Modified high-order adaptive algorithm for decentralized control of multi-agent structurally uncertain plants under disturbance

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Abstract – The subject of this paper is solving the adaptive dynamic controller design problem for multi-agent systems with structural and parametric uncertainty for the case when derivatives of input and output parameters cannot be measured. The operability of the designed control systems subject to non-measurable bounded disturbances is presented. Only the measurable variables of the local subsystems are used to generate the control actions, therefore control is completely decentralized.

I. INTRODUCTION

The problem of adaptive control with scalar input and output has become one of the classical problems of modern control theory [1-3]. Plenty of methods for adaptive control design have been developed including high-order adaptive algorithms, the augmented error method, shunting method and others. The original motivation of the research in this field can be explained by the fact that output control is a way to reduce spending on design and development of the sensors, which in turn increase the order of the models and introduce additional measurement errors. Let’s also keep in mind that for many real engineering systems, suitable practical solutions to the problem of measurements of the state variables of controlled systems have not yet been found. Most of the existing adaptive control methods use high order regulators and their order may be a few times bigger than the order of the systems [3].

The high order adaptive methods [4] used the same additional filters as the filters in the method of the augmented error. All those methods generate both the vectors of the control parameters and their derivatives. There are three major drawbacks of the high order adaptive methods [4]. First, these methods use “pure integral” tuning algorithms which are not robust to the external perturbations. Second, the algorithms use non-steady-state filters i.e. these algorithms become the algorithms with a dynamic high order which is not desirable from the implementation point of view. Third, to implement a high order adaptive algorithm we have to know the upper estimate of the gain coefficient of the original system and this restricts applicability of the methods in comparison with the traditional first-order algorithms.

To eliminate the limitations and drawbacks of these methods, the robust adaptive high-order algorithms with more simple structure should be designed. This problem have been solved in [5, 6] where the estimations of the error derivatives and different estimations filters are used. More details on the existing methods can be found in [3].

A modified high order algorithm is proposed in [7]. The algorithm uses the estimates of the derivatives of the control action which allows to reduce the order of the closed system by excluding the additional filters. Applications of the method to the interconnected systems are considered in [8, 9].

Complex interconnected systems which are characterized by a large number of control and controlled variables play significant role in the modern control theory [10-15]. These systems are considered as a special class of systems because of the more complex analysis in comparison with the systems with one control channel. To overcome the difficulties of designing such systems, the systems should be decentralized [16]. Decentralized algorithms are more “natural” for the large interconnected systems [17] because such plants imply spatially distributed components. Decentralized approach allows to produce more reliable and quality control systems due to deep and natural integration between the control system and the plant.

It is important to note that almost all the suggested methods are based on an assumption that the structure of a plant is known i.e. the order of a system of differential equations is known and parametric and external disturbances are known. There are a few studies related to the problems of control with an unknown order [18-20]. Sources [18, 19] consider control problems of linear, stationary systems with an unknown and constant order of numerator and denominator for their transfer functions. Source [20] considers a wider class of systems with disturbances that are able to influence both the parameters of the system and its order. Applications of the method to the interconnected systems are considered in [21, 22].

This paper considers the problem of adaptive control of the multivariable plants under the external and parametric uncontrolled disturbances. The order of the systems is unknown and scalar input and output signals can only be measured. A modified high-order control algorithm is proposed to solve the problem. Only the measurable variables of the local subsystems are used for the control, therefore control is completely decentralized.

II. PROBLEM STATEMENT

Let us consider an interconnected system with local subsystems’ dynamic processes described by the following equations:  

\[
Q_j(P)y_j(t) = k_j R_j(P) u_i(t) + G_{ij}(P)f_j(t) + \sum_{j=1}^{i-1} S_{ij}(P)y_j(t), \quad i \neq j, \quad i = 1, k, 
\]

(1)

where \( P = \frac{d}{dt} \) is the differential operator; \( Q_j(P), R_j(P), S_{ij}(P), G_{ij}(P) \) are the linear differential operators with unknown constants parameters; \( u_i(t) \) is a scalar control action; \( y_j(t) \) is a scalar controlled variable in
the \(i\)-subsystem which can be measured; \(f_i(t)\) is an uncontrolled disturbance.

Decentralized adaptive control for such a system is defined as a problem of finding \(k\) local adaptive control blocks, each of which can access only current information about their subsystems [23]. The required quality of transition processes in a subsystem is defined by the equations of the local reference models

\[
Q_{mi}(P)y_{mi}(t) = r_i(t), \quad i = 1, k.
\]  

Here \(Q_{mi}(P)\) are the linear differential operators; \(k_{mi} > 0\); \(r_i(t)\) are the scalar bounded reference signal.

