HYBRID UNKNOWN INPUT OBSERVER FOR ACTUATOR FAULT DETECTION AND COMPENSATION

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Abstract: The problem of unknown input estimation and compensation is studied for actuated nonlinear systems with noisy measurements. The proposed solution is based on high order sliding-mode differentiation and discrete-time optimization technique. Accuracy of the proposed hybrid estimation scheme is evaluated and stability of the compensating mechanism is established. It is shown that the fault detection delay as well as the smallest detectable fault magnitude can be estimated. The proposed procedure is applied to the problem of actuators fault detection and compensation under feedback control in an aeronautical application.

1. INTRODUCTION

Fast and efficient Fault Detection and Isolation (FDI) techniques are an increasingly important issue for many industrial systems operation in order to attain a high degree of availability and safety. For example, failure to detect, in an early stage, certain aircraft system failures has been cited as a contributing factor in several accidents (Palmer and Abbott, 1994). Many researchers have focused on development of the methodologies to detect and isolate faults (see Zhang and Jiang, 2003) for a recent survey), so that appropriate actions could be taken to accommodate their effects.

Industrial processes usually operate under feedback control, and any FDI algorithm must be applied to the controlled system. In such situations, faults in the open loop system may be covered by control actions which makes the problem of online detection and isolation more difficult. However, in the literature, design of FDI systems is frequently considered from an open-loop point of view, even if the resulting FDI unit is supposed to supervise the plant under closed-loop feedback configuration. If, for example, an observer based scheme is used for FDI design (Blanke, et al., 1997; Blanke, et al., 2003; Chen and Patton, 1999), then the control and the output signals directly influence the FDI algorithm, desensitizing the residual signals and deteriorating the FDI unit’s ability to detect incipient faults. Going back to the aircraft example, current automated flight monitoring systems do not indicate the presence of a failure until a parameter value has exceeded an alert limit (Boskovic and Mehra, 2002). However, the permissible time window in these systems can be quite narrow. More advanced model based FDI techniques, if they generate fault indicating signals based on output sensor measurements, would suffer the same drawback. The objective of this paper is to propose a design setup for FDI under feedback control with posterior fault compensation based on a paradigm differing from a pure observer design.

Let us consider the model of a nonlinear SISO system:

\[ y^{(k)}(t) = F(t, y, ..., y^{(k-1)}, u), t \geq 0, \quad (1) \]

where \( y \in \mathbb{R} \) is the system output, \( u \in \mathbb{R} \) is the system control, \( F: \mathbb{R}^{n+1} \rightarrow \mathbb{R} \) is locally Lipschitz continuous uniformly in \( t \) and continuously differentiable with respect to the argument \( u \). The nonlinear dependence of the system (1) on the input \( u \) takes into account the actuators presence, that is distinctive for industrial applications. It is assumed that

\[ \psi(t) = y(t) + \nu(t), \quad t \geq 0 \quad (2) \]

is the signal available for measurements, where \( \nu: \mathbb{R} \rightarrow [-v_m, v_m] \) is the bounded noise, \( 0 < v_m < +\infty \), and

\[ u(t) = u_0(t) + f(t), \quad t \geq 0 \quad (3) \]

is the control signal, where \( u_0: \mathbb{R} \rightarrow \mathbb{R} \) is the nominal part of the control (the part known for the FDI unit) and \( f: \mathbb{R} \rightarrow \mathbb{R} \) corresponds to the effect of faults on the control signal (off-nominal part). Thus, it is assumed that the faults affect directly the control through the actuator mechanism. In this case, being accordingly designed the control \( u_0 \) ensures certain level of insensitivity or robustness of the system output \( y \) with respect to the faults \( f \). Then, even in the presence of \( f(t) \neq 0 \) (for small amplitudes of \( f \)) the output of the system \( y \) may be close to some reference, that prevents an early stage fault detection from the output level. This is why in this work we will not consider the output observer based approaches (Blanke, et al., 1997; 2003; Chen and Patton, 1999).

