Compensation of Harmonic Disturbance for Nonlinear Plant with Parametric and Functional Uncertainty

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Abstract: This paper devoted to the disturbance cancellation problem for the nonlinear plant. We present a new output control approach of harmonic disturbance compensation. The proposed control law is easier to implement and outperform known analogs by some characteristics.

Keywords: Adaptive control, disturbance rejection, nonlinear systems, parametric uncertainty.

1. INTRODUCTION

In this paper we consider new output control algorithm for nonlinear plant with parametric and functional uncertainty disturbed by the biased harmonic signal \( \vec{w}(t) = \vec{a} + \vec{a} \sin(\omega t + \vec{\phi}) \). Many different approaches exist for adaptive identification of unknown sinusoid, see, for example, (Bobtsov, 2008 a,b; Bobtsov et al., 2008, 2009, 2010, 2011; Hou, 2005; Hsu et al., 1999; Marino et al., 2003; Nikiforov, 1998; Pyrkin et al., 2010, 2010 a, b; Xia, 2002). Some of these approaches are not restricted to the case of a single sinusoid, in particular, a biased sinusoidal signal is considered in (Bobtsov et al., 2005; Pyrkin et al., 2010, 2011; Bobtsov, 2008; Hou, 2005), and the case of multiple sinusoids with different frequencies is presented in (Xia, 2002; Bobtsov et al., 2010).

In (Marino et al., 2003), there is submitted the control approach for linear plant with known parameters disturbed by biased harmonic signal. The given algorithm has (2n+1)-order. The observer’s synthesis method is difficult to implement and needs complex computations. Also the minimum value of \( \vec{a} \) has to be known. Developing (Marino et al., 2003) in (Bobtsov and Kremlev, 2005), the control law having (n+4)-order is proposed.

Also it has less complex structure (comparing to Marino et al., 2003) and there is no need to obtain the minimum value of the frequency. Control law for non-minimum phase plant with known parameters is presented in (Bobtsov, 2008).

However approaches obtained in (Marino et al., 2003; Bobtsov et al., 2005 a, 2009; Pyrkin et al., 2010, 2010 a, b) are applicable only for linear plants with known parameters.

There are some works dealing with robust control of linear and nonlinear systems with unknown parameters should be noted (Bobtsov, 2002, 2005; Bobtsov and Nikolaev, 2005; Furtat et al., 2008, 2009, 2010) where the similar approach of stabilization was developed. More complicated problems like harmonic disturbance compensation for plants with parametric uncertainty are considered in (Bobtsov, 2008, 2009). The works presented in (Bobtsov et al., 2009) describe unknown disturbance observer for output signal.

In (Pyrkin et al., 2010, 2010 a, b) approaches of biased harmonic disturbance compensation with delay in control channel are described, notably in (Pyrkin et al., 2010 a, b) unstable plants have been considered. Algorithms proposed in (Bobtsov, 2008, 2009) exclude control plant parameterization. However approach presented in (Bobtsov, 2008) provides output convergence to zero only, but not the asymptotic convergence of equilibrium which could be inappropriate in certain cases. In (Bobtsov, 2009) this disadvantage has been eliminated. Nevertheless approaches of compensation of harmonic disturbance submitted in (Bobtsov, 2008, 2009) are applicable only for linear plants.

The problem of appliance this approaches for nonlinear plants is solved in this paper basing on results presented in (Bobtsov et al., 2009, 2011).
2. PROBLEM STATEMENT

Consider nonlinear plant
\[
\dot{z}(t) = Fz(t) + g_1u(t) + g_2\varphi(y),
\]
\[
y(t) = h^Tz(t),
\]
where the vector of state variables \(z(t)\in \mathbb{R}^n\) is not measured, \(u(t)\) is an scalar control signal, \(y(t)\) is a scalar output variable, \(\varphi(y)\) is an unknown nonlinear function. The input disturbance \(\overline{w}(t)\) is
\[
\overline{w}(t) = \overline{\sigma}_0 + \sigma \sin(\omega t + \varphi),
\]
which is biased sinusoid with unknown amplitude \(\overline{\sigma}\), frequency \(\omega\), phase \(\varphi\) and bias \(\overline{\sigma}_0\).

Together with the model (1) consider an input-output model
\[
y(t) = \frac{b(p)}{a(p)}u(t) + \frac{c(p)}{a(p)}\overline{w}(t) + \frac{d(p)}{a(p)}\varphi(y),
\]
where \(p = d/dt\) is the differentiation operator, \(a(p), b(p), c(p)\) and \(d(p)\) are corresponding polynomials, resulting from difference equation to state space form transformation, and the polynomial \(a(p)\) is n-order.