It is necessary to design a control system such that

\[
\lim_{t \to 0^+} \left| y_i(t) - y_{mi}(t) \right| < \delta,
\]

where \(\delta\) is a positive and small enough value. According to \(\lambda\) and \(\gamma\) the \(i\)-subsystem which can be measured; \(y_{mi}(t)\) is an uncontrolled disturbance.

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\]

where \(\delta\) is a positive and small enough value. According to the statement of the problem, we cannot use the variables of one subsystem to control other subsystems.

**Assumptions:**

i) \(R_i(\lambda), Q_i(\lambda), R_{mi}(\lambda)\) are Hurwitz polynomials (\(\lambda\) is a complex variable in Laplace transformation);

ii) the orders of \(\deg Q_i = n_i; \deg R_i = m_i; \deg S_{ij} = n_{ij}\); \(n_{ij} < n_i - 1\) are unknown and relative degree of the local system \(\gamma_i = n_i - m_i > 1\);

iii) the upper bound \(\gamma_{ui} \geq \gamma_i\) of the relative order \(\gamma_i\) as well the upper bound of the operator \(Q_i\) are defined, i.e. \(n_i \leq \overline{n}_i\);

iv) the orders of the polynomials \(Q_{mi}\) are equal to \(\gamma_{ui}\);

v) we know the coefficients’ sings \(k_i\) and assume that \(k_i > 0\);

vi) the coefficients of the operators \(R_i(\lambda), Q_i(\lambda)\) depend on the vector of unknown parameters \(\xi \in \Xi\), where \(\Xi\) is a bounded set;

vii) reference signal \(r_i(t)\) and disturbance \(f_i(t)\) are bounded functions;

viii) we cannot use the derivatives \(y_i(t), u_i(t), r_i(t)\) of the signals.

**III. METHOD OF SOLUTION**

Let us first write the \(Q_i(P), R_i(P)\) in the following form:

\[
Q_i(P) = Q_{0i}(P) + \Delta Q_i(P), \quad R_i(P) = R_{0i}(P) + \Delta R_i(P),
\]

where \(Q_{0i}(P)\) is an arbitrary linear differential operator, such that the polynomial \(Q_{0i}(\lambda)\) is Hurwitz polynomial, \(\deg Q_{0i} = \overline{n}_i\). Then the operator \(\Delta Q_i(P)\) is the difference \(Q_i(P) - Q_{0i}(P)\) and \(\deg \Delta Q_i \leq \overline{n}_i\), i.e. if \(\deg Q_i < \deg Q_{0i}\) then \(\deg \Delta Q_i = \deg Q_{0i}\), and if \(\deg Q_i = \deg Q_{0i}\) then \(\deg \Delta Q_i = \overline{n}_i - 1\). Introduce the arbitrary linear differential operator \(R_{0i}(P), \deg R_{0i} = \overline{n}_i - \gamma_{ui}\) such that the polynomial \(R_{0i}(\lambda)\) is Hurwitz polynomial. The structure of \(\Delta R_i(P)\) is such that if \(m_i < \overline{n}_i - \gamma_{ui}\) then \(\deg \Delta R_i = \overline{n}_i - \gamma_{ui}\), and if \(m_i > \overline{n}_i - \gamma_{ui}\) then \(\deg \Delta R_i = m_i\). This means that the above decomposition of the operators \(Q_i(P), R_i(P)\) is correct because \(\Delta Q_i(P)\) and \(\Delta R_i(P)\) either have non-zero coefficients or an appropriate amount of their components are equal to zero. The decomposition [20] is different from the known methods of parameterization of the equations.

We can transform the equation of the system (1) as follows

\[
y_i(t) = \frac{k_i}{Q_{0i}(P)} \left( u_i(t) - \frac{\Delta Q_i(P)}{k_iM_i(P)} y_i(t) + \frac{\Delta R_i(P)}{R_{0i}(P)} u_i(t) + \frac{G_{ii}(P)}{k_iR_{0i}(P)} f_i(t) + \sum_{j=1,i \neq j}^{k} \frac{S_{ij}(P)}{k_iR_{0i}(P)} y_j(t) \right),
\]

since \(Q_{0i}(P)\) and \(R_{0i}(P)\) are arbitrary operators, we can choose them so that the following condition is obeyed:

\[
R_{0i}(\lambda) = \frac{1}{Q_{0i}(\lambda)},
\]

