Robust decentralized control with scalar output of multivariable structurally uncertain plants with state delay

Elizaveta Parsheva

Abstract— The problem of a robust control system design for interconnected systems with structural and parametrical uncertainty was solved for the case where derivatives of input and output parameters cannot be measured. The order of the mathematical model may change over time. Operability of the designed control systems in the case of non-measurable and bounded disturbances acting on the controlled plant was demonstrated. Only the measurable variables of the local subsystems are used to generate the control actions, that is, control is completely decentralized.

I. INTRODUCTION

The problem of control with scalar input and output has become one of the classical problems of modern control theory and plenty of methods for robust control design have been developed. The key developments in robust control theory, as well as a comprehensive bibliography, can be found in [1, 2]. In monograph [3] the classification of disturbances of various types and two key methods of their compensation are given. Under the first approach, the structure and parameters of the controlling systems are chosen in such a way that they would provide insensitivity of the system to disturbance (invariant systems). The second approach is based on a dynamic compensation of internal and external disturbances, when the control of adjusting a device suppresses the influence of disturbances on the parameters of the system. In addition, the study [3] gives a general statement of the problem and proposes a few methods of designing the invariant systems that are based on the algebraic structure of mathematical models of plants.

In [3-5] an internal model of disturbances is used to solve the problem whereas [6, 7] use the methods of the theory of robust and adaptive systems. The approach to the synthesis of static, robust controllers for linear systems that is based on the linear-quadratic problem that is in turn based on the parameterization of Lurie–Riccati equations is presented in [8]. Robust systems with compensation of disturbances that use these methods are studied in [9, 10].

A simple robust control algorithm that remains the same for various types of plants is proposed in [1]. It is shown that the algorithm compensates for parametric and external disturbances with a given accuracy. A closed system works here as an implicitly given nominal model whose parameters are used in control.

An increasing variety of challenging applied problems in the adaptive decentralized control theory forces the researchers to work with the plants with uncertain parameters, time delay and other issues which have to be taken into account when designing control systems. The key feature of the control systems for delay systems [12-16] is the dependency of the state of the controlled process on the previous states (history). Ignoring the influence of that delay may lead to the quality degradation of the control system and, more over, to inability to perform control functions.

It is important to note that almost all the suggested methods are based on an assumption that the structure of a plant is known i.e. the order of a system of differential equations is known and parametric and external disturbances are unknown. There are various studies devoted to the problems of control with an unknown order [17-19]. Sources [17, 18] consider control problems of linear, stationary systems with an unknown and constant order of numerator and denominator for their transfer functions. Source [19] considers a wider class of systems with disturbances that are able to influence both the parameters of the system as well as its order.

This paper considers the problem of robust control for interconnected state delay systems with uncertain parameters which are subject to the uncontrolled external and parametric disturbances. These disturbances may change the order of a system in unpredictable ways. This means that the order of a system is unknown and scalar input and output signals can only be measured. To solve the problem, a simple robust control algorithm is proposed that compensates for this class of uncertainties with a given accuracy and a finite time. Only the measurable variables of the local subsystems are used for the control i.e. control is completely decentralized.

II. PROBLEM STATEMENT

Let us consider an interconnected system whose local subsystems’ dynamic processes are described by the following equations

\[ Q_i(P, t) y_i(t) + G_i(P, t) y_j(t - \tau_j) = k_i(t) R_i(P, t) u_i(t) + f_j(t) + \sum_{j=1}^{M} S_{ij}(P, t) y_j(t), \quad i \neq j, \quad i = 1, M, \]  

where \( P = d/dt \) – differential operator;

\[ Q_i(P, t) = q_{n_i ti}(t) P^{n_i -1} + \ldots + q_{0i}(t), \]

\[ G_i(P, t) = g_{n_i ti}(t) P^{n_i -1} + \ldots + g_{0i}(t), \]

\[ k_i(t) R_i(P, t) = r_{n_i ti}(t) P^{n_i} + r_{n_i ti}(t) P^{n_i -1} + \ldots + r_{0i}(t), \]

\[ S_{ij}(P, t) = s_{n_i tj}(t) P^{n_j} + s_{n_i tj}(t) P^{n_j -1} + \ldots + s_{0j}(t) \] – linear differential operators with unknown parameters; \( f_j(t) \) – an uncontrolled disturbance; \( u_i(t) \) – a scalar control action; \( y_i(t) \) – a scalar controlled variable in the i-subsystem which can be measured.

