x_k) \]  \quad (1b)

where the \( x_k \in \mathbb{R}^n \) is the state, \( y_k \in \mathbb{R}^m \) is the output. We assume that the map \( F \) is a diffeomorphism. We assume, also, that the pair \((h, F)\) satisfies the rank observability condition. Thus, the following 1-forms:

\[ d(hoF^{i-1}) \quad 1 \leq i \leq n \]

are linearly independent, where \( F^{i-1} = F_{oF...oF} \) is the \((i-1)\)th composition of \( F \).

Under this assumption see for e.g. (Sundarapandian [2006]) the dynamical systems can be rewritten in new coordinates as follows:

\[ x_{k+1}^1 = f_1(x_k) \]  \quad (2a)
\[ x_{k+1}^2 = f_2(x_k) \]  \quad (2b)
\[ y_k = x_k^2 \]  \quad (2c)

where vector \( x_k = \begin{pmatrix} x_k^1 \\ x_k^2 \end{pmatrix} \in \mathbb{R}^n \) and \( x_k^1 \in \mathbb{R}^{n-m} \) and \( x_k^2 \in \mathbb{R}^m \) denote the global state, the unmeasured state and the measured state respectively.

It is easy to see that the pair \((h, F)\) satisfies to the observability rank condition if and only if the pair \((f_1, f_2)\) as well satisfies the observability rank condition.

References

[Sundarapandian [2004] and Sundarapandian [2006]]
Now, let us define the so-called reduced order observer.

**Definition 1.** We say that the following discrete dynamical system
\[\dot{x}_{k+1}^1 = f_1(x_k^1, x_k^2) \tag{3}\]
is a reduced order observer for the discrete time dynamical system (1) if the error \(e_k = \tilde{x}_k^1 - x_k^1 \to 0\) as \(k \to +\infty\).

In the case where maps \(f_1\) and \(f_2\) are linear, Thus, if we set
\[f_1(x_k^1) = A_{11}x_k^1 + A_{12}x_k^2\]
\[f_2(x_k^2) = A_{21}x_k^1 + A_{22}x_k^2\]
yielding
\[e_k = A_{11}\tilde{x}_k^1 + A_{12}\tilde{x}_k^2\]
it's easy to see that the reduced order observer is as follows:
\[\dot{x}_{k+1}^1 = A_{11}\tilde{x}_k^1 + A_{12}\tilde{x}_k^2 + K(2A_{11}\tilde{x}_k^1 - A_{21}\tilde{x}_k^1) \tag{4}\]

Then the observation error dynamics behaves as follows:
\[e_{k+1} = (A_{11} - KA_{21})e_k\]
which is stabilizable as long as the pair \((A_{11}, A_{21})\) is observable.

We will end this section by a remark.

**Remark 1.** The reduced order observer (4) introduced by Leuenberger (Luenderger [1964]) enables us to overcome on the redundancy of measurement. However, the new output \(A_{21}x_k^1\) can be contains more information that we are needing for example:
\[x_{k+1}^1 = f_1(x_k^1, x_k^2) = \begin{bmatrix} x_{1,k+1}^1 = y_k \\ x_{2,k+1}^1 = x_{1,k} \\ x_{3,k+1}^1 = x_{2,k} \end{bmatrix}\]
\[x_{k+1}^2 = f_2(x_k^1, x_k^2) = \begin{bmatrix} \xi_{k+1} = x_{3,k} + \xi_k \\ \eta_{k+1} = ax_{1,k} + bx_{2,k} + Cx_{3,k} \end{bmatrix}\]
yielding
\[y_k = x_k^2 = \begin{bmatrix} \xi_k \\ \eta_k \end{bmatrix}\]

Thus
\[A_{21} = \begin{pmatrix} 0 & 0 & 1 \\ a & b & c \end{pmatrix} \]

This shows that we need to use only the dynamic \(\xi_{k+1} = x_{3,k} + \xi_k\) to design a reduced order observer. Thereafter, we will take into account this fact in the proposed normal form.