We can rewrite the last equation as:

\[
y_i(t) = \frac{k_i}{Q_{0i}(P)} \left( u_i(t) - \frac{\Delta Q_i(P)}{k_iM_i(P)} y_i(t) + \frac{\Delta R_i(P)}{R_{0i}(P)} u_i(t) + \frac{G_{ii}(P)}{k_iR_{0i}(P)} f_i(t) + \sum_{j=1,i \neq j}^{k} \frac{S_{ij}(P)}{k_iM_i(P)} y_j(t) \right),
\]

where \(M_i(P) = R_{0i}(P)T_i(P): T_i(P)\) are linear differential operators with constant coefficients such that the polynomials \(T_i(\lambda)\) are Hurwitz polynomials, \(\deg T_i = \gamma_{ui} - 1\). We have to choose the polynomials \(T_i(\lambda)\) so that

\[
\frac{k_{mi} T_i(\lambda)}{Q_{0i}(\lambda)} = \frac{k_{mi}}{\lambda + a_{mi}}, \text{ where } a_{mi} > 0.
\]

To obtain the main result of the paper, we will use the well-known approach [3, 7] when the derivatives of the input and output signals are measurable. We will choose the control law in the form

\[
u_i(t) = T_i(P)g_i(t).
\]

Here \(g_i(t)\) is an additional control action. Let us write the equation for the error \(e_i(t) = y_i(t) - y_{mi}(t)\) by subtracting (2) from (4) and taking into account (5),

\[
e_i(t) = \frac{k_i}{P + a_{mi}} \left( g_i(t) - \frac{\Delta Q_i(P)}{k_iM_i(P)} y_i(t) + \frac{\Delta R_i(P)}{R_{0i}(P)} u_i(t) - \frac{k_{mi} T_i(P)}{k_iR_{0i}(P)} r_i(t) + \frac{G_{ii}(P)}{k_iR_{0i}(P)} f_i(t) + \sum_{j=1,i \neq j}^{k} \frac{S_{ij}(P)}{k_iM_i(P)} \left( e_j(t) + y_{mj}(t) \right) \right).
\]

Then we will define the filters for the each subsystem:
\[
\dot{y}_i = F_1 y_i + b_0 y_i, \quad \eta_{1i} = d_{0i} y_i + d_{1i}^T y_i,
\]
\[
\dot{v}_u = F_2 v_u + b_0 \varphi_i, \quad \eta_{2i} = d_{2i}^T v_u,
\]
\[
\dot{v}_r = F_3 v_r + b_0 \varphi_i, \quad \eta_{3i} = L_{rT} v_r,
\]
where \(v_{yi} \in \mathbb{R}^{T-1}, v_{ui} \in \mathbb{R}^{T-\gamma_u}, v_{ri} \in \mathbb{R}^{\gamma_r-1}, F_1, F_2, F_3 \) are Hurwitz matrices corresponding to the polynomials \(M_i(\lambda), R_i(\lambda), T_i(\lambda)\) and defined in Frobenius form; \(L_{ri} = [1,0,...,0]; \) all \(b_0^T = [0,...,0,1]\) have an appropriate order in each equation.

Introduce the regression vector \(\omega_{ik} = \text{col}(y_{ji}, v_{yi}, v_{ui}, \eta_{ri})\) and unknown vector of parameters that depends on the polynomial coefficients \(\Delta Q_i(\lambda), R_i(\lambda), M_i(\lambda)\) and parameter \(k_i:\)

\[
C_{0i} = \text{col} \left( d_{0i}, d_{1i}, d_{2i}, \frac{k_{mi}}{k_i} \right),
\]
then according to (6) we receive the error equation:

\[
e_i(t) = \frac{k_i}{P + d_{mi}} \left( \beta_i(t) - C_{0i}^T \omega_{i}(t) + \varphi_{i}(t) + \sum_{j=1, j \neq i}^{k} \frac{S_j(P)}{k_j M_j(P)} \psi_{j}(t) \right) \quad i = 1,k
\]

where \(\varphi_{i}(t) = G_i(P) f_i(t) + \sum_{j=1, j \neq i}^{k} \frac{S_j(P)}{k_j M_j(P)} \psi_{j}(t)\) are the bounded functions because \(M_i(\lambda)\) are Hurwitz polynomials and \(\eta_{ri}(t)\) complies with the assumption vii).

Consider now the expression under the sum sign in (9).