1 This work was supported by the Russian Foundation for Basic Research, project no. 09-08-00237
Decentralized control for such a system is defined as the problem of finding $M$ local control blocks, each of which only can access current information about a system [20]. Required quality of transition processes in a subsystem is defined by equations of the local nominal models

$$Q_{ni}(P)y_{ni}(t) = k_{ni}r_{i}(t), \quad i = 1,M.$$  

(2)

Here $Q_{ni}(P)$ are linear differential operators; $k_{ni} \neq 0$; $r_{i}(t)$ are the scalar bounded control actions. It is necessary to design a control system for which the following condition will be satisfied:

$$\lim_{t \to \infty} k_{i}(t) = \lim_{t \to \infty} y_{ni}(t) - y_{mi}(t) < \delta \quad \text{if} \quad t \geq T.$$  

(3)

Here $\delta$ is the accuracy of the dynamic error $e_{i}(t)$. $T$ is the time beyond which the dynamic error should not exceed the value $\delta$. It is forbidden to use measurable parameters of one subsystem in other local subsystems.

Assumptions:

i) $Q_{ni}(\lambda)$ are Hurwitz polynomials ( $\lambda$ – complex variable in Laplace transformation);

ii) the operator $R_{i}(P,t)$ is stable, i.e. trivial solution of equation $R_{i}(P,t)u_{i}(t) = 0$ is asymptotically stable. For the fixed value $t_{i}$ polynomial $R_{i}(\lambda,t_{i})$ is Hurwitz;

iii) the orders of polynomials $deg Q_{ni} = n_{i}, \quad deg R_{ni} = m_{i}, \quad deg S_{i} = n_{i}, \quad n_{i} < n_{i} - 1$ are known and relative degree of a local subsystem $\gamma_{i} = n_{i} - m_{i} > 1$;

iv) the upper bound $\gamma_{ui} \geq \gamma_{i}$ of relative degree $\gamma_{i}$ is known as the upper bound of the degree of the polynomial $Q_{ni}$, i.e. $n_{i} \leq \pi_{i}$;

v) the order of the polynomials $Q_{mi}$ is equal to $\gamma_{ui}$;

vi) the coefficients’ signs $k_{i}(t)$ are known and $k_{i}(t) > 0$;

vii) the operators coefficients $k_{i}(t)R_{i}(P,t), Q_{i}(P,t), G_{i}(P,t), S_{i}(P,t)$ are bounded functions; the non-zero coefficients of high orders of operators $R_{i}(P,t)$ and $Q_{i}(P,t)$ are positive functions;

viii) the coefficients of differential operators $k_{i}(t)R_{i}(P,t), Q_{i}(P,t)$ depend on vector of unknown parameters $\xi \in \Xi$, where $\Xi$ is a known bounded set;

ix) the actions $r_{i}(t)$ are bounded functions;

x) the signal of local nominal model $y_{ni}(t)$ and its derivatives $y_{ui}(t)$ are bounded functions;

xi) the signal of local nominal model $y_{ni}(t)$ and its derivatives $y_{ui}(t)$ are bounded functions;

xii) it is prohibited to use the derivatives of signals $y_{i}(t), u_{i}(t), r_{i}(t)$.

Based on the assumptions it is possible to conclude that the dynamic order of the system (1) is unknown and subject to change as the result of parametric disturbances. For instance if $q_{n_{i}}(t) = 0$ and $q_{n-1}(t) \neq 0$ then $deg Q_{i}(P,t) = n_{i} - 1$; if $q_{n_{i}}(t) = q_{n-1}(t) = 0$ and $q_{n-2}(t) \neq 0$ then $deg Q_{i}(P,t) = n_{i} - 2$ etc. The requirement to know the signs of the non-zero coefficients of high orders of operators $R_{i}(P,t), Q_{i}(P,t)$ (assumption viii) is related to knowing the sign of a high-frequency gain of the system (1).