### 3. Normal Form And Its Reduced Order Observers Design

In this section we will give a nonlinear normal form which admits a reduced order observer. We consider a multivariable discrete-time nonlinear system described by:
\[z_{k+1} = Az_k + \alpha(y_k)zo_k + \beta(y_k) \tag{5a}\]
\[\xi_{k+1} = \gamma_1(y_k)zo_k + \gamma_2(y_k) \tag{5b}\]
\[\eta_{k+1} = \mu(z_k, y_k) \tag{5c}\]
\[y_k = (\xi_k, \eta_k)^T = (\tilde{y}_k^1, \tilde{y}_k^2)^T \tag{5d}\]

where \(z_{0,k} = Cz_k = (z_1^1, z_2^2, z_3^m, \ldots z_m^m) \in \mathbb{R}^m\) is the unmeasured state,
\[\eta_k = (\eta_{1,k}, \ldots, \eta_{p,k})^T \in \mathbb{R}^p\]
are the measurable outputs,
\[z_{0,k} = Cz_k = (z_1^1, z_2^2, z_3^m, \ldots z_m^m) \tag{5b}\]
and \(A = diag(A_1, \ldots, A_p)\)

We will assume that \(\gamma_1(y_k)\) is an invertible map such that from (5b) we obtain:
\[z_0,k = Cz_k = (\gamma_1(y_k))^{-1}(\xi_{k+1} - \gamma_2(y_k)) \tag{6}\]
Therefore the pair \((C, A)\) is observable.

Now, let us consider the following system
\[\xi_{k+1} = N\xi_k + \Psi(y_k, y_{k-1}) \tag{7a}\]
\[\hat{z}_k = \xi_k + G(y_k, y_{k-1}) \tag{7b}\]

Where \(N\) is a matrix of appropriate dimension, \(G\) and \(\Psi\) are nonlinear maps which must be determined such that (7) is an asymptotic observer for system (5). One can see that the form (7) is a generalization of the functional and reduced order observers forms considered in (Darouch [2000]) for example.

We have the following result.

**Proposition 1.** For the canonical form (5), where the pair \((C, A)\) is observable; there always exists a gain \(\kappa\) such that the observer (7) is asymptotically stable, with:
\[N = A - \kappa C\]
is a stability matrix
\[L(y_k) = \alpha(y_k) + \kappa \tag{8a}\]
\[G(y_k, y_{k-1}) = L(y_{k-1})\gamma_1^{-1}(y_{k-1})(\xi_k - \gamma_2(y_{k-1})) \tag{8b}\]
\[\Psi(y_k, y_{k-1}) = \hat{G}(y_k, y_{k-1}) + \beta(y_k) \tag{8d}\]