Consider the following assumptions concerning system (1), (2).

A. 1. Assume that only signal \(y(t)\) is measurable.

A. 2. Coefficients of matrix \(F\) and vectors \(g_1, g_2, g_3\) and \(h\) are unknown.

A. 3. Pair \(F, g_1\) completely controllable and pair \(F, h\) completely observable.

A. 4. Polynomial \(b(p) = b_n p^n + \ldots + b_1 p + b_0\) is Hurwitz and \(b_n > 0\).

A. 5. Polynomial \(c(p)\) doesn’t have roots \(\pm j\omega\), where \(j = \sqrt{-1}\).

A. 6. Nonlinear function \(\varphi(y)\) fulfils the following limitation (Bobtsov, 2005)
\[
(\varphi(y))^2 \leq Cy\psi(y),
\]
where function \(\psi(y)\) is known, but positive constant \(C\) is unknown.

The objective is to develop an algorithm which will fulfill the following purpose of control with any initial conditions
\[
\lim_{t \to \infty} y(t) = 0.
\]

3. ADAPTIVE CONTROL DESIGN

Model (2) can be presented as
\[
a(p)y(t) = b(p)[u(t) + w(t)] + d(p)\varphi(y)
\]
where signal \(w(t) = \frac{c(p)}{b(p)}\overline{w}(t) = \sigma_0 + \sigma \sin(\omega t + \varphi) + \varepsilon(t)\) is the sum of harmonic signal (by reason of polynomial \(b(p)\) is Hurwitz) with bias \(\sigma_0\), amplitude \(\sigma\), frequency \(\omega\), phase \(\varphi\) exponential decaying function \(\varepsilon(t)\) which will be neglected.

It is known (Bobtsov, 2008, 2009) that signal \(w(t) = \sigma_0 + \sigma \sin(\omega t + \varphi)\) can be presented as follows
\[
p^3w(t) = -\omega^2 pw(t) = -\theta pw(t),
\]
where \(\theta = \omega^2\).

Neglecting exponentially decaying terms for (6) we obtain
\[
(p + 1)^3w(t) = (3p^2 + 3p + 1 - \theta p)w(t)
\]
\[
(p + 1)^4b(p)w(t) = (3p^2 + 3p + 1 - \theta p)b(p)w(t).
\]

This means that for (5) and (7) we find
\[
(p + 1)^3[a(p)y(t) - b(p)u(t) - d(p)\varphi(y)] =
\]
\[
= (3p^2 + 3p + 1 - \theta p)[a(p)y(t) - b(p)u(t) - d(p)\varphi(y)].
\]

Consider linear third order filter:
\[
\hat{x}(t) = \frac{1}{(p + 1)^3}u(t).
\]

Then neglecting exponentially decaying components of (9) we have
\[
(p + 1)^3 + \theta p)a(p)y(t) = (p + 1)^4b(p)[u(t) - 3\hat{x}(t) - 3\hat{x}(t) - \xi(t) + \theta x(t)] + (p + 1)^3 + \theta p)d(p)\varphi(y).
\]
Let us assume that control law is as follows
\[
u(t) = 3\dot\xi(t) + 3\ddot\xi(t) + \dot\xi(t) - \tilde\theta(t)\ddot\xi(t) - \gamma_1(t)\psi(y) - \gamma_3(t)y(t),
\]
(11)
where functions \(\tilde\theta(t), \gamma_1(t),\) and \(\gamma_3(t)\) are calculated by the following rules:
\[
\dot\gamma_1(t) = \gamma_{i1}y(t)\psi(y),
\]
(12)
\[
\dot\gamma_3(t) = \gamma_{i3}y^2(t),
\]
(13)
\[
\tilde\theta(t) = k_a\ddot\xi(t)y(t),
\]
(14)
and parameters \(\gamma_{i1}, \gamma_{i3},\) and \(k_a\) are random positive numbers.