III. METHOD OF SOLUTION

Let us write the operators $Q_{i}(P,t), R_{i}(P,t)$ as

$$Q_{i}(P,t) = Q_{0}(P) + \Delta Q_{i}(P,t),$$

$$R_{i}(P,t) = R_{0}(P) + \Delta R_{i}(P,t),$$

where $Q_{0}(P)$ is an arbitrary linear differential operator, such that polynomial $Q_{0}(\lambda)$ is Hurwitz polynomial, $deg Q_{0} = \pi_{i}$. Then $\Delta Q_{i}(P,t)$ is the difference $Q_{i}(P,t) - Q_{0}(P)$ and $deg \Delta Q_{i} \leq \pi_{i}$, i.e. if $deg Q_{i} < deg Q_{0}$, then $deg \Delta Q_{i} = deg Q_{0}$, and if $deg Q_{i} = deg Q_{0}$, then $deg \Delta Q_{i} \leq \pi_{i} - 1$. $R_{0}(P)$ is an arbitrary linear differential operator $deg R_{0} = \pi_{i} - \gamma_{ui}$ such that polynomial $R_{0}(\lambda)$ is Hurwitz. Regarding structure $\Delta R(P,t)$ it’s possible to say that if $m_{i} < \pi_{i} - \gamma_{ui}$, then $deg \Delta R_{i} = \pi_{i} - \gamma_{ui}$, and if $m_{i} > \pi_{i} - \gamma_{ui}$, then $deg \Delta R_{i} = m_{i}$. Thus it is always possible to guarantee correctness of the mentioned decomposition of the operators $Q_{i}(P,t), R_{i}(P,t)$, as in case operators $\Delta Q_{i}(P,t)$ and $\Delta R_{i}(P,t)$ have all coefficients non-zero, in other case the correspondent number of components are nonzero. The decomposition [19], that allows to solve the problem, differs from known methods of parameterization equations of control plants.

Let us transform the equation of a system (1):

$$y_{i}(t) = \frac{k_{i}R_{0}(P)}{Q_{0}(P)}u_{i}(t) + \frac{\Delta R_{i}(P,t)}{k_{i}R_{0}(P)}u_{i}(t) - \frac{G_{i}(P,t)}{k_{i}R_{0}(P)}y_{i}(t - \tau_{i}) + \frac{1}{k_{i}R_{0}(P)}f_{i}(t) + \frac{\Delta Q_{i}(P,t)}{k_{i}R_{0}(P)}y_{i}(t) + \sum_{j=1}^{M} \frac{S_{ij}(P,t)}{k_{i}R_{0}(P)}y_{j}(t)$$  

(4)

since operators $Q_{0}(P) and R_{0}(P)$ are arbitrary, we can choose them in order that the following condition is obeyed

$$\frac{R_{0}(\lambda)}{Q_{0}(\lambda)} = \frac{1}{Q_{mi}(\lambda)}.$$  

(5)

Let us write the equation for error $e_{i}(t) = y_{i}(t) - y_{mi}(t)$, subtracting (2) from (4), and taking into consideration (5),

$$Q_{mi}(P)e_{i}(t) = k_{i}u_{i}(t) + \frac{\Delta R_{i}(P,t)}{R_{0}(P)}u_{i}(t) + \frac{1}{R_{0}(P)}f_{i}(t) - k_{mi}r_{i}(t) - \frac{\Delta Q_{i}(P,t)}{R_{0}(P)}y_{i}(t) - G_{i}(P,t)y_{i}(t - \tau_{i}) + \sum_{j=1}^{M} \frac{S_{ij}(P,t)}{R_{0}(P)}y_{j}(t).$$  

(6)

To obtain the main result, let’s use the approach [11], which allows to compensate disturbance. Let choose a local control law in the following form

$$u_{i}(t) = \alpha_{i}\vartheta_{i}(t).$$  

(7)
where $\alpha > 0$; \(\beta(t)\) is an additional control action. Then the following equation of error can be derived from (6)

\[
Q_{mi}(P)e_i(t) = \partial_i(t) + \phi_i(t),
\]

(8)

\[
\phi_i(t) = \frac{1}{R_{o}(P)} \left( \Delta R_i(P, t) u_i(t) \right) - \frac{1}{R_{o}(P)} \left( G_i(P, t) y_i(t - \tau_i) \right) - \frac{1}{R_{o}(P)} \left( \Delta Q_i(P, t) y_i(t) \right) + \frac{1}{R_{o}(P)} f_i(t) - k_m r_i(t) + (k, \alpha_i - 1) \phi_i(t).
\]

Signal \(\phi_i(t)\) contains all components action of which in the error needs to be compensated. It is necessary to extract the signal.

