Abstract: Driven by the satellite’s features such as closed-loop, limited storage space and computing capacity constraint, a fault detection scheme based on the coprime factorization and Youla parameterization is proposed for the on-orbit satellite attitude control system. Based on the SISO dynamic of the satellite in pitch direction and the MIMO one in roll/yaw direction, the residuals equivalent to \( y(k) - \hat{y}(k) \) are generated by a simple computation using the plant input and control error signals without running the state observer in parallel, therefore the computing expense is reduced to a certain extent, and the related theorems are provided synchronously. Simulation results are presented to show the performance of the proposed fault detection algorithm.

1. INTRODUCTION

With the development of space technologies, different types of satellites have been constructed and used for various space missions, such as position location, Earth observation, atmosphere data collection, and communication (Qing Wu and Mehrdad Saif, 2007). Equipment fault leading to scientific mission failure and high costs has been common with the evolution of technology, and the proposed design procedure involves eigenstructure assignment. Directional nonlinear observers are used to detect and isolate faults in small satellites’ actuators in (Jensen and Wisniewski 2002); however, as it is outlined by the authors, there exist some conditions regarding the distribution functions of the disturbances and faults, which basically rely on rank conditions. A particle filtering method combined with a bank of Kalman filters is proposed to address the problem of FDI in spacecraft and planetary rovers in (De Freitas N., 2002). In (Qing Wu and Mehrdad Saif, 2007), a nonlinear observer which synthesizes second order sliding mode techniques and wavelet networks is proposed for abrupt and incipient fault in a multiple satellite formation flying system. In (Qing Wu and Mehrdad Saif, 2007), an iterative learning observer is designed to achieve estimation of time-varying thruster faults in satellite system.

However, two reasons prevent these methods from applying to the satellite in reality. One reason is that some of the aforementioned methods are based on an open-loop model of the monitored system, although the FDI scheme is placed in a feedback loop and faults may be covered by control actions (D.Henry, 2008)). This motivates the integrated design of control and diagnosis schemes in which the design of controllers and fault detectors are formulated as a standard optimization problem (C. N. Nett, 1988; A. Marcos and G. J. Balas, 2005; H. H. Niemann and J. Stoustrup, 1997; H. H. Niemann and J. Stoustrup, 2003; M. L. Tyler and M. Morari, 1994; Schultalbers et al, 2010; S.X.Ding, 2009). However, because of the fact that the already-in-place attitude control system is certified and thus cannot be removed, this solution is hard to be applied here. The other one is that instead of some complex methods, the limited storage space and computing capacity constraint of the satellite computer need simple but effective fault diagnosis approaches that can monitor the satellite operating condition without any extra online computation.

In this paper, because of the fact that the dynamics in pitch direction decouples from that in roll/yaw direction in linear model on the small angles assumption, the corresponding SISO model and the MIMO model are obtained respectively by considering the satellite kinematic and dynamic model simultaneously. Based on the models, instead of running the state observer, the coprime factorization of the plants and the Youla parameterization of the stabilizing controllers are used to conclude that the residual equivalent to \( y(s) - \hat{y}(s) \) is generated by a simple computation using the plant input and control error signals that are available in the feedback control loop without any computing expense.

2. SATELLITE DESCRIPTION

A satellite is assumed to be a rigid body with the momentum wheel providing torques. The linear model of the satellite dynamic is presented as follows (Yang Ciann-dong and Sun Yun-ping, 2002):

\[
\begin{align*}
I_1 \dot{\phi} &- n(I_1 - I_2 + I_3) \dot{\psi} + 4n^2(I_2 - I_3) \phi = T_1 \\
I_2 \dot{\theta} + 3n^2(I_1 - I_3) \theta &= T_2 \\
I_3 \dot{\psi} + n(I_1 - I_2 + I_3) \phi + n^2(I_2 - I_3) \psi &= T_3
\end{align*}
\]

where \( I_1, I_2, I_3 \) are the principal moments of inertia, \( \phi, \theta, \psi \) are the roll angle, the pitch angle and the yaw angle respectively, and \( T_1, T_2, T_3 \) are the body-axis components of the external torques acting on the satellite, which usually
contain control torques and environmental disturbance torques, however in this paper they only contain control torques, \( n \) is the orbital rate of the satellite. It is noticed that the pitch dynamics decouples from the roll/yaw dynamics in linear model on the small angles assumption, so the residuals of the pitch channel and the roll/yaw channel can be designed separately.