**Proof 1.** Let \(e_k = z_k - \hat{z}_k = z_k - \xi_k - G(y_k, y_{k-1})\) to be the estimation error, then its dynamic is:
\[e_{k+1} = z_{k+1} - \xi_{k+1} - G(y_{k+1}, y_k)\]
\[= -N\xi_k - \Psi(y_k, y_{k-1}) - G(y_{k+1}, y_k) + Az_k + \alpha(y_k)zo_k + \beta(y_k) \tag{5a}\]
By using the fact that \(-\xi_k = e_k - z_k + G(y_k, y_{k-1})\), we obtain
\[e_{k+1} = N\xi_k - N\xi_k + NG(y_k, y_{k-1})\]
\[= -\Psi(y_k, y_{k-1}) + Az_k + \alpha(y_k)zo_k + \beta(y_k) - G(y_{k+1}, y_k)\]
If (8b) is satisfied, we obtain
\[ e_{k+1} = Ne_k - (N - A + \kappa C)z_k + \beta (yk) - G(y_{k+1}, y_k) - \Psi(y_k, y_{k-1}) + \gamma_2(y_k) \]
and by induction we define the following family of vector fields:
\[ \tau_{i,j}(p) = Ad_{f^{j-1}(p)}(\tau_{i,j-1}(F^{-1}(p))) \quad \text{for} \quad 2 \leq j \leq r_i \]
From the rank observability condition of the pair \((f_1, H)\) we can see that the family of vector fields \(\tau = (\tau_{i,j})\) are independent.
Before to go ahead we must check if the vector fields \(\tau_{i,j}\) commute thus: for \(1 \leq i \leq m, 1 \leq j \leq r_i\) and \(1 \leq l \leq r_s\),
\[ [\tau_{i,j}, \tau_{s,t}] = 0. \quad (11) \]
where \([\cdot, \cdot]\) is the Lie bracket.
As will be shown later, equations (11) are necessary for the existence of change of coordinates. Under these conditions we construct \(\sigma_1, ..., \sigma_m\) and \(\nu_1, ..., \nu_p\) vector fields such that:
\begin{align*}
(1) & \quad d\bar{\xi}_{i,k}(\sigma_j) = \delta^j_i \quad \text{for} \quad i = j \quad \text{and} \quad d\bar{\xi}_{i,k}(\nu_j) = 0 \\
(2) & \quad d\bar{\eta}_{i,k}(\sigma_j) = 0 \quad \text{and} \quad d\bar{\eta}_{i,k}(\nu_j) = \delta^j_i \quad \text{for} \quad i = j \quad \text{and} \quad 0 \quad \text{for} \quad i \neq j \\
(3) & \quad \{\tau, \sigma_j, \nu_j\} \quad \text{is a basis of the} \quad \text{hall space} \\
(4) & \quad [X, Y] = 0 \quad \text{where} \quad X, Y \in \{\tau, \sigma_j, \nu_j\} 
\end{align*}
Now consider the following matrix:
\[ \Lambda = \begin{bmatrix}
\theta & d\bar{\xi}_{i,k} \\
\frac{\partial}{\partial \bar{\eta}_{i,k}} & (\tau, \sigma_j, \nu_j).
\end{bmatrix} \]
Next we need the following important definition introduced by Lee and Nam in [Lee and Nam [1990]].
Definition 2. Let \(\sigma : U_1 \rightarrow U_2\) be a diffeomorphism between to be a vector field on \(U_1\) and \(U_2\) and let \(X\) be a vector field on \(U_1\), we define \(Ad_{\sigma}X\) to be a vector field on \(U_2\) such that:
\[ Ad_{\sigma}X(p) = D\sigma|_{\sigma^{-1}(p)}X(\sigma^{-1}(p)), \]
where \(D\sigma\) is the Jacobian of \(\sigma\).
Hereafter, we will make the following assumption:
Assumption 1. We will assume within this work that \(F = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}\) is a diffeomorphism. Where \(f_2 = \begin{bmatrix} \bar{\xi}_{k+1} \\ \bar{\eta}_{k+1} \end{bmatrix}. \)
Now, for \(i = 1 : m\) we define the vector fields \(\tau_{i,j}\) by the following equations:
\begin{align*}
\theta_{i,r_i}(\tau_{1,1}) &= 1, \quad \text{for} \quad 1 \leq i \leq m \\
\theta_{i,l}(\tau_{1,1}) &= 0, \quad \text{for} \quad 1 \leq l \leq r_i - 1 \\
\theta_{j,l}(\tau_{1,1}) &= 0, \quad \text{for} \quad j < i \quad \text{and} \quad 1 \leq l \leq r_i \\
\theta_{j,l}(\tau_{1,1}) &= 0, \quad \text{for} \quad j > i \quad \text{and} \quad 1 \leq l \leq r_j 
\end{align*} \quad (10)
By the rank condition observability it easy to see that the matrix Λ is invertible. Therefore, we can consider the following multi 1-forms:

\[ \omega = \Lambda^{-1} \begin{bmatrix} \theta \\ \frac{d\xi_k}{d\eta_k} \end{bmatrix}. \]

Set by definition that \( : \omega \begin{bmatrix} \tau \\ \sigma_i \\ \nu_j \end{bmatrix} = I_{n \times n} \) and let the multi 1-forms be \( \omega = \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} \) where \( \omega_1 \) is formed by the m first 1-forms of \( \omega \).

Then we have the following result.

**Theorem 1.** There exists a diffeomorphism

\[
\begin{bmatrix} z_k \\ \xi_k \\ \eta_k \end{bmatrix} = \begin{bmatrix} \phi(x_k, \xi_k, \eta_k) \\ \xi_k \\ \eta_k \end{bmatrix}
\]

which transforms the dynamical system (9) into the canonical form (5) if and only if the following conditions are satisfied:

1. the commutativity conditions (11) are fulfilled,
2. \( Ad_{f_1}(\tau_{i,j}) \in \ker \omega_1 \) for \( 1 \leq i \leq m \), \( 1 \leq j \leq r_i \)
3. \( Ad_{f_1}(\tau_{i,r_i}) = V(y) \) span\( \nu_j \), for \( 1 \leq i \leq m \), \( 1 \leq j \leq r_i \)
4. \( \theta_{j,1}(\tau_{i,j}) = 0 \), for \( j > i \), \( 1 \leq i \leq m \), \( r_j + 1 \leq l \leq r_i \) where \( V(y) \) is a smooth map and span\( \nu_j \) modulo a combination of vector fields \( \nu_j \).