Let’s define the additional loop \(Q_{mi}(P)\bar{e}_i(t) = \partial_i(t)\)

(10)

and write the equation with the error signal \(\zeta_i(t) = e_i(t) - \bar{e}_i(t)\):

\[
Q_{mi}(P)\zeta_i(t) = \phi_i(t).
\]

If the derivatives \(\gamma_{ui}\) of the output signal \(y_j(t)\) can be measured then defining the variation law of the additional control action in the following form

\[
\partial_i(t) = -Q_{mi}(P)\zeta_i(t) = -\phi_i(t),
\]

we will get the following equation of the closed loop system using the error equation (8)

\[
Q_{mi}(P)e_i(t) = 0.
\]

(12)

Let’s show that all the signals in the closed loop system are bounded. It is necessary for the efficiency of the algorithm which will be described later. Equation (12) shows that the signal \(y_j(t)\) and its derivatives \(\gamma_{ui}\) are bounded due to assumption x). Then from conditions of the assumptions deg \(\Delta Q_i(P) = \bar{n}_i\) and because \(R_{o}(\lambda)\) is Hurwitz polynomial of \(\bar{n}_i - \gamma_{ui}\) degree we can conclude that

\[
\phi_i(t) = \frac{1}{R_{o}(P)} f_i(t) - k_m r_i(t) - \frac{1}{R_{o}(P)} \left( \Delta Q_i(P, t) y_i(t) \right) - \frac{1}{R_{o}(P)} \left( G_i(P, t) y_i(t - \tau_i) - \sum_{j=1}^{M} S_j(P, y_i(t)) \right)
\]

is a bounded value. It is necessary to show that the chosen control action is bounded. For that purpose let’s substitute \(\phi_i(t)\) in (11) with the statement above and resolve derived equation for \(\partial_i(t)\):

\[
\partial_i(t) = -\frac{1}{k, \alpha_i} \left( \phi_i(t) + \frac{1}{R_{o}(P)} \left( \Delta R_i(P, t) u_i(t) \right) \right).
\]

(13)

Let us substitute \(\partial_i(t)\) in equation (9) and resolve it for \(u_i(t)\), taking into consideration following parameterization \(k_j(t)R_j(P) = k_{j0}R_j(P) + \Delta R_j(P, t)\):

\[
k_j(t)R_j(P) u_i(t) = -\phi_i(t).
\]

From condition of assumption ii) and boundedness of \(\phi_i(t)\) boundedness of local control action \(u_i(t)\) is followed.

Because we cannot measure the derivatives, let’s formulate the local law of additional control action \(\partial_i(t)\) in the following form

\[
\partial_i(t) = -g_{mi}(P)\zeta(t),
\]

(14)

where \(g_{mi}(P) = [\gamma_{m1}, \ldots, \gamma_{m1}, 1] \) vector composed with polynomial coefficients

\[
Q_{mi}(\lambda) = \lambda^m + \gamma_{m1} \lambda^{m-1} + \ldots + \gamma_{m1} \lambda,
\]

\(\zeta(t) = \zeta_i(t) = e_i(t) - \bar{e}_i(t)\) is estimation of derivatives \(P^k \zeta_i(t)\), obtained from filters

\[
\zeta_{ik}(t) = \frac{1}{\mu} F_i z_{ik}(t) + \frac{1}{\mu} b_k P^k \zeta_i(t), \quad \zeta_{ik}(t) = L_{0i} z_{ik}(t), \quad i = 1, M, \quad k = 1, y_{ui}.
\]

(15)

Where \(z_{ik} \in R^w); \ L_{0i} = [1, 0, \ldots, 0]; \ b_k = [0, \ldots, 0, 1]\)

\(\mu > 0\) is small number.

If we use (14) and (15) in Laplace transformation we’ll get the following

\[
\partial_i(t) = -\frac{Q_{mi}(\lambda)}{(\mu \lambda + 1)^w} \zeta_{ik}(\lambda).
\]

Taking into consideration (10) and statement for error signal \(\zeta_{ik}(t) = e_i(t) - \bar{e}_i(t)\) we have

\[
\partial_i(\lambda) = -\frac{Q_{mi}(\lambda)}{(\mu \lambda + 1)^w} e_i(\lambda).
\]

Substituting \(\partial_i(t)\) in equation (7) with the obtained statement and using the original of Laplace transformation we’ll get control algorithm. Obviously that control law now is technically feasible since it contains only known or measurable variables.