For the pitch channel, by defining the state vector \( x = [\theta \ \dot{\theta}]^T \) and the output vector \( y = x \), we have the following SISO state-space representation of the equations:

\[
\begin{bmatrix}
\dot{\theta} \\
\ddot{\theta}
\end{bmatrix} = \begin{bmatrix}
0 & I \\
-3I_3^2n^2(I_1-\dot{I}_3) & 0
\end{bmatrix} \begin{bmatrix}
\theta \\
\dot{\theta}
\end{bmatrix} + \begin{bmatrix}
I_3^2
\end{bmatrix} \begin{bmatrix}
T_2
\end{bmatrix} 
\]

\[
\theta = \begin{bmatrix}
I \\
0
\end{bmatrix} \begin{bmatrix}
\dot{\theta}
\end{bmatrix}
\]  

(2)

For the roll/yaw channel, let \( Q = [\phi \ y]^T \) and rewrite Eq.(1) as the following matrix form:

\[
A_3 \dot{Q} + A_1 \dot{Q} + A_2 Q = B_1 T_1 + B_2 T_3
\]

(3)

where

\[
A_0 = n^2 \begin{bmatrix}
4(I_2-I_3) & 0 \\
0 & I_2-I_1
\end{bmatrix}, \quad A_2 = \begin{bmatrix}
I_1 \\
0
\end{bmatrix}, \quad B_1 = \begin{bmatrix}
I \\
0
\end{bmatrix}, \\
B_2 = \begin{bmatrix}
0 \\
I
\end{bmatrix}, \quad A_3 = n\begin{bmatrix}
0 \\
-I_1+I_2-I_3
\end{bmatrix}
\]

By defining the state vector \( x = [Q \ \dot{Q}]^T \) and the output vector \( y = x \), we have the following MIMO state-space representation of the equations:

\[
\begin{bmatrix}
\dot{Q} \\
\ddot{Q}
\end{bmatrix} = \begin{bmatrix}
0 & I \\
-A_3^2 A_0 & -A_3^2 A_1
\end{bmatrix} \begin{bmatrix}
Q \\
\dot{Q}
\end{bmatrix} + \begin{bmatrix}
0 & 0 \\
A_3^2 B_1 & A_3^2 B_2
\end{bmatrix} \begin{bmatrix}
T_1 \\
T_3
\end{bmatrix}
\]

(4)

3. FAULT DETECTION WITH COMPUTATION CONSTRAINT

3.1 Fault detection of SISO plant

Without lost of generality, we consider the following SISO plant shown in Fig.1:

\[
\begin{align*}
\dot{x}(t) &= A x(t) + B u(t) \\
y(t) &= C x(t) + D u(t)
\end{align*}
\]

(5)

where \( x(t) \), \( u(t) \) and \( y(t) \) are the plant states, inputs and outputs. \( A \), \( B \), \( C \) and \( D \) are the related matrix.

\[
Xs \dot{Q} + A_1 \dot{Q} + A_2 Q = B_1 T_1 + B_2 T_3
\]

(3)

where

\[
A_0 = n^2 \begin{bmatrix}
4(I_2-I_3) & 0 \\
0 & I_2-I_1
\end{bmatrix}, \quad A_2 = \begin{bmatrix}
I_1 \\
0
\end{bmatrix}, \quad B_1 = \begin{bmatrix}
I \\
0
\end{bmatrix}, \\
B_2 = \begin{bmatrix}
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I
\end{bmatrix}, \quad A_3 = n\begin{bmatrix}
0 \\
-I_1+I_2-I_3
\end{bmatrix}
\]

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\dot{Q}
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4(I_2-I_3) & 0 \\
0 & I_2-I_1
\end{bmatrix}, \quad A_2 = \begin{bmatrix}
I_1 \\
0
\end{bmatrix}, \quad B_1 = \begin{bmatrix}
I \\
0
\end{bmatrix}, \\
B_2 = \begin{bmatrix}
0 \\
I
\end{bmatrix}, \quad A_3 = n\begin{bmatrix}
0 \\
-I_1+I_2-I_3
\end{bmatrix}
\]

By defining the state vector \( x = [Q \ \dot{Q}]^T \) and the output vector \( y = x \), we have the following MIMO state-space representation of the equations:

\[
\begin{bmatrix}
\dot{Q} \\
\ddot{Q}
\end{bmatrix} = \begin{bmatrix}
0 & I \\
-A_3^2 A_0 & -A_3^2 A_1
\end{bmatrix} \begin{bmatrix}
Q \\
\dot{Q}
\end{bmatrix} + \begin{bmatrix}
0 & 0 \\
A_3^2 B_1 & A_3^2 B_2
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T_3
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\end{align*}
\]