Introducing the variables \(\eta_{sij} = \frac{S_j(P)}{k_j M_j(P)} e_j, s_j\) and the vectors \(\eta_{si} = \text{col}(\eta_{s1i},...,\eta_{sk})\), \(s_i = \text{col}(s_{1i},...,s_{ki})\) we get the following equations

\[
\dot{s}_j = F_i s_j + B_i e_j, \quad \eta_{si} = L_{sij} s_j,
\]

where \(s_j \in \mathbb{R}^m; \eta_{sij} \in R; F_i, B_i, L_{sij} \) are the matrices of the minimal realization in the state space of transfer function \(S_j(\lambda); L_{sij} = [1,0,...,0]\). If we introduce the vector \(e = \text{col}(e_1,...,e_k)\) and the matrices \(F_i = \text{diag}[F_{1i},...,F_{ki}], L_{sij} = \text{diag}[L_{s11},...,L_{sk}], B_{s1} = \text{diag}[0, B_{s12},...,B_{s1k}], B_{s2} = \text{diag}[B_{s21},B_{s22},...,B_{s2j-1},0,B_{s2j+1},...,B_{s2k}]\), then we get the following equations

\[
\dot{s}_i = F_i s_i + B_i e_i, \quad \eta_{si} = L_{sij} s_j.
\]

Let us define the control law of the additional control action as

\[
\dot{\vartheta}_{i}(t) = C_{0i}^T \vartheta_{i}(t) + \eta_{sij} = \frac{\partial C_{0i}}{\partial t} \vartheta_{i}(t),
\]

\[
\frac{dC_i}{dt} = -\rho_i \vartheta_{i}(t)e_i - \alpha_i e_i^2(t) C_{i}(t),
\]

where \(C_i\) is the vector of configurable parameters that has the structure similar to \(C_{0i}\): \(\rho_i > 0, \alpha_i > 0\). Using the error equation (9), we can write the equation for the closed system:

\[
\dot{e}_i = -a_{mi} e_i(t) + k_i (C_i(t) - C_{0i})^T \vartheta_{i}(t) + k_i E_i \eta_{sij}(t) + k_i \varphi_{i}(t), \quad i = 1,k
\]

\[
\dot{s}_i = F_i s_i(t) + B_i e_i(t), \quad \eta_{sij}(t) = L_{sij} s_j(t),
\]

where \(E_i\) is the matrix row with the order \((1\times k)\) and whose elements are equal to 1.

Let us demonstrate that the systems (7), (11), (12) are dissipative and there is a number \(\rho_{0i}\) such that if \(\rho_i \geq \rho_{0i}\) then the condition \(\lim_{t \to \infty} |e_i(t)| < \delta\) is true. We will use Lyapunov function in the form:

\[
V_i = \sum_{i=1}^{k} \left( h_i e_i^2(t) + s_i^T P_i s_i(t) \right) + \frac{k_i}{\rho_{0i}} (C_i(t) - C_{0i})^T (C_i(t) - C_{0i}),
\]

where \(h_i > 0, \rho_i = h_i \rho_{0i} \); \(P_i \) is a positive defined symmetric matrix \(P_i = \text{diag}[P_{s1},...,P_{sk}]\). Using the equations (12) and (11) we obtain the full derivative of Lyapunov function

\[
\dot{V}_i = \sum_{i=1}^{k} \left(-2a_{mi} h_i e_i^2 + 2h_i e_i (\varphi_i + E_i \eta_{sij}) + 2s_i^T P_i B_i e_i + \right.
\]

\[
+ s_i^T \left( P_i F_i + F_i^T P_i \right) k_i \left( (C_i - C_{0i})^T e_i^2 C_i \right).
\]

Taking into account the fact that the matrices \(P_i\) and \(F_i\) are also block diagonal matrices we can conclude that for each subsystem positive the defined matrices \(P_i\) will satisfy the following conditions

\(P_i F_i + F_i^T P_i = -2Q_{si} - \rho_{si} I_{n_i-1}\),

where \(Q_{si}\) are arbitrary positive-defined symmetric matrices; \(\rho_{si} > 0; I_{n_i-1}\) are unitary matrices of the \((n_i - 1)\times(n_i - 1)\) order. Keeping in mind that the matrices \(F_i\) are Hurwitz matrices, we can conclude that such matrices do exist [1].

Taking into consideration the block diagonal structure of matrices we can use the following estimations:

\[-2(C_i - C_{0i})^T C_i \leq -\eta_{min}(Q_{si}) \|v_i\|^2 \leq -\eta_{max}(Q_{si}) \|v_i\|^2,\]

\[-2e_i^T h_i \eta_{sij} \leq \eta_{sij}^T Q_{sj} \eta_{sij} \leq 2h_i e_i \|\eta_{sij}\|,\]

\[-s_i^T P_i s_i \leq -\lambda_{min}(P_i) \|s_i\|^2,\]

\[-s_i^T P_i s_i \leq -\lambda_{min}(P_i) \|s_i\|^2,\]

\[
s_i Q_{sj} s_i \leq \|s_i^T P_i Q_{sj} \|^2 = \|s_i^T P_i B_{sij} \|^2,\]

\[
2s_i^T P_i B_{sij} e_j \leq 2 \|s_i^T P_i B_{sij} \| \|e_j\|.
\]