**Remark 1.** It should be noted that functions \(\gamma_i(t)\) and \(\gamma_3(t)\) are tuning coefficients and also estimation of several ideal parameters of \(\gamma_i' > 0\) and \(\gamma_3' > 0\). According to theorem of pacification of linear systems (Bobtsov, 2005; Fradkov et al. 1999; Bobtsov et al. 2005) the parameter \(\gamma_3\) provide the transfer function
\[
H(p) = \frac{(p + 1)b(p)}{(p + \theta p)a(p) + \gamma_3(p + 1)b(p)}
\]
being strict real positive. The parameter \(\gamma_i'\) will be calculated further and is function of unknown constant \(C\) from inequality (3), also it depends on \(\theta\) and parameters of polynomial \(d(p)\). It ought to be known that there is no need in accurate estimations of \(\gamma_i'\) and \(\gamma_3'\). Tuning coefficients \(\gamma_i(t)\) and \(\gamma_3(t)\) have to be greater than or equal \(\gamma_i'\) and \(\gamma_3'\). Fulfillment of these conditions will be demonstrated in the following proving.

Therefore (10) will be as follows
\[
(p^3 + \theta p)a(p)\psi(y) = \]
\[
= (p + 1)b(p)[-\gamma_1\psi(y) + \tilde\gamma_1\psi(y) - \tilde\gamma_2y + \ddot\xi d(p)\phi(y),
\]
(15)
where
\[
\tilde\gamma_1(t) = \gamma_1 - \gamma_1(t),
\]
(16)
\[
\tilde\gamma_2(t) = \gamma_3 - \gamma_3(t),
\]
(17)
\[
\tilde\theta(t) = \theta - \tilde\theta(t).
\]
(18)
From (15) we have
\[
y(t) = H(p)[\tilde\gamma_1y + \ddot\gamma_2\psi(y) + \ddot\gamma_1\psi(y)] +
\]
\[
+ W(p)ph(y),
\]
(19)
where
\[
W(p) = \frac{(p + 1)\psi(y)d(p)}{(p^3 + \theta p)a(p) + \gamma_3(p + 1)b(p)}.
\]

Let us present input-output model (19) in state-space from:
\[
\dot{x} = Ax + b[y + \ddot\gamma_2\psi(y) + \ddot\gamma_1\psi(y)] + d\phi(y),
\]
(20)
\[
y = c'x.
\]
(21)

Here \(x \in \mathbb{R}^n\) is vector of state variables (20), (21); \(A, b, c,\) and \(d\) are corresponding matrix and vectors obtained during transformation into state space form.

**Remark 2.** On account of strict real positiveness of transfer function \(H(p)\) for (20), (21) following matrix equations are fulfilled:
\[
A'P + PA \leq -Q,
\]
(22)
\[
Pb = c,
\]
(23)
where \(P = P^T > 0\) and \(Q = Q^T > 0\).

**Proposition 1.** Let us assume that non-linear system (1) disturbed by signal \(\bar{w}(t) = \bar{\sigma}_a + \sigma\sin(\omega t + \bar{\theta})\) with unknown amplitude \(\sigma\), frequency \(\omega\), phase \(\bar{\theta}\), and bias \(\sigma\), fulfill assumptions 1 – 6. Hence control law (11) – (14) fulfill the purpose of control (4).

The proof of the Proposition 1 is presented in Appendix. For efficiency illustration of proposed control algorithm the following example is considered.

4. EXAMPLE

Let us assume the following non-linear plant
\[
y(t) = \frac{b_1p + b_2}{(p + a_1p + a_2)}u(t) + \frac{c_1p + c_2}{(p + a_1p + a_2)}\bar{w}(t) +
\]
\[
+ \frac{d_1p + d_2}{(p + a_1p + a_2)}\phi(y),
\]
(24)
where \(u(t)\) is control signal, \(y(t)\) is output variable, \(\bar{w}(t) = \bar{\sigma}_a + \sigma\sin(\omega t + \bar{\theta})\) is output disturbance, parameters \(a_1, a_2, b_1, b_2, c_1, c_2, \sigma, \bar{\sigma}, \sigma,\) and \(\bar{\sigma}\) are unknown.

Keeping to assumption 6 we will consider that unknown non-linear function \(\phi(y)\) fulfills inequality (4) and function \(\psi(y)\) is known. Assume that \(\phi(y) = 2y^2\sin y\) and \(\psi(y) = y^3\). Choosing the following coefficients \(\gamma_{i0} = 1, \gamma_{i3} = 1,\) in control law (11) – (14) \(k_a = 1\) we simulate the system.

Fig. 1 – 3 show simulation results of compensation algorithm proposed in the paper for non-linear system (24) with parameters \(a_1 = -2,\) \(a_2 = 0,\) \(b_1 = 1,\) \(b_2 = 1,\) \(c_1 = 2,\) \(c_2 = 2,\) \(d_1 = 1,\) \(d_2 = 1\), and disturbance \(\bar{w}(t) = 3 + 3\sin(2t + 1)\).
Fig. 1. Transient for $\tilde{\theta}(t)$, $\gamma_1(t)$ and $\gamma_2(t)$.