(5)

where \( x(t) \), \( u(t) \) and \( y(t) \) are the plant states, inputs and outputs. \( A \), \( B \), \( C \) and \( D \) are the related matrix.
\[ \dot{X}(s)u(s) - \dot{Y}(s)y(s) = u(s) - F\dot{x}(s) \]  \hspace{1cm} (16)

Since \( \dot{x}(s) = (sI - A)^{-1}(Bu(s) + L(y(s) - \hat{y}(s))) \), we get

\[ \dot{X}(s)u(s) - \dot{Y}(s)y(s) = u(s) - F(sI - A)^{-1}(Bu(s) + L(y(s) - \hat{y}(s))) \]  \hspace{1cm} (17)

\[ = (I - F(sI - A)^{-1}B)u(s) - F(sI - A)^{-1}L(y(s) - \hat{y}(s)) \]

For the formula on the right side of Eq.(12), by using \( \tilde{N}(s) = D + C(sI - A + LC)^{-1}(B - LD) \) and \( \tilde{M}(s) = I - C(sI - A + LC)^{-1}L \) shown in Eq.(10), it leads to

\[ Q(s)N(s)u(s) - \tilde{M}(s)y(s) = Q(s)((D + C(sI - A + LC)^{-1}(B - LD))u(s) \]

\[ - (I - C(sI - A + LC)^{-1}L)y(s)) \]

\[ = Q(s)(Du(s) + C\hat{x} - y(s)) \]

\[ = Q(s)u(s) \]

Considering Eq.(12), Eq.(17) and Eq.(18) at the same time, we obtain:

\[ u(s) = (I - F(sI - A)^{-1}B)^{-1}(F(sI - A)^{-1}L - Q(s))(y(s) - \hat{y}(s)) \]  \hspace{1cm} (19)

Since the following transition

\[ (I - F(sI - A)^{-1}B)^{-1}F(sI - A)^{-1}L \]

\[ = \begin{bmatrix} A & B \\ -F & I \end{bmatrix}^{-1} \begin{bmatrix} A \\ F \end{bmatrix} \]

\[ = \begin{bmatrix} A + BF \\ F \end{bmatrix} \begin{bmatrix} A \\ F \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} A \\ F \end{bmatrix} \]

and choosing nonsingular matrices \( T = \begin{bmatrix} I & -I \\ 0 & I \end{bmatrix} \), one can yield the transformations of Eq. (20) as follows

\[ (I - F(sI - A)^{-1}B)^{-1}(F(sI - A)^{-1}L) = \begin{bmatrix} A + BF \\ BF \\ F \end{bmatrix} \begin{bmatrix} A \\ 0 \\ 0 \end{bmatrix} \]  \hspace{1cm} (21)

In term of the reference (Z.W. Gao and Albert T.P. So, 2003), we get

\[ (I - F(sI - A)^{-1}B)^{-1}(F(sI - A)^{-1}) = \begin{bmatrix} A + BF \\ BF \\ F \end{bmatrix} \begin{bmatrix} A \\ 0 \\ 0 \end{bmatrix} \]  \hspace{1cm} (22)

then the following equation is obtained

\[ u(s) = (Y(s) - M(s)Q(s))(y(s) - \hat{y}(s)) \]  \hspace{1cm} (23)

Considering the feedback control system shown in Fig. 1, the relationship between \( e(s) \) and \( u(s) \) exists

\[ e(s) = (K(s))^{-1}u(s) \]  \hspace{1cm} (24)

and the residual is generated by the combination of \( e(s) \) and \( u(s) \)

\[ r(s) = au(s) + be(s) = [aI + bK(s)]^{-1}u(s) \]  \hspace{1cm} (25)

Substituting Eq.(7) into Eq.(26), we get

\[ r(s) = y(s) - \hat{y}(s) \]  \hspace{1cm} (26)

This completes the proof of Theorem 1.

The main steps to get the residual for the fault detection are as follows:

(1) Given the system plant with the matrices \( A,B,C,D \)

known in Eq.(5), the double coprime factorization parameters \( M(s), N(s), X(s), Y(s), \tilde{M}(s), \tilde{N}(s), \tilde{X}(s), \tilde{Y}(s) \) of the plant are obtained according to the Eq. (9) and Eq. (10). It is noted that all the double coprime factorization parameters have the unknown parameters \( F \) and \( L \).

(2) Based on the double coprime factorization parameters, the controller \( K(s) \) that internally stabilized the feedback system is obtained by Eq. (11), but the parameter \( Q(s) \) in controller \( K(s) \) is unknown.