Another approach to fault detection for (1)–(3) is based on input observers design. In this approach the input observer is a dynamical system with an output that converges asymptotically or in finite-time to \( f(t) \). There exists many solutions in this area based on sliding-mode (Bejarano, et al., 2009; Ferreira, et al., 2007; Floquet, et al., 2007; Fridman, et al., 2008; Saif and Xiong, 2003; Tan and Edwards, 2003; Yan and Edwards, 2007) linear systems techniques (Aldeen and Sharma, 2008; Chen et al., 2000; Hou and Patton, 1998; Xiong and Saif, 2003) or adaptive observer approach (Wang and Daley, 1996). The advantages and popularity of the above mentioned sliding-mode approaches are due to certain level of robustness against measurement corruption, excellent scalarability and finite time of convergence. Unfortunately, for the best of the authors knowledge, all existing input observers are devoted to the input estimation under assumption...
that the input enters linearly in the system equations (see the references above). However, the system (1) has nonlinear dependence on \( u \) and, hence, on the fault signal \( f \), to be estimated, that is originated by actuators presence in (1)–(3). Thus, the main goal of this paper is to present an approach for the input estimation in completely nonlinear case under measurement noise presence.

The proposed solution for the input estimation is based on High-Order Sliding Mode (HOSM) differentiation (Levant, 1998; 2003) with the complementary discrete-time system, that resolves on-line the nonlinear equation (1) providing an estimate of the fault signal \( f \). In this case the HOSM differentiator and the system (1)–(3) constitute “continuous-time” subsystem and the equation (1) resolving system is the “discrete” one. Operation of the obtained hybrid system is presented in section 5.

2. PRELIMINARIES

Euclidean norm for a vector \( \mathbf{x} \in \mathbb{R}^n \) will be denoted as \( ||\mathbf{x}|| \), and for a measurable and locally essentially bounded input \( u : \mathcal{R}_+ \rightarrow \mathcal{R} \) \( (\mathcal{R}_+ = \{\tau \in \mathcal{R} : \tau \geq 0\}) \) the symbol \( ||u||_{[t_0,T]}\) denotes \( L_{\infty} \) norm, if \( T = +\infty \) then we will simply write \( ||u|| \). We will denote as \( L_{\infty} \) the set of all such Lebesgue measurable inputs \( u \) with property \( ||u|| < +\infty \). Further we will assume that the inputs \( u_0, v \) and \( f \) are from \( L_{\infty} \). Denote the sequence \( 1,...,k \) as \( \overline{1,k} \).

Taking measurements (2) of the \( n \)-th times differentiable variable \( y(t) \) its derivatives can be estimated by HOSM differentiator (Levant, 2003):

\[
\begin{align*}
\dot{z}_0 &= v_0, \quad v_0 = -\lambda_0 | z_0 - \psi(t) |^{(n+1)} \text{sign}(z_0 - \psi(t)) + z_1; \\
\dot{z}_i &= v_i, \quad i = 1,n-1, \\
v_i &= -\lambda_i | z_i - v_{i-1} |^{(n+i-1)} \text{sign}(z_i - v_{i-1}) + z_{i+1}; \\
\dot{z}_n &= -\lambda_n \text{sign}(z_n - v_{n-1}),
\end{align*}
\]

(4)

where \( \lambda_i, \quad k = 0,n \) are positive parameters to be tuned. Note that for the case \( n = 1 \) the HOSM algorithm (4) can be reduced to the well known super-twisting differentiator (Levant, 1998):

\[
\begin{align*}
\dot{z}_0 &= -\lambda_0 | z_0 - \psi(t) | \text{sign}(z_0 - \psi(t)) + z_1, \\
\dot{z}_1 &= -\lambda_1 \text{sign}(z_1 - \psi(t)). 
\end{align*}
\]

Theorem 1 (Levant, 2003). Let \( y : \mathcal{R}_+ \rightarrow \mathcal{R} \) be \( n \)-th times continuously differentiable and \( v \in L_{\infty} \) in (2), then there exist \( 0 \leq T < +\infty \) and some constants \( \mu_k > 0, \quad k = 0,n \) (depending on \( \lambda_k \), \( k = 0,n \) only) such that in (4) for all \( t \geq T \):

\[ |z_k(t) - y^{(k)}(t)| \leq \mu_k ||v||^{(n-k)(n+1)}, \quad k = 0,n. \]

In particular, this result means that if \( v(t) \equiv 0 \) for all \( t \geq 0 \), then the differentiators (4) or (5) ensure exact derivatives estimation in a finite time. Application of HOSM differentiators for input estimation and compensation for linear systems has been studied in (Ferreira, et al., 2007). Development of this theory to nonlinear case is studied below.

3. ACTUATED INPUT ESTIMATION

Let us start with an estimation algorithm design for the fault signal \( f \) observation.

Assumption 1. Let \( (y(t),...,y^{(n-1)}(t)) \in Y \subset \mathbb{R}^n \) for all \( t \geq 0 \).