**Proposition.** If assumptions i) - xii) are obeyed then there are numbers \(\mu_0 > 0, \ T_0 > 0\) such that under conditions \(\mu \leq \mu_0, \ T \geq T_0\) control algorithm

\[
\left( (\mu P + 1)^w - 1 \right) v_i(t) = -\alpha Q_{mi}(P) e_i(t)
\]

(16)

guarantees that target condition (3) is obeyed, where \(\alpha > 0\).

It is necessary to note that the described algorithm remains invariant if there is state delay in a system as well as in the case when a system is in a steady state with unknown parameters with known boundaries.

**Proposition proof.** Let’s consider vectors of the estimation error of derivatives \(P^k \zeta_i(t)\)

\[
\eta_{ik}(t) = z_{ik}(t) + F_i^{-1} b_k P^k \zeta_i(t), \quad k = 1, y_{ui}, \quad i = 1, M.
\]

Here the vector \(F_i^{-1} b_k\) has first component equal to -1. If to prove that the value \(\eta_{ik}(t)\) is small, then from condition

\[
\left| \zeta_{ik}(t) - P^k \zeta_i(t) \right| < \eta_{ik}(t)
\]

it follows that estimation \(\bar{\zeta}_{ik}(t)\) is
near to \( P^k \zeta(t) \). From (15) we’ll get the equation of dynamic for vectors \( \eta_{ik}(t) \):

\[
\dot{\eta}_{ik}(t) = \frac{1}{\mu} F_i \eta_{ik}(t) + h_i P^k \zeta(t) + F_i^{-1} h_i P^{k+1} \zeta(t) = \\
= \frac{1}{\mu} F_i \eta_{ik}(t) + h_i P^{k+1} \zeta(t),
\]

\( \Delta_{ik}(t) = L_i \eta_{ik}(t), \quad i = 1, M, \quad k = 1, \gamma_m. \)

Taking into account that the additional control action is formulated as (14), we can transform the equation of error into the following form

\[
Q_{mi}(P) e_i(t) = -q^T_{mi} \Delta_{i}(t),
\]

where \( q^T_{mi} = \left[ q_{mi,1}, \ldots, q_{mi,1} \right] \); \( \Delta_{i}(t) = \text{col}(\Delta_{i1}(t), \Delta_{i2}(t), \ldots, \Delta_{i\gamma_m}(t)) \);

\( \Delta_{ik}(t) = \tilde{\zeta}_{ik}(t) - P^k \zeta(t) \). Let’s transform equation (17) into vector-matrix form. As a result we’ll get the following equations set of closed loop system:

\[
\begin{align*}
\dot{e}_i(t) &= A_{mi} e_i(t) + b_q q^T_{mi} \Delta_{i}(t), \quad e_i(t) = L_i e_i(t), \\
\mu_i \dot{\eta}_{i}(t) &= F_i \eta_{i}(t) + \mu_i h_i P^{k+1} \zeta(t), \\
\Delta_{i}(t) &= L_i \eta_{i}(t), \quad i = 1, M, \quad k = 1, \gamma_m,
\end{align*}
\]

where \( \mu_i = \mu_1 = \mu \). We’ve got singularly perturbed system as \( \mu \rightarrow 0 \). Let us use Lemma [16].

**Lemma** [16]. If a system is defined by the equation \( \dot{x} = f(x, \mu_1, \mu_2) \), \( x \in \mathbb{R}^m \), where \( f(t) \) is a continuous function that is Lipschitz function with respect to \( x \) and in the case when \( \mu_2 = 0 \) it has a bounded closed region of dissipation \( \Omega = \{x : F(x) < \tilde{C} \} \), where \( F(x) \) – positive defined continuous piecewise smooth function, then there is \( \mu_0 > 0 \) such that under \( \mu_2 \leq \mu_0 \) the initial system has the same dissipative region \( \Omega_1 \), if for some numbers \( \tilde{C}_1 \) and \( \mu_1 \) for \( \mu_2 = 0 \) following condition is obeyed

\[
\text{sup}_{x \in \Omega} \left( \frac{\partial F(x)}{\partial x} \right)^T f(x, \mu_1, 0) \leq -\tilde{C}_1, \quad \text{if} \quad F(x) = \tilde{C}. \tag{19}
\]