(3) The parameters \( F, L \) and \( Q(s) \) are computed by comparing the controller \( K(s) \) with the existed controller \( K_0(s) \) (like the already-in-place controller in the satellite).

(4) With the parameters \( F, L, Q(s) \) and \( M(s), N(s), X(s), Y(s), \tilde{M}(s), \tilde{N}(s), \tilde{X}(s), \tilde{Y}(s) \) known, the relationship between \( u(s) \) and \( y(s) - \hat{y}(s) \) as well as that between \( e(s) \) and \( y(s) - \hat{y}(s) \) is obtained.

(5) The Eq. (6) is used to get the parameter \( a \) and \( b \), and then we get the residual \( r(s) = y(s) - \hat{y}(s) \) which is essential to realize the fault detection of the system.

### 3.2 Fault detection of MIMO plant

Without lost of generality, we consider the following MIMO plant:

\[ \dot{x}(t) = Ax(t) + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \]  \hspace{1cm} (27)

\[ \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} x(t) + \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \]  \hspace{1cm} (28)

Theorem 2. Given the feedback control system with the plant model \( G(s) \) and the controller \( K(s) \) parameterized in Eq. (34), then

\[ r(s) = R_1u(s) + R_2e(s) = y(s) - \hat{y}(s) \]  \hspace{1cm} (29)
where \( R_1 \) and \( R_2 \) satisfy the following form:
\[
[R_1 I + R_2 (K(s))^{-1}] [U(s) - \Psi(s)Q(s)] = I
\]
(30) and \( \hat{y}(s) \) is an estimate of \( y(s) \) delivered by the following observer:
\[
\hat{\dot{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - \hat{y}(t))
\]
\[
\hat{y}(t) = C\hat{x}(t) + Du(t)
\]
(31) where the related parameters are shown as follows:
\[
\begin{bmatrix}
\Psi(s) & U(s) \\
\Omega(s) & V(s)
\end{bmatrix} = \begin{bmatrix}
A + B_1 F & B_2 & I \\
C_2 + D_2 F & D_2 & I
\end{bmatrix} I
\]
(32)
\[
\begin{bmatrix}
\Psi(s) & \Omega(s) \\
\tilde{U}(s) & \tilde{V}(s)
\end{bmatrix} = \begin{bmatrix}
-A - LC_2 & -L - B_2 - LD_2 \\
C_2 & I
\end{bmatrix} I
\]
(33)
\[
K(s) = (U(s) - \Psi(s)Q(s)(V(s) - \Omega(s)Q(s))^{-1})
\]
(34)

**Proof.** It is similar to the Proof of Theorem 1.

### 3.3 Residual evaluation

In the residual evaluation stage, an evaluation function and a threshold should be provided for the purpose of fault decision. Here, as in most contributions, we adopt the following residual evaluation function and the threshold
\[
J_k = \left\{ \sum_{i=0}^{k} r_{i-1}^2 \right\}^{1/2}
\]
\[
J_m = \sup_{i=0}^{m} E[J_k]
\]
(35) And the decision logic can be defined as
\[
J_k \geq J_m \text{ alarm for fault}
\]
\[
J_k < J_m \text{ no fault}
\]
(36)

### 4. SIMULATION RESULTS

In this section, the proposed fault detection scheme is applied to detect actuator and sensor faults in a microsatellite attitude control system. Considering the rigid body dynamics only, the typical motion equations of the satellite with faults are shown in (Qing Wu and Mehrdad Saif, 2005)