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where \( \lambda_{\text{min}}, \lambda_{\text{max}} \) are the minimal and maximal characteristic numbers of the defined matrices, 
\[
\chi_{ij} = \frac{1}{B_{ij}^{\text{eq}} P_{C} Q_{mi} P_{C} R_{ij}}.
\]
From (13) we will get:
\[
V_1 \leq -\sigma_1 V_1 + \sum_{i=1}^{k} \left( -0.5 a_{mi} h_i e_i^2 + \frac{\alpha_i k_i}{\rho_i} e_i^2 \left| C_{0i} \right| \right) + \frac{h_i k_i^2 \left| \varphi_{i} \right|^2}{0.5 a_{mi}} - e_i^2 \left( 0.5 h_i a_{mi} - \frac{k_i^2}{\lambda_{\text{min}}(\rho_i)} - \sum_{j=1}^{k} \frac{1}{\chi_{ij}} \right)
\]
where \( \sigma_1 = \min \{ \alpha_i, 0.5 a_{mi}, \frac{\lambda_{\text{min}}(Q_i)}{\lambda_{\text{max}}(P_i)} \} \). If we select
\[
0.5 h_i a_{mi} - \frac{k_i^2}{\lambda_{\text{min}}(\rho_i)} - \sum_{j=1}^{k} \frac{1}{\chi_{ij}} > 0
\]
we obtain
\[
V_1 \leq -\sigma_1 V_1 + \sum_{i=1}^{k} e_i^2 \left( 0.5 a_{mi} h_i - \frac{\alpha_i k_i}{\rho_i} \left| C_{0i} \right| \right) + \sigma_2,
\]
where \( \sigma_2 = \sup \frac{h_i k_i^2 \left| \varphi_{i} \right|^2}{0.5 a_{mi}} \). Choosing a small \( h_i \) we can decrease \( \sigma_2 \). If \( h_i, \rho_i \) are defined according to
\[
0.5 a_{mi} h_i - \frac{\alpha_i k_i}{\rho_i} \left| C_{0i} \right| > 0
\]
we obtain
\[
\lim_{t \to \infty} V_1 \leq \frac{\sigma_2}{\sigma_1}, \quad \text{and hence} \quad \lim_{t \to \infty} h_i e_i^2 \leq V_1,
\]
then
\[
\left| e_i^2 \right| < \frac{\sigma_2}{\lim_{t \to \infty} V_1} \quad \text{and} \quad \lim_{t \to \infty} e_i \left( t \right) < \delta.
\]
It is necessary to show that all the variables in (7) are bounded functions. Taking into consideration i) and iv) and Hurwitz matrices \( F_{ii} \) and \( F_{ii} \), we can conclude that the vectors \( V_{yi}, V_{ri} \), and \( \left( C(t) - C_0 \right)^T \omega \) are bounded. Transform the second equation (7):
\[
\dot{V}_{ui} = \left( F_{ui} + b_{ui} d_{2i}^T \right) V_{ui} + b_{ui} \left( C_{0i} - C_0 \right) \gamma_0 + b_{ui} \left( d_{0i} y_i(t) + d_{1i} y_{ti} + \frac{k_m}{k_i} \eta_{ri}(t) \right).
\]
Matrix \( F_{ui} + b_{ui} d_{2i}^T \) has the characteristic polynomial \( R_{ui}(\lambda) \) which is Hurwitz polynomial (according to assumption i) and hence \( V_{ui}(t) \) is a bounded vector, i.e. the vector \( \omega_i(t) \) is also bounded. The vector \( C_i(t) \) is bounded as well as follows from the equation (11). It follows from (7), (11) that the vectors \( \dot{\omega}_i(t) \) and \( \dot{C}_i(t) \) are bounded. It means that if the system starts working in the region \( \Omega_0 \), then there is the region
\[
\Omega_0 = \left[ e_i(t), \omega_i(t), C_i(t), \dot{\omega}_i(t), \dot{C}_i(t) : \left| \omega_i(t) \right| \leq k_1, \left| C_i(t) \right| < k_2, \left| \dot{\omega}_i(t) \right| < k_3, \left| \dot{C}_i(t) \right| < k_4 \right]
\]
with some region of attraction \( \Omega_1 \) where \( \lim_{t \to \infty} e_i(t) < \delta \).