In the case of \( \mu_2 = 0 \) in (18) we have asymptotically stable system for variables \( e_i(t) \) and \( \eta_{ik}(t) \), since \( A_{mi}, F_i \) are Hurwitz matrices. It is the same situation which we had for measuring the derivatives i.e. \( \lim_{t \rightarrow \infty} e_i(t) = 0 \). It was proved that if this condition is obeyed all the signals in the system are bounded. It means that there is a certain region

\[
\Omega = \{e_i(t), \eta_{ik}(t), \zeta(t) : \\
\begin{align*}
&\left| P^{k+1} \zeta(t) \right| \leq \delta_{3k}, \quad \left| \dot{\zeta}(t) \right| \leq \delta_2, \\
&\left| \eta_{ik}(t) \right| \leq \delta_{3k}, \quad F(e_i, \eta_{ik}) < C_1, \quad k = 1, \gamma_m,
\end{align*}
\]

where signals \( e_i(t), \eta_{ik}(t), \zeta(t) \) are within their boundaries for some initial conditions from \( \Omega_0 \).

Let us consider two vectors

\[
\begin{align*}
\theta^T_i(t) &= \left[ \zeta_i(t), \ldots, \zeta_i(t) \right], \\
\eta^T_i(t) &= \left[ \eta_{i1}(t), \eta_{i2}(t), \ldots, \eta_{i\gamma_i}(t) \right],
\end{align*}
\]

and block-diagonal matrices with \( \gamma_m \) diagonal blocks

\[
F_{ii} = \text{diag}\{F_1, F_2, \ldots, F_i\}, \quad B_i = \text{diag}\{h_1, h_2, \ldots, h_i\},
\]

\( C_i = \text{diag}\{L_1, L_2, \ldots, L_i\} \),

then equations (18) will take the following form

\[
\begin{align*}
\dot{e}_i(t) &= A_{mi} e_i(t) + b_q q^T_{mi} \Delta_{i}(t), \quad e_i(t) = L_i e_i(t), \\
\mu_i \dot{\eta}_{i}(t) &= F_i \eta_{i}(t) + \mu_i B_i \theta_i(t), \\
\Delta_{i}(t) &= C_i \eta_{i}(t), \quad i = 1, M.
\end{align*}
\]

Evidently that condition (19) was obeyed if to take Lyapunov function for \( F_i \)

\[
V(e_i(t), \eta_{i}(t)) = \sum_{i=1}^{M} \left( e_i^T(t) H_{ii} e_i(t) + \eta_{i}^T(t) H_{ii} \eta_{i}(t) \right),
\]

where the positive defined symmetric matrices \( H_{ii}, H_{ii} \) are determined from equations solution

\[
H_{ii} A_{mi} + A_{mi}^T H_{ii} = -\rho_1 \eta_{ii} - Q_{ii}, \quad H_{ii} F_i + F_i^T H_{ii} = -\rho_2 \eta_{ii} - Q_{ii},
\]

where \( \rho_1 > 0, \rho_2 > 0, \quad Q_1 = Q_{ii}^T > 0, \quad Q_2 = Q_{ii}^T > 0 \). Thus in accordance with Lemma [16], there is \( \mu_0 > 0 \) such that if \( \mu < \mu_0 \) then \( \Omega \) remains dissipative region of system (18).

However it is necessary to note that keeping the dissipative region doesn’t guarantee that the set of attraction \( \Omega_1 \) remains the same in a singularly perturbed system.