In industrial applications in fault-free mode and for the properly designed control \( u_0 \) this is a mild assumption and, typically, the set \( Y \) is known and predefined during the design phase. According to theorem 1, for the system (1)–(4) there exists \( 0 \leq T < +\infty \) such that \( y^{(k)}(t) = z_k(t) + e_k(t) \) and \( |e_k(t)| \leq \mu_k ||v||^{(n-k)(n+1)} \) for all \( t \geq T \) with some \( \mu_k > 0, \quad k = 0,n \) uniformly in \( v \). Then the system (1) can be presented as follows

\[
z_0(t) + e_n(t) = F(t,\psi(t),z_1(t) + e_1(t),...,z_{n-1}(t) + e_{n-1}(t),u(t),t), \quad t \geq 0. \quad (6)
\]

Let \( Y_t \subset \mathbb{R}^n \) be the neighborhood of the set \( Y \) \( (Y \subset Y_t) \) such that if \( ||z_k - y^{(k)}|| \leq \mu_k ||v||^{(n-k)(n+1)}, \quad k = 0,n \) and \( (y^{(0)},...,y^{(n-1)}) \in Y_t \) then necessarily \( (z_0,z_1,...,z_{n-1}) \in Y_t \). Since the function \( F \) is locally Lipschitz continuous then for all \( (\psi,z_1,...,z_{n-1}) \in Y_t \), there exists \( L > 0 \) such that

\[
|F(t,\psi,z_1 + e_1,...,z_{n-1} + e_{n-1}(t),u(t)) - F(t,\psi,z_1,...,z_{n-1},u(t))| \leq L \sum_{i=1}^{n-1} |e_i(t)|. 
\]

According to theorem 1 we have that

\[
\sum_{i=1}^{n-1} |e_i(t)| \leq \sum_{i=1}^{n-1} \mu_i ||v||^{(n-i)(n+1)} \quad \text{for all} \quad t \geq T. \quad \text{Therefore, from (6) we can define the augmented error}
\]

\[
\delta(t) = z_n(t) - F(t,\psi(t),z_1(t),...,z_{n-1}(t),u_0(t) + f(t)) \quad (7)
\]

with the property

\[
|\delta(t)| \leq \rho(||v||) \quad \text{for all} \quad t \geq T, \\
\rho(s) = \mu_n s^{(n+1)} + L \sum_{i=1}^{n} \mu_i s^{(n-i)(n+1)}.
\]

All variables in the right hand side of (7) are available for measurements except the fault signal \( f(t) \). In the left hand side of (7) we have the augmented error \( \delta \), that is not measurable and it is proportional to the measurement noise \( v \) amplitude (this error becomes zero in the finite time \( T \) for the case of exact measurements). Let \( f(t) \) be a solution of the
equation (7) for the case $\delta(t) = 0$, i.e.
$$w = F(t, \psi(t), z_1(t), \ldots, z_{n-1}(t), u_0(t) + \tilde{f}(t)),$$
then substituting (8) in (7) we get
$$\delta(t) = F(t, \psi(t), z_1(t), \ldots, z_{n-1}(t), u_0(t) + \tilde{f}(t)) - F(t, \psi(t), z_1(t), \ldots, z_{n-1}(t), u_0(t) + f(t)).$$
Define the gradient of the function $F$ with respect to $u$:
$$\nabla_u F(t, y, \ldots, y^{(n-1)}, u) = \partial F(t, y, \ldots, y^{(n-1)}, u)/\partial u,$$
then by the Mean value theorem there exists a function $c : R_+ \to [0,1]$ such that for all $t \geq 0$
$$\delta(t) = g(t)[\tilde{f}(t) - f(t)],$$
$$g(t) = \nabla_u F(t, \psi(t), z_1(t), \ldots, z_{n-1}(t), u_0(t) + [1-c(t)][\tilde{f}(t) + c(t) f(t)].$$
Assumption 2. Let
$$\int_0^1 g(t) |\tilde{f}(t) - f(t)| \, dt \geq g_{min}^{1/p} |\tilde{f}(t) - f(t)|^p$$
for all $t \geq T$ and some $g_{min} > 0$, $0 < p < +\infty$. \square