Before giving the proof let us explain these conditions in the following

**Remark 2.**

1. Conditions (1) on commutativity of vector fields ensure the existence of a diffeomorphism \( z_k = \phi(x_k, \xi_k, \eta_k) \) such that:

\[ \phi \ast (f_1) = D\phi(f_1) \]

and

\[ \phi\circ f_1 = A\phi + \beta(y, z_{0,k}) \]

2. Condition (2) implies that \( \beta \) is linear in \( z_{0,k} \).
3. Condition (3) implies that \( z_{r_i, k} \) doesn’t depend on \( z_{r_j, k} \) for \( l < i \).

The proof which we will give here uses the same materials as in (Krener and Respondek [1985], Xiao and Gao [1988], Boutat et al. [2007], Boutat et al. [2009]) for continuous time. We will follow the method of (Lee and Nam [1990]) for discrete time.

**Proof 2.**

Necessity: Indeed, if (9) can be transformed into (5) via the diffeomorphism \( z_k = \phi(x_k, \xi_k, \eta_k) \), then \( \tau_{i,j} = \frac{\partial}{\partial z_{i,k}^j} \).

And it is easy to check that all conditions of Theorem 1 are satisfied.

Sufficiency: The evaluation of the differential of \( \omega \) for any \( X, Y \in \{ \tau, \sigma_i, \nu_j \} \) give

\[
d\omega(X, Y) = L_X\omega(Y) - L_Y\omega(X) - \omega[X, Y]
\]

because \( \omega(X) \) and \( \omega(Y) \) are constant. As \( \omega \) is an isomorphism, this implies the equivalence between

\[
[X, Y] = 0
\]

and

\[
d\omega = 0
\]

According to theorem of Poincaré (Poincaré [1892]), \( d\omega \) implies that there exists a local diffeomorphism \( \phi \) such that \( \omega = D\phi \). Therefore we can write:

\[
D\phi(\tau_{i,j}) = \frac{\partial}{\partial z_{i,k}^j}
\]

Now let us clarify the effect of the diffeomorphism \( \phi \) on the global map \( F = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \) defined in (9a). To do so for \( 1 \leq i \leq m \) and \( 1 \leq j \leq r_i - 1 \), we get

\[
\frac{\partial(\phi_s(F))}{\partial z_{i,j}} = \begin{bmatrix}
\omega_1(Ad_{f_1}(\tau_{i,j}) + \omega_1(Ad_{f_1}(\tau_{i,j})) \\
\omega_2(Ad_{f_1}(\tau_{i,j}) + \omega_2(Ad_{f_1}(\tau_{i,j}))
\end{bmatrix}
\]

From condition (3) we have \([\tau_{i,j}, f_2] \in \ker \omega_1 \) for \( 1 \leq i \leq m \) and \( 1 \leq j \leq r_i - 1 \) implies that \( \omega_2(Ad_{f_1}(\tau_{i,j}) = 0 \).

By integrating we obtain:

\[
\phi_0 f_1\phi_0^{-1} = A z_k + \theta(y, z_{0,k})
\]

Now by the condition (2), we have

\[
\frac{\partial(\phi_0 f_0 \phi_0^{-1})}{\partial z_{i,k}^j} = D\phi(\tau_{i,r_i+1})
\]

\[
= D\phi(\sum_{j=1}^{r_i} V_{i,j}(y_k) \tau_{i,j})
\]

\[
= \sum_{j=1}^{r_i} V_{i,j}(y_k) D\phi(\tau_{i,j})
\]

\[
= \sum_{j=1}^{r_i} V_{i,j}(y_k) \frac{\partial}{\partial z_{i,k}^j}
\]

which means \( \theta(y, z_{0,k}) \) in (12) can be decomposed as:

\[
\theta(y, z_{0,k}) = \beta(y, z_{0,k}) + \gamma(y_k)
\]

and this implies that (9a) is transformed to form (5a) via \( \phi(x_k) \).

Finally, let us prove that the diffeomorphism \( \phi(x_k) \) will transform (9b) to (5b).