According to the statement of the problem we cannot measure the derivatives. Let us write the control law in the following form:
\[
u_i(t) = T_i(P)\tilde{y}_i(t),
\]
\[
\dot{\theta}_i(t) = C_i(t)\gamma_i(t),
\]
\[
d\xi_i = -\rho_i \gamma_i(t) e_i - \alpha_i e_i^2 \left| C_i(t) \right|,
\]
where \( \tilde{y}_i(t) \) is an estimation of function \( y_i(t) \). To implement the control law (17), we have to estimate \( \tilde{y}_i(t) \) and its \( \gamma_{ai} - 1 \) derivatives. To estimate the derivatives, we will use the observer from [24]:
\[
\dot{\zeta}_i = F_{0i}\zeta_i + H_i(\theta_i - \tilde{\theta}_i), \quad \tilde{\theta}_i = L_{0i}\zeta_i, \quad i = 1, k. \quad (18)
\]
where \( \zeta_i \in \mathbb{R}^{r_{ai}}, \quad L_{0i} \equiv [1, 0, \ldots, 0]; \quad H_i^T = \left[ \frac{h_{ii}}{\mu}, \ldots, \frac{h_{ii}}{\mu^{r_{ai}}} \right]. \]
\[
F_{0i} = \begin{bmatrix} 0 & I_{r_{ai} - 2} \\ 0 & 0 \end{bmatrix}. \]
The vector \( H_i \) is chosen such that the matrix \( F_i = F_0i + H_i^T L_{0i} \) is Hurwitz matrix, where \( H_i^T = [-h_{ii}, \ldots, -h_{ii}^{r_{ai} - 1}]; \quad I_{r_{ai} - 2} \) is a unitary matrix of order \( (r_{ai} - 2) \times (r_{ai} - 2); \quad \mu > 0 \) is a small number. It is obvious now that the control law is technically feasible because it contains only known or measurable variables.

Now introduce two more vectors:
\[
\theta_i(t) = \left[ \tilde{\theta}_i(t), \theta_0(t), \ldots, \theta_k(t) \right], \quad \gamma_i(t) = \left[ \Gamma_i^{-1}(\zeta_i(t) - \theta_0(t)) \right],
\]
where the block diagonal matrix \( \Gamma_i = \text{diag} \left[ \mu^{r_{ai} - 2}, \mu^{r_{ai} - 3}, \ldots, \mu, 1 \right] \).

From the equation (18) we obtain the equation for the normalized deviations \( \eta_i(t) \)
\[
\dot{\eta}_i(t) = \Gamma_i^{-1}(F_{0i} - H_i L_{0i})\eta_i(t) + 1_i b_{0i} \tilde{\theta}_i(t).
\]
The structure of the matrices \( F_{0i}, b_{0i}, \Gamma_i, H_i \) is such that
\[
\Gamma_i^{-1}(F_{0i} - H_i L_{0i}) \Gamma_i = \frac{1}{\mu} F_i, \quad \Gamma_i^{-1} b_{0i} = b_{0i}.
\]

Taking into account \( \zeta_i(t) - \theta_0(t) = \Gamma_i \eta_i(t) \), the last equation can be transformed as follows:
\[
\dot{\eta}_i(t) = \frac{1}{\mu} F_i \eta_i(t) + b_{0i} \tilde{\theta}_i(t), \quad \Delta \eta_i(t) = \tilde{\theta}_i(t) - \theta_0(t) = \mu^{r_{ai} - 2} L_{0i} \eta_i(t). \quad (19)
\]

Let us transform this equation into equivalent with respect to the output equation \( \Delta \tilde{\theta}_i(t) \)
\[
\tilde{\eta}_i(t) = \frac{1}{\mu} F_i \tilde{\eta}_i(t) + b_{0i} \tilde{\theta}_i(t), \quad \Delta \tilde{\theta}_i(t) = \mu^{r_{ai} - 2} L_{0i} \tilde{\eta}_i(t), \quad (20)
\]
where \( b_{0i} = [1, \mu^{r_{ai} - 2}, 0, \ldots, 0] \), \( \tilde{\eta}_i(t) = \eta_i(t) \). Because the control is formed according to (17), we can rewrite (12) as...
\[ \dot{e}_i = -a_{\text{mi}}e_i(t) + k_i(C_i(t) - C_{\text{mi}})^T \omega_i(t) + \mu^* L_{\text{mi}} \eta_i(t) + k_i E_i \eta_i(t) + k_i \phi_i(t), \quad i = 1, k, \]
\[ \dot{s}_i = F_i s_i(t) + B_i e_i(t), \quad \eta_i(t) = L_{\text{mi}} s_i(t). \tag{21} \]

**Proposition.** Under the assumptions i) - viii) and (7) there is the number \( \mu_0 \) such that if \( \mu \leq \mu_0 \) then the system (7), (17), (20), (21) is dissipative if the system starts in the region \( \Omega_0 \) and the target condition (3) is true.