Let us calculate the full derivative of function (21) on system’s trajectories (20), taking into account equation (22) and assigning \( \mu_1 = \mu_2 = \mu \):

\[
\begin{align*}
\dot{V}(e_i(t), \eta_{i}(t)) &= \sum_{i=1}^{M} \left( -\rho_1 \left| e_i(t) \right|^2 + \rho_2 \left| \eta_{i}(t) \right|^2 \\
+ 2 \epsilon_i^T(t) H_{ii} b_q q^T_{mi} \Delta_{i}(t) - \frac{\rho_2}{\mu} \left| \eta_{i}(t) \right|^2 - \frac{1}{\mu_0} \left| \eta_{i}(t) \right|^2 \right)
\end{align*}
\]

Let us use estimations

\[
\begin{align*}
2 \epsilon_i^T(t) H_{ii} b_q q^T_{mi} \Delta_{i}(t) &\leq \left| \epsilon_i(t) \right|^2 + \rho_2 \left| \eta_{i}(t) \right|^2, \\
2 \eta_{i}^T(t) H_{ii} B_i \theta_i(t) &\leq \frac{1}{\mu_0} \left| \eta_{i}(t) \right|^2 + \mu_0 \rho_4, \\
- \epsilon_i^T(t) Q_2 \epsilon_{ii}(t) &\leq -\lambda_{\min}(Q_2) \left| \epsilon_i(t) \right|^2 \\
- \eta_{i}^T(t) Q_2 \eta_{ii}(t) &\leq -\lambda_{\min}(Q_2) \left| \eta_{i}(t) \right|^2
\end{align*}
\]

also

\[
\begin{align*}
\epsilon_i^T(t) \left( H_{ii} \right) \epsilon_{ii}(t) &\leq \lambda_{\max}(H_{ii}) \left| \epsilon_i(t) \right|^2, \\
- \eta_{i}^T(t) \left( H_{ii} \right) \eta_{ii}(t) &\leq -\lambda_{\max}(H_{ii}) \left| \eta_{i}(t) \right|^2
\end{align*}
\]
are the minimal and maximal characteristic numbers of the mentioned matrixes. Using those estimations into (23) we’ll get

\[ \dot{V}(e_i(t), \eta_i(t)) \leq -\sigma_0 V + \sum_{i=1}^{M} \left( \left( \rho_{2i} - 1 \right) \| e_i(t) \|^2 - \frac{\rho_{2i}}{\mu_0} \| e_i(t) \|^2 + \mu_0 \rho_{4i} \right), \]

where \( \sigma_0 = \min \left\{ \frac{\lambda_{\min}(Q_{ii})}{\lambda_{\max}(H_{ii})}, \frac{\lambda_{\min}(Q_{2i})}{\lambda_{\max}(H_{2i})} \right\} \). If to choose \( \rho_{ii}, \rho_{2i} \) from conditions

\[ \rho_{ii} - 1 > 0, \quad \rho_{2i} - 1 \frac{\mu_0}{\mu_0} > 0, \quad (24) \]

the following inequality is correct:

\[ \dot{V}(e_i(t), \eta_i(t)) \leq -\sigma_0 V + \sum_{i=1}^{M} \frac{\mu_0 \rho_{4i}}{\sigma_0}. \]

If we solve the inequality

\[ V(e_i(t), \eta_i(t)) \leq V(0)e^{-\sigma_0 t} + \sum_{i=1}^{M} \frac{\mu_0 \rho_{4i}}{\sigma_0} \]

we can see that if to choose \( \mu_0 \) small enough we get the following region of attraction:

\[ \Omega_2 = \{ e_i(t), \eta_i(t) : \dot{V}(e_i(t), \eta_i(t)) \leq \sum_{i=1}^{M} \frac{\mu_0 \rho_{4i}}{\sigma_0} \} \]

Inserting the required value \( T_0 \) from the target condition (3) into the right part and taking into consideration the inequalities

\[ \| e_i(t) \|^2 \leq \| e_i(t) \|^2 \leq \frac{V(0)e^{-\sigma_0 t} + \sum_{i=1}^{M} \frac{\mu_0 \rho_{4i}}{\sigma_0}}{\lambda_{\min}(H_{ii})}, \]

we get the estimation of the value \( \delta \) in the target condition (3)

\[ \delta \leq \frac{1}{\lambda_{\min}(H_{ii})} \left( \frac{V(0)e^{-\sigma_0 t} + \sum_{i=1}^{M} \frac{\mu_0 \rho_{4i}}{\sigma_0}}{\lambda_{\min}(H_{ii})} \right), \]

that shows that there are numbers \( \mu_0 \) and \( T_0 \) guaranteeing that target condition will be obeyed. Thus for \( \mu \leq \mu_0 \) varying \( \rho_{ii} \) in (24) and \( \mu \), we can get the required value \( \delta \) in the target condition (3).