As we know

\[
\frac{\partial}{\partial z_{i,k}^j} H_j \circ \phi = dH_j(\tau_{i,j}) = \theta_{j,1}(\tau_{i,j})
\]

According to the definition of \( \tau_{i,1} \) in (10), we get
\[ \theta_{j,1}(\tau_{i,l}) = \theta_{j,1}([\tau_{i,l-1}, f]) = \theta_{j,2}(\tau_{i,l-1}) = \ldots = \theta_{j,l}(\tau_{i,1}) = 0 \quad \text{for } j < i \text{ and } 1 \leq l \leq r_j. \]

For \( j < i \), the above procedure can be repeated. Following the same procedure, we have \( \theta_{j,1}(\tau_{i,l}) = 0 \), for \( j > i \) and \( 1 \leq l \leq r_j \). Combined with condition (3) in Theorem 1, we have

\[ \theta_{j,1}(\tau_{i,l}) = \begin{cases} 1, & i = j, l = r_i \\ 0, & \text{otherwise} \end{cases} \]

which implies that (9b) is transformed to form (5b) through the diffeomorphism. 

Let us consider an example to highlight our propose.

5. EXAMPLE

This section gives an academic example in order to highlight the proposed results. For this, consider the following nonlinear discrete time dynamical system:

\[
\begin{cases}
x_{1,k+1} &= \bar{\eta}_k + \bar{\xi}_k x_{2,k} - x_{1,k} (x_{1,k} + \bar{\xi}_k x_{2,k})^2 \\
x_{2,k+1} &= x_{1,k} + \bar{\xi}_k x_{2,k} \\
\bar{\xi}_{k+1} &= x_{1,k} + \bar{\xi}_k x_{2,k} \\
y_k &= (\bar{\xi}_k, \bar{\eta}_k)
\end{cases}
\] (13)

The dynamic of this system is given by the following diffeomorphism:

\[
F \left( \begin{array}{c} x_{1,k} \\ x_{2,k} \\ \bar{\xi}_k \\ \bar{\eta}_k \end{array} \right) = \left( \begin{array}{c} \bar{\eta}_k + \bar{\xi}_k x_{2,k} - x_{1,k} (x_{1,k} + \bar{\xi}_k x_{2,k})^2 \\
\quad x_{1,k} + \bar{\xi}_k x_{2,k} \\
\bar{\xi}_{k+1} \\
y_k = (\bar{\xi}_k, \bar{\eta}_k) \end{array} \right)^2
\]

Let

\[
x_k = \left( \begin{array}{c} x_{1,k} \\ x_{2,k} \\ \bar{\xi}_k \\ \bar{\eta}_k \end{array} \right) \quad \text{and} \quad \chi_k = \left( \begin{array}{c} X_{1,k} \\ X_{2,k} \\ X_{3,k} \\ X_{4,k} \end{array} \right)
\]

Its inverse map is given by:

\[
x_k = F^{-1}(\chi_k) = \left( \begin{array}{c} X_{2,k} - X_{3,k} (X_{4,k} - X_2, k) \\
X_{3,k} - X_{4,k} \\
X_{1,k} - (X_{4,k} - X_2, k) X_{3,k} + X_{3,k} X_{2,k} \end{array} \right)
\]

The differential of the part \( f_1 = (x_{1,k+1} x_{2,k+1}) \) represented by the two first dynamics is

\[
Df_1(x_k) = \left( \begin{array}{c} -2x_{2,k} (x_{1,k} + \bar{\xi}_k x_{2,k})^2 \\
\quad 2 \bar{\xi}_k x_{2,k} \end{array} \right)
\]

then the differential of \( f_1 \) at \( F^{-1}(\chi_k) \) is:

\[
Df_1 (F^{-1}(\chi_k)) = \left( \begin{array}{c} -2X_{2,k}X_{3,k} \left( \frac{(X_{4,k} - X_{2,k}) - X_{2,k}^2}{-4(X_{4,k} - X_{2,k}) X_{3,k} X_{2,k}} \right) \\
\quad 1 \\
\end{array} \right)
\]

A simple calculation gives the 1-forms of observability as follows:

\[
\theta_{1,1} = dx_{2,k}, \quad \theta_{1,2} = dx_{1,k} + \hat{\xi}_k x_{2,k} dx_{2,k} + x_{2,k}^2 d\hat{\xi}_k,
\]