Roughly speaking, the function $g : R_+ \to R$ norm has strictly separated from zero average value for all $t \geq T$. This property also can be considered as a variant of the well known in the estimation theory persistency of excitation condition (Iannou and Sun, 1996; Narendra and Annaswamy, 1989). Then under assumption 2 from (9) for all $t \geq T$
$$\rho(||v||) \geq \int_0^1 \delta(t) |dt - \tilde{g}(t) - f(t)| \, dt \geq g_{min}^{1/p} |\tilde{f}(t) - f(t)|^p$$
and finally,
$$|\tilde{f}(t) - f(t)| \leq [g_{min}^{1-p} \rho(||v||)]^{1/p},$$
that implies boundedness of the discrepancy $\tilde{f}(t) - f(t)$ for all $t \geq T$. Consequently, under assumption 2 the problem of fault detection and isolation can be handled finding a solution $\tilde{f}$ of the equation (8), the penalty for such FDI problem reduction is proportional to $||v||$ ($\rho(0) = 0$).

The equation (8) is a nonlinear one, for each $t \geq 0$ it may have a single solution $\tilde{f}(t)$ or, in general case, $\tilde{f}(t) \in S$, where for all elements $s \in S$ the equation $\tilde{w}(t) = F(t, \psi(t), z_1(t), \ldots, z_{n-1}(t), u_0(t) + s)$ holds. It could be the case that for some $t \geq 0$ this equation has no solutions with respect to $\tilde{f}(t)$. Thus some regularizing conditions have to be imposed.

Assumption 3. Let $\nabla_u F(t, z_0, z_1, \ldots, z_{n-1}, u) \neq 0$ for all $(z_0, z_1, \ldots, z_{n-1}) \in Y_0$, $u \in R$ and $t \geq 0$. \square

This assumption states that the gradient of the function $F$ with respect to the last argument $u$ in the domain $Y$ (specified by assumption 1) and onto its neighborhood proportional to $||v||$ is restricted from zero. Under these restrictions the equation (8) has the single solution $\tilde{f}(t)$. Then a gradient descent method (Deuflhard, 2004) can be applied to find an estimate $\hat{f}(t)$ on $\tilde{f}(t)$ for $t \geq 0$:
$$d\hat{f}(t)/dt = \gamma \varphi(\theta \tilde{f}(t)) \cdot (10)$$
$$\varphi(s) > 0 \text{ for all } s \neq 0, \|v\| < +\infty,$$
$$\theta \tilde{f}(t) = F(t, \psi(t), z_1(t), \ldots, z_{n-1}(t), u_0(t) + f(t))\nabla_u F(t, \psi(t), z_1(t), \ldots, z_{n-1}(t), u_0(t) + f(t)),$$
where $\gamma > 0$ is a design parameter and $t \geq 0$ is an independent time. For each fixed $t \geq 0$ the execution of (10) in the time $t$ ensures convergence of $\tilde{f}(t)$ to $\tilde{f}(t)$ (more precisely this claim will be formulated later).

The singular perturbation theory \textit{cannot} be used to unite the systems (1)-(4) and (10) written in different time scales (this theory requires some kind of continuity and differentiability of the system equations (Khalil, 2002), that is clearly not the case for the system (4)). Another approach consists in the hybrid systems techniques application.

The first idea consists in discretization of (10), when the estimate $\tilde{f}(t_k)$ is generated discretely for some sequence of strictly increasing sample instants $t_k$, $k \geq 0$ ($t_0 = 0$) having accumulation point at infinity only. Then the discrete representation of (10) can be written as follows for any $k \geq 0$:

$$\theta_0 = \tilde{f}(t_k), \quad \tilde{f}(t_k) = \tilde{f}_0; \quad \theta_{t+1} = \theta_t + \gamma \varphi(\theta_{t+1}, \theta_t),$$

where $\gamma > 0$, $N > 0$ and $\tilde{f}_0 \in R$ are design parameters. The operation of (11) can be expressed as follows, at each sampling time $t_k$ the algorithm takes the initial value $\theta_0 = \tilde{f}(t_k)$ (or some guess $\theta_0 = \tilde{f}_0$ on the first step $k = 0$), then $N$ steps of the discrete minimization procedure (10) are computed, the output of the algorithm (11) is $\tilde{f}(t_{k+1}) = \theta_N$. The number $N$ is bounded by available computational power for (11) realization. The system (11) period or the shift between the sample instants $t_{k+1} - t_k$, $k \geq 0$ depends on the time that is required to perform $N$ steps of (11) and the fault detection minimum time specifications.