Fig. 2. Transient for control signal $u(t)$.

Fig. 3. Transient for output variable $y(t)$.

Fig. 4 – 6 show simulation results for non-linear system (24) with parameters $a_0 = 2$, $a_1 = 1$, $b_0 = 1$, $b_1 = 2$, $c_0 = 3$, $c_1 = 1$, $d_0 = 4$, $d_1 = 3$, and disturbance $\bar{w}(t) = 2 + 4\sin(t + 5)$.

Fig. 1 – 6 show the fulfillment of control purpose (4).

5. CONCLUSION

The new output control algorithm for nonlinear plant with parametric and functional uncertainty disturbed by biased harmonic signal is designed. Proposed result extends approaches presented in (Marino et al., 2003; Bobtsov et al., 2008, 2009, 2005a). Comparing to (Marino et al., 2003; Bobtsov et al., 2005a) in this work the plant with parametric uncertainty is considered. Opposing to (Bobtsov, 2008, 2009) we designed the control algorithm for nonlinear plant with functional uncertainty.
Consider the Lyapunov function
\[ V = x^T P x + k_1^1 \tilde{\theta}^2 + \gamma_0^1 \tilde{y}_1^2 + \gamma_0^2 \tilde{y}_2^2. \] \tag{A.1}

After differentiation (A.1) we obtain
\[ \dot{V} = x^T (A^T P + PA) x + 2x^T P \dot{\theta} \tilde{y}_2 y + \tilde{\theta} \dot{\theta} - \gamma_1 \psi(y) + \tilde{\gamma}_1 \psi(y) + x^T P \ddot{\psi}(y) + 2k_a^1 \tilde{\theta} \dot{\theta} + 2\gamma_0^1 \tilde{y}_1^2 + 2\gamma_0^2 \tilde{y}_2^2 \leq -x^T Q x + 2\gamma_0^1 \tilde{y}_1^2 + 2\gamma_0^2 \tilde{y}_2^2, \] \tag{A.2}

Substituting (22) and (23) in (A.2) we have
\[ \dot{V} \leq -x^T Q x + 2\gamma_0^1 \tilde{y}_1^2 + 2\gamma_0^2 \tilde{y}_2^2, \] \tag{A.3}

where \( \delta > 0 \) is a small number so that for the next square form the following inequality fulfilled
\[ -x^T Q x + \delta x^T P d x^T P x \leq -x^T Q_o x, \]

where \( Q_o = Q_o^T > 0 \).

After differentiation (16) – (18) in account (12) – (14) we obtain
\[ \dot{\gamma}_1(t) = \gamma'_1(t) - \gamma_1(t) = 0 - \gamma_0(y(t) \psi(y), \] \tag{A.4}
\[ \dot{\gamma}_2(t) = \gamma'_2 - \gamma_2(t) = 0 - \gamma_0(y(t), \] \tag{A.5}
\[ \dot{\tilde{\theta}}(t) = \tilde{\theta} - \tilde{\theta}(t) = 0 - k_2 \tilde{\gamma}_2(t) \psi(y(t)). \] \tag{A.6}

Considering assumption 6 and substituting (A.4) – (A.6) in inequality (A.3) we have
\[ \dot{V} \leq -x^T Q_o x - 2\gamma_0^1 y \psi(y) + \delta^{-1} Cy \psi(y). \]

Assuming \( \gamma_1 \geq \frac{1}{2} \delta^{-1} C \), hence
\[ \dot{V} \leq -x^T Q_o x. \] \tag{A.7}

From inequality (A.7) arise Lyapunov stability of equilibriums \( x=0, \tilde{\theta}=0, \tilde{\gamma}_1=0, \tilde{\gamma}_2=0 \) and fulfillment of condition
\[ \int_0^\infty x^T Q_o x dt < \infty. \] \tag{A.8}

Hence from Lyapunov stability of equilibriums \( x=0, \tilde{\theta}=0, \tilde{\gamma}_1=0, \tilde{\gamma}_2=0 \) and from inequity (A.8) it can be proved (Fradkov et al., 1999) that equilibrium \( x=0 \) has asymptotic stability. Therefore according to asymptotic stability of the equilibrium \( x=0 \) and (21) purpose of control (4) is fulfilled.

REFERENCES