Moreover, we can determine the following vector fields:

\[
\tau_{1,1} = \frac{\partial}{\partial x_{1,k}}, \quad \tau_{1,2} = \frac{\partial}{\partial x_{2,k}} - 2x_{2,k} \hat{\xi}_k \frac{\partial}{\partial x_{1,k}},
\]

\[
\sigma_1 = \frac{\partial}{\partial \hat{\xi}_k} - x_{2,k} \frac{\partial}{\partial \hat{\eta}_k}, \quad \sigma_2 = \frac{\partial}{\partial \hat{\eta}_k} - x_{2,k} \frac{\partial}{\partial \hat{\xi}_k}.
\]

Thus one has

\[
\Lambda = \left[ \begin{array}{cccc} \theta_{1,1} & \theta_{1,2} & \theta_{1,3} & \theta_{1,4} \\
\theta_{2,1} & \theta_{2,2} & \theta_{2,3} & \theta_{2,4} \\
\theta_{3,1} & \theta_{3,2} & \theta_{3,3} & \theta_{3,4} \\
\theta_{4,1} & \theta_{4,2} & \theta_{4,3} & \theta_{4,4} \end{array} \right] = \left( \begin{array}{cccc} 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \end{array} \right).
\]

Then

\[
\omega = \Lambda^{-1} \left[ \begin{array}{c} \theta_1 \\
\theta_2 \\
\theta_3 \\
\theta_4 \end{array} \right] = \left( \begin{array}{c} 0 \\ 1 \\ 0 \\ 0 \end{array} \right) \left[ \begin{array}{c} \theta_{1,1} \\
\theta_{1,2} \\
\theta_{1,3} \\
\theta_{1,4} \end{array} \right] = \left( \begin{array}{c} 0 \\ 1 \\ 0 \\ 0 \end{array} \right) \left[ \begin{array}{c} \theta_{1,1} \\
\theta_{1,2} \\
\theta_{1,3} \\
\theta_{1,4} \end{array} \right].
\]

Set the following change of coordinates:

\[
\begin{align*}
z_{1,k} &= x_{1,k} + \bar{\xi}_k x_{2,k} \\
z_{2,k} &= x_{2,k}
\end{align*}
\]

Therefore the canonical form is:

\[
\begin{align*}
z_{1,k+1} &= \eta_k + \bar{\xi}_k z_{2,k} \\
\eta_{k+1} &= \bar{\xi}_k + z_{1,k} \\
y_k &= (\bar{\xi}_k, \eta_k)
\end{align*}
\]

\( \eta_k \) and \( \bar{\xi}_k \) are invariant.

Now we design an observer for system (14). Using the notations of proposition 1 we have:

\[
A = \left( \begin{array}{cc} 0 & 0 \\
1 & 0 \end{array} \right), \quad \alpha(\eta_k) = \left( \begin{array}{c} 1 \\
0 \end{array} \right) \left[ \begin{array}{c} \bar{\xi}_k \\
\eta_k \end{array} \right]
\]

\[
C = \left( \begin{array}{c} 0 \\ 1 \end{array} \right), \quad \beta(\eta_k) = \left( \begin{array}{c} 0 \\
1 \end{array} \right) \left[ \begin{array}{c} \bar{\xi}_k \\
\eta_k \end{array} \right].
\]

\[ \gamma_1(y_k) = 1, \quad \gamma_2(y_k) = 0, \quad \Psi(z_k, y_k) = \bar{\xi}_k + z_{1,k} \]
and
\[ \kappa = \begin{bmatrix} \kappa_1 \\ \kappa_2 \end{bmatrix}, \quad N = \begin{bmatrix} 0 & N_{12} \\ 1 & N_{12} \end{bmatrix} \]

\[ L(y_k) = \begin{bmatrix} \xi_k + \kappa_1 \\ \kappa_2 \end{bmatrix} \]

\[ G(y_k, y_{k-1}) = \begin{bmatrix} \xi_{k-1} \xi_k + \ldots \end{bmatrix} \]


6. CONCLUSION

In this paper a reduced-order observer design procedure is proposed for nonlinear discrete-time systems. The approach relies on the equivalence to a canonical observer form through coordinates change. Numerical example was given to show the applicability of our approach.

REFERENCES