**Proof.** Consider (20) and (21) as
\[ \dot{e}_i = -a_{\text{mi}}e_i(t) + k_i(C_i(t) - C_{\text{mi}})^T \omega_i(t) + \mu^* L_{\text{mi}} \eta_i(t) + k_i E_i \eta_i(t) + k_i \phi_i(t), \]
\[ \dot{s}_i = F_i s_i(t) + B_i e_i(t), \quad \eta_i(t) = L_{\text{mi}} s_i(t). \tag{22} \]
\[ \mu^* \eta_i(t) = F_i \eta_i(t) + \mu^* \tilde{b}_0 \dot{\tilde{\eta}}_i(t), \]
\[ \dot{\tilde{\eta}}_i(t) = -\rho \alpha \omega_i(t) e_i(t) - \alpha \omega_i^2(t) C_i(t), \quad i = 1, k, \]
where \( \mu_1 = \mu_2 = \mu \). We will use lemma [25].

**Lemma [25].** If a system is described by the equation
\[ \dot{x} = f(x, \mu_1, \mu_2), \quad x \in \mathbb{R}^m, \] where \( f(t) \) is a continuous function and Lipschitz with respect to \( x \), and for \( \mu_2 = 0 \) it has the bounded and closed region of dissipation \( \Omega_1 = \{x \mid F(x) < \tilde{C} \} \), where \( F(x) \) is a positive-defined, continuous, and piecewise smooth function, then there exists \( \mu_0 > 0 \) such that for \( \mu_2 < \mu_0 \) the original system has the same dissipative region \( \Omega_1 \), if for some \( \tilde{C}_1 \) and \( \tilde{\mu}_1 \) for \( \mu_2 = 0 \) the following statement is true:
\[ \sup_{|x| \leq \tilde{C}_1} \left( \frac{\partial F(x)}{\partial x} \right)^T f(x, \mu_1, 0) \leq -\tilde{C}_1, \text{ for } F(x) = \tilde{C}. \tag{23} \]
\[ \text{Let us consider Lyapunov function } \]
\[ V_2 = \sum_{i=1}^{k} \eta_i(t) H_{2i} \eta_i(t), \]
where \( H_{2i} = H_{2i}^T > 0 \) is the solution of the equation
\[ H_{2i} F_i + F_i^T H_{2i} = -Q_{2i}, \]
where \( Q_{2i} = Q_{2i}^T > 0 \). Taking into consideration (22) we can obtain the following:
\[ \dot{V}_2 = -\sum_{i=1}^{k} \frac{1}{\mu_1} \eta_i(t) Q_{2i} \eta_i(t) \quad \text{for } \mu_2 = 0. \]

For \( \mu_2 = 0 \) the initial equation system (12) contains the additional independent equation \( \mu^* \eta_i(t) = F_i \eta_i(t) \) with asymptotically stable variable \( \eta_i(t) \). This gives us the dissipative region \( \Omega \) with the attraction region \( \Omega \).

Using Lyapunov function as \( F(x) \) we obtain
\[ F = \sum_{i=1}^{k} \left( h_i e_i^2(t) + s_i^2(t) P_i s_i(t) + \frac{k_i}{\rho_i} (C_i(t) - C_{\text{mi}})^T (C_j(t) - C_{\text{mi}}) + V_j^T(t) H_{2j} V_{2j}^T(t) + \right. \]
\[ \left. + V_{j0}^T(t) H_{2j} V_{j0}^T(t) + \tilde{\eta}_i^2(t) H_{2i} \tilde{\eta}_i(t) \right), \]
where \( h_{j0} > 0, H_{2j}, H_{j0}, H_{2j} \) are positive-defined symmetric matrices. Let us choose \( \tilde{C} \) such that the bounded and closed surface \( F(x) = \tilde{C} \), where \( x^T(t) = [e_1, s_1, \eta_1, V_1, V_{j0}] \) coincides with the boundary of the region \( \Omega \) by the variables \( x(t) \). The system is dissipative because the attraction set \( \Omega_1 \) belongs to the open region \( V(x) < \tilde{C} \). The variables \( x(t) \) will converge to the attraction region \( \Omega_1 \), and there exists \( \tilde{C}_1 \) that satisfies to the equation (23). Only the variables \( \tilde{\eta}_i(t) \) will depend on the choice of \( \mu_1 \). According to Lemma [25], there is \( \mu_0 > 0 \) such that for \( \mu < \mu_0 \) \( \Omega \) remains the dissipative region for the systems (7), (17), (20), (21).