For the pitch channel, the already-in-place attitude controller is:
\[
K_0 = K_p + \frac{K_i}{s} \quad (K_p = 20, K_i = 0.02)
\]
(37) and with the assumption that \( F = [f_1, f_2] \) and \( L = [l_1, l_2] \), then we have the double coprime factorization parameters in the following format
\[
\begin{bmatrix}
M(s) & Y(s) \\
N(s) & X(s)
\end{bmatrix}
= \frac{1}{s^2 - f_1 s - f_2} \begin{bmatrix}
(f_1 + f_2) & f_1 + f_2 \\
0.0549(s) & 0.0549(s)
\end{bmatrix}
\]
(38) Based on the parameters \( M(s), N(s), X(s), Y(s) \), and according to Eq. (11), the controller \( K(s) \) that internally stabilized the feedback system with the free parameter \( q \) is as follows
\[
K(s) = \frac{-q s^2 + (f_1 l_1 + f_2 l_2)s + f_1 l_1}{s^2 + (0.0549 l_1 - f_1)s + 0.0549 l_2 f_2 - f_2 - 0.0549}
\]
(39) In order to ensure that the controller \( K(s) \) is equal to the given PI-controller, the following equation is obtained
\[
-\frac{q s^2 + (f_1 l_1 + f_2 l_2)s + f_1 l_1}{s^2 + (0.0549 l_1 - f_1)s + 0.0549 l_2 f_2 - f_2 - 0.0549} = \frac{K_p}{s}
\]
By solving (39), we have the following parameters
\[
\begin{cases}
\{ f_1 = 0, f_2 = 18.2149 \} & \{ l_1 = 0, l_2 = 1.2067 \}, q = -K_p = -20.
\end{cases}
\]
Moreover, according to the theorem 1, Eq.(24) and Eq.(25),
\[
u(s) = \frac{K_p s^2 + 0.0549 K_p^2 + K_p}{s^2 + 0.0549 K_p^2 + K_p} \quad \begin{cases}
a = 0.0025 \\
b = -0.00027
\end{cases}
\]
\[
e(s) = \frac{s^2 + 0.0549 K_p s + 0.945(0.0549 K_p^2 + K_p)}{s^2 + 0.0549 K_p^2 + K_p}
\]
(40) Then, the residual used to detect actuator and sensor faults is obtained
\[
r_2 = 0.0025\nu(s) - 0.00027e(s) \quad \text{(The pitch channel)}
\]
(41) Following the similar steps, we also have the corresponding parameters in roll/yaw channel.

With residuals known, simulation results are represented.

### (1) Performance evaluation of the proposed fault detection method

In this section, the performance of the proposed fault detection scheme is evaluated in pitch direction and in roll/yaw direction respectively.

#### (a) The pitch direction

For the purpose of simulating the actuator and sensor faults in pitch direction, three scenarios are considered:

(a) The sensor saturation fault happens
\[ y_2 = \begin{cases} y_{2,\text{normal}} & t < t_f \\ y_{2,\text{max}} & t \geq t_f \end{cases} \quad (42) \]

where \( y_{2,\text{normal}} \) is the sensor output when it is in the normal operating condition. \( y_{2,\text{max}} \) is the maximal output of the sensor. \( t_f \) is the time of the fault happens. The control torque, the measured pitch angle, the residual obtained by the proposed method and the residual evaluation function are shown in (a) of Fig. 2. In Fig. 2, the green line represents the result in the faulty condition and the blue dotted line denotes that in the normal condition, which are similar to that in the following figures. Paying attention to the residual evaluating function \( J \), you can find that the fault is very easily detected.

\[ u_2 = \begin{cases} u_{2,\text{normal}} & t < t_f \\ D & t = t_f \\ 0 & t > t_f \end{cases} \quad (44) \]

where \( u_{2,\text{normal}} \) is the actuator output when it is in the normal operating condition. \( D \) is a very large value. The simulation results are shown in (c) of Fig. 2.

(2) The roll/yaw direction

Because of the coupled relationship, the fault detections in roll and yaw directions are considered together and the following four scenarios are considered:

(a) The sensor saturation fault happens in roll direction
(b) The sensor locking fault happens in yaw direction
(c) The actuator stalling fault happens in roll direction

The simulation results are shown in Fig.3.

(2) Evaluation of the consistency relationship between \( r(s) \) and \( y(s) - \hat{y}(s) \)

According to Eq.(6) in Theorem 1, it is known that using the proposed fault detection scheme, we can obtain the residual \( r(s) \) which is equal to \( y(s) - \hat{y}(s) \) theoretically, therefore in this section, the following four scenarios in pitch direction are used to judge if the fact \( r(s) = y(s) - \hat{y}(s) \) exists:

(a) The sensor saturation fault happens in roll direction
(b) The sensor locking fault happens in yaw direction
(c) The actuator stalling fault happens in roll direction
(d) The actuator stalling fault happens in yaw direction

The simulation results are shown in Fig.4.

5. CONCLUSION

In this paper, to meet the satellite requirements, and with the aid of the coprime factorization and Youla parameterization, we have proposed a simple fault detection scheme to get the residuals in the format that only related to the plant input and the control error, which saves some computing expense. It is concluded that the proposed fault detection method is very efficient from the simulation results above. Our future work will be dedicated to the extension of this study to the feedback control systems with model uncertainties.

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Fig. 3 The simulation results in roll/yaw direction

Fig. 4 The relationship between \( r(\epsilon) \) and \( y(k) - \hat{y}(k) \)