Structural scheme of the designed control system is shown in Figure 1.

The drawback of the proposed algorithm is a lack of analytically proved choice of parameters \( \mu \) and \( \alpha_i \). However they can be easily matched during the modeling phase. For a system (1) minimally possible coefficients of operators \( k_i(t)R_i(P,t), Q_i(P,t), G_i(P,t), S_i(P,t) \) are used and maximally possible values of \( f_i(t), r_i(t) \) are used for the input. Constant components don’t matter. Numbers \( \mu \) and \( \alpha_i \) are selected in order to guarantee a given dynamic error. Number \( \mu \) is usually varying within 0.005 to 0.05. Error will not exceed a given value for other parameters values and values of external actions from given class of uncertainty.

**IV. EXAMPLE**

Let us consider a dynamic system of sixth order represented as two subsystems

\[
\begin{align*}
\dot{x}_1(t) &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 5 + 6 \cos 3t & 5 + 5 \cos t & 5 + \sin 2t \end{bmatrix} x_1(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} y_1(t-2) + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} u_1(t), \\
\dot{x}_2(t) &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 + 3 \cos 2t & 4 + 4 \cos t & 4 + 2 \sin t \end{bmatrix} x_2(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} y_2(t-3) + \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} u_2(t),
\end{align*}
\]

where \( x_1 \) and \( x_2 \) are the state vectors of the subsystems, \( y_1 \) and \( y_2 \) are the measurable scalar outputs of the subsystems, \( u_1 \) and \( u_2 \) are the scalar control actions whose law of variation is generated according to Eq. (16), and

\[ f_1(t) = \sin 2t, \quad f_2(t) = 2 \sin 3t \]
are the disturbances. The parameters of the local reference models (2) are taken as $Q_{mi}(P) = (P+1)^2$ and $k_{mi} = 1$, $i=1,2$, the reference signals $r_1$ and $r_2$ are as follows:

$$r_1(t) = 1 + 2 \sin t, \quad r_2(t) = 1 + 2 \sin 0.5t.$$ 

We represent the considered plant using (1), where

$$k_i(t)R_i(P,t) = k_i + \Delta k_i(t),$$

$$Q_i(P,t) = P^3 + (q_{1i} + \Delta q_{1i}(t))P^2 + (q_{2i} + \Delta q_{2i}(t))P + q_{3i} + \Delta q_{3i}(t)$$

The class of uncertainty is defined by the inequalities

$$1 \leq \Delta k_i \leq 10, \quad |\Delta q_{1i}| \leq 10, \quad |\Delta q_{2i}| \leq 10, \quad l = 1, 2, 3.$$

The controller (16) consists of two cascaded blocks with the following transfer functions

$$W_1(\lambda) = \frac{1}{\mu} + \frac{1}{\mu \lambda}, \quad W_2(\lambda) = \frac{(\lambda + 1)^2}{\mu^2 \lambda^2 + 3 \mu \lambda + 3},$$

and amplifier with gain of $\alpha_i$.

Given that $\delta = 0.05$ in the target condition (3), the values $\mu = 0.01, \quad \alpha_1 = 1.5, \quad \alpha_2 = 2$ allow to achieve the required accuracy.

![Fig.2. Error trajectories $e(t) = y(t) - y_{ad}(t)$](image)

Computer-aided modeling demonstrated good operability of the designed systems.

V. CONCLUSION

Paper considers the problem of decentralized control with an nominal model for interconnected systems with unknown parameters and an unknown order when derivatives of input and output signals of the local subsystems cannot be measured. Considered robust control system allows compensating parametric and external disturbances with given accuracy $\delta$ for the period of time $T$. Values $\delta$ and $T$ can be small enough using the appropriate parameters of the closed loop system. It is necessary to note that the closed loop system is functioning as an implicitly defined nominal model and parameters of the model are used in control algorithm.

It is important to note that considered algorithm remains the same if there is state delay in a system as well as in the case when a plant is stationary with unknown parameters which values are limited by a certain bounded set. Besides, the advantage of the suggested algorithm consists in the fact that the structure of a local controller is coincided with the structure of a local controller of single-connected system. This gives an advantage for the control of spatially distributed systems. The drawback of the algorithm is a lack of an analytically proved method of selection of the parameters of the controller.

REFERENCES