Theorem 2. Let assumptions 1-3 hold, then in the system (1)-(4), (11) for any $\varphi^* > 0$ there exist $\gamma^* > 0$ and $N^* \geq 0$ such that for any $k > 0$ with $t_k \geq T$ (where $T \geq 0$ is the time of the derivatives estimation from theorem 1)
$$|\tilde{f}(t_{k+1}) - \tilde{f}(t_k)| \leq \varphi^* [g_{min}^{-1} \rho(||v||)]^{1/p}$$
provided that $0 < \gamma < \gamma^*$, $N \geq N^*$ for any initial conditions, $v \in L_\infty$ and continuous $f \in L_\infty$. \quad \blacksquare

All proofs are excluded due to space limitations. The result of theorem 2 claims that for any desired accuracy $\varphi^* > 0$ there exists some maximum adaptation rate $\gamma^* > 0$ and maximum number of steps $N^* \geq 0$ such that the fault value $f(t_k)$,
\( t_k \geq T \) for all such \( k \geq 0 \) is estimated by the algorithm (11) output \( \hat{f}(t_{k+1}) \) with the worst case accuracy 
\[
\left[ g_{\min}^{-1}(\|v\|)^{|1/p + \varepsilon^*}} \right]^{1/p + \varepsilon^*}. \]
In the absence of the measurement noise \( v \) the accuracy \( \varepsilon^* \) is achievable. The theorem does not restrict the sampling rate in the system (the delay \( t_{k+1} - t_k \), \( k \geq 0 \) can be chosen in accordance with computational constraints). There exist a casual time shift in the algorithm response \( (\hat{f}(t_{k+1}) \rightarrow f(t_k)) \) due to calculations in (11) performed on the interval \([t_k, t_{k+1})\), the estimate on the value \( f(t_k) \) is always obtained on the next step \( t_{k+1} \) only.

In particular, for FDI purposes, if \( 0 < t_{k+1} - t_k \leq T_0 \) \( (T_0 > 0) \) is the maximal sample time of the algorithm (11) operation), then theorem 2 guarantees that for time instants \( t_k \geq T \), \( k \geq 0 \) the signal \( \hat{f}(t_k) \) detects all faults with amplitudes bigger than \( \left[ g_{\min}^{-1}(\|v\|)^{|1/p + \varepsilon^*}} \right]^{1/p + \varepsilon^*} \) after period \( T_0 \) (in other words, \( T + T_0 \) is the fault detection time and \( \left[ g_{\min}^{-1}(\|v\|)^{|1/p + \varepsilon^*}} \right]^{1/p + \varepsilon^*} \) represents the amplitude of the smallest detectable fault.

Remark 1. The time \( T \) from theorem 1 can be estimated \( a \) \( \text{priori} \) (analytically using results of the works (Levant, 1998; 2003)) or in real time verifying the conditions
\[
 z_0(t) = \psi(t), \ z_i(t) = v_{i-1}(t), i = \overline{1,n},
\]
that have to hold for all \( t \geq T \) for the system (4). For the system (5) the conditions above are reduced to \( z_0(t) = \psi(t), t \geq T, \) that is easy to check.

The second idea for (10) realization is that we can use the continuous optimization algorithm (10) in the same time \( t \), but for frozen on the intervals \([t_k, t_{k+1})\), \( k \geq 0 \) values of the incoming variables in the right hand side of (10):
\[
 \hat{f}(t) = \gamma \Phi(k[t, \hat{f}(t)]), \ t \in [t_k, t_{k+1}), k \geq 0; \ \hat{f}(0) = \hat{f}_0(0), \]
where \( \gamma > 0 \), \( \hat{f}_0 \in R \) are design parameters. The system (12) is also a “discrete” one, since its right hand side is updated discretely at time instants \( t_k \), \( k \geq 0 \). Such “discrete” approximation of the algorithm (10) has to ensure the same property as the pure discrete version (11), i.e. \( f(t_{k+1}) \rightarrow f(t_k) \).

Theorem 3. Let assumptions 1–3 hold and \( t_{k+1} - t_k \geq \tau_0 > 0 \) for all \( k \geq 0 \), then in the system (1)–(4), (12) for any \( \varepsilon^* > 0 \) there exists \( \psi^* > 0 \) such that for any \( k > 0 \) with \( t_k \geq T \) (where \( T \geq 0 \) is the time of the derivatives estimation from theorem 1)
\[
|\hat{f}(t_{k+1}) - f(t_k)| \leq |\varepsilon^* + \left[ g_{\min}^{-1}(\|v\|)^{|1/p + \varepsilon^*}} \right]^{1/p}|
\]
provided that \( \gamma > \psi^* \) for any initial conditions, \( v \in \mathbb{L}_\infty \) and continuous \( f \in \mathbb{L}_\infty \).