However it is necessary to note that keeping the dissipative region doesn’t guarantee that \( \Omega_1 \) remains the same in a singularly perturbed system.

Let’s assume that \( \mu_1 = \mu_2 = \mu_0 \) in the equation (22) and the system starts in the set of the initial conditions \( \Omega_0 \) and hence all trajectories of the system will stay in the dissipation region \( \Omega \). Consider the Lyapunov function
\[ V = V_1(e_1, s_1, (C_j - C_{\text{mi}})) + V_{j0}(\tilde{\eta}_i) \]
and define matrix \( H_{2j} \) from the equation
\[ H_{2j} F_i + F_i^T H_{2j} = -3I, \]
where \( I \) is a unitary matrix with corresponding dimensions. We can calculate the full derivative of Lyapunov function using (22) and (14):
\[ \dot{V} \leq -\sigma_1 V_1 + \sigma_2 + \sum_{i=1}^{k} \left( -e_i^2 \left( 0.5 a_{\text{mi}} h_i - \frac{\alpha \kappa_i}{\rho_i} \right) C_{\text{mi}}^2 \right) + 2 e_i(t) h_i \mu_0 \gamma_0 L_{\text{mi}} \eta_i(t) - 3 \mu_0 \eta_i(t) 2 + \right. \]
\[ \left. + 2 \eta_i(t) H_{2j} \tilde{\eta}_i \left( C_i^T(t) \tilde{\phi}_i(t) + C_j^T(t) \tilde{\phi}_j(t) \right) \right), \tag{25} \]
As the system’s trajectories are in the region \( \Omega \):
\[ \Omega = \{e_1(t), \omega_i(t), C_i(t), \tilde{\phi}_i(t) \mid \tilde{\phi}_j(t) \leq k_j, \tilde{\omega}_i(t) \leq k_2, |C_j(t)| < k_3, |C_j(t)| < k_4 \}, \]
then the following estimates are correct:
\[ 2 \| \eta^T \| K_0, \]
\[ 2 \| \eta \| K_0, \]
where \( K_0 = \| H_{2j} \tilde{\eta}_i \| k_4 k_3 + k_3 k_4 \). Using this in (25):
\[ \dot{V} \leq -\sigma_1 V + \sigma_2 + \sum_{i=1}^{k} \left( -e_i^2 \left( 0.25 a_{\text{mi}} h_i - \frac{\alpha \kappa_i}{\rho_i} \right) C_{\text{mi}}^2 \right) - \]
where $\sigma_3 = \min \left\{ \sigma_1, \frac{1}{\mu_0} \right\}$. If $\mu_0, h, \rho_{li}$ are chosen according to

$$0.25a_{mi} h_i \mu_0^2 h_i^2 \mu_0^{2(r_u-1)} > 0,$$

$$\rho_{2i} = 0.25a_{mi} h_i - \frac{\alpha_k k_l}{\rho_{li}} C_{0i}^2 > 0,$$

then the square form

$$0.25a_{mi} h_i e_i^2(t) - 2e_i(t) \mu_0 h_i \mu_0^{2(r_u-1)} \tilde{p}_i(t) + \frac{1}{\mu_0} \tilde{p}_i(t)^2$$

will be the positive-defined and

$$\dot{V} \leq -\sigma_3 V + \sum_{i=1}^{k} (\rho_{3i} e_i^2 + \sigma_2 + \mu_0^2 K_{0i}^2)$$

This means that $\dot{V} < -\sigma_3 V$ for the region $e_i(t) > \sqrt{\frac{\sigma_2 + \mu_0^2 K_{0i}^2}{\rho_{2i}}}$ and the system is asymptotically stable in the region for variables $e_i(t), \tilde{p}_i(t)$. These variables will decrease until condition $e_i(t) < \sqrt{\frac{\sigma_2 + \mu_0^2 K_{0i}^2}{\rho_{2i}}}$ is satisfied. Keeping $\mu \leq \mu_0$ we can change $\mu$ and $\rho_{li}$ in (26) to obtain the required value of $\sigma$ in the condition (3).

IV. CONCLUSION

The paper considered the problem of adaptive control with a nominal model for interconnected systems with unknown parameters and order when derivatives of input and output signals of the local subsystems cannot be measured. A modified high order output control algorithm is proposed and its efficiency is validated. The number of tunable parameters of the algorithm is equal to the number of unknown parameters of the system. Unlike other high-order methods, the proposed method does not use the filter for the regression vector which allows to reduce the order of the system.

REFERENCES