The results of theorems 2 and 3 are the same except the algorithm (12) provides a continuous estimate \( \hat{f}(t) \) of the fault signal \( f(t) \). The continuity of (12) becomes important in the case when the variable \( \hat{f} \) is used for compensation. Substitution of the discretely updated variable \( \hat{f} \) in (1) in order to attenuate the presence of \( f \) makes the signal \( \hat{y}^{(n)} \) in (1) discontinuous, that prevents application of differentiators (4) or (5) (see theorem 1 conditions). Consequently, theorem 2 and the algorithm (11) are more suitable for pure estimation, in this case discrete optimization scheme (11) directly describes the system operation. The algorithm (12) and theorem 3 are used below in the compensation part.

4. INPUT COMPENSATION

For this purpose the control (3) takes the form
\[
u(t) = u_0(t) + f(t) - \hat{f}(t), t \geq 0\]
where \( u_0 \) and \( f \) are as before, \( \hat{f} \) is the fault compensation signal generated by the following modified version of the algorithm (12):
\[
\hat{f}(t) = \gamma \text{Proj}_{[-F,F]}(\Phi(k[t, \hat{f}(t)])), \]
where \( \gamma > 0 \), \( \text{Proj}_{[-F,F]} \) ensures the values \( \hat{f}(t) \) projection onto the interval \([-F,F], F > 0 \). In this case the system (1), (2), (3), the differentiator (4) and the estimation algorithm (12), (13) are feedback interconnected. Therefore, assumption 1 is in general not longer valid and we cannot analyze stability of the algorithms (11) or (12) separately as we did for estimation. To take into account dynamics of all coupled subsystems we need new assumptions.

Theorem 4. Let

A. The control \( u_0 \) for any \( f \in \mathbb{L}_\infty, \|f\| < \Phi \), \( 0 < \Phi < +\infty \) ensures for the system (1), (3) that
\[
(y(t),...,y^{(n-1)}(t)) \in Y \subset R^n \text{ for all } t \geq 0, \text{ and for all } (y(0),...,y^{(n-1)}(0)) \in Y \text{ there exists } \Psi > 0 \text{ such that } \|y(t)\| \leq \phi(\Psi,t) + \sigma(\|f\|), \in \mathbb{K}, \sigma \in \mathbb{K}.
\]
B. \( \nabla_y F(t, z_0, z_1, ..., z_{n-1}, u) \neq 0 \) for all \( (z_0, z_1, ..., z_{n-1}) \in Y_t, u \in R \) and \( t \geq 0 \), where the set \( Y_t \) is defined with respect to the set \( Y \) as previously.
C. Assumption 2 is satisfied.
D. \( 0 < \tau_0 \leq t_{k+1} - t_k \leq T_0 \), \( k \geq 0 \) and \( \|f\| < \Phi / 2 \), \( \sup_{[t_0-t_0] \leq \tau_0} |f(t) - \hat{f}(t)| \leq \Delta, \Delta > 0 \) for all \( t \geq 0 \).

Then the system (1), (2), (3), (13) with \( (y(0),...,y^{(n-1)}(0)) \in Y \), \( v \in \mathbb{L}_\infty \) and the estimation algorithms (4), (14) have bounded solutions and for any \( \varepsilon^* > 0 \) there exist \( \psi^* > 0 \) and \( T_0 \geq T + T_0 \) (where \( T \geq 0 \) is the time of the derivatives esti-
motion from theorem 1) such that
\[ |y(t)| \leq \sigma(\varepsilon^* + \left[ \min_{t \geq t_0} \rho(||v||) \right]^{1/p} + \Delta), \quad t \geq t_0, \]
provided that \( \gamma > \gamma^* \), \( F = \Phi / 2 \).

The theorem conditions say that the control \( u_0 \) ensures robustness with respect to certain faults \( f \) and asymptotical decreasing to zero of \( y \) in the fault-free case.

5. SIMULATION RESULTS
Let us consider the problem of oscillatory failure detection and compensation in an aircraft (Alcorta-Garcia, et al., 2011; Goupil, 2010). Some failures of the electronic flight control system may result in an unwanted control surface oscillation, generating unacceptably high loads on the carrier structure. Usual monitoring techniques cannot always guarantee safeguarding with acceptable robustness, thus, a specific OFC detection algorithm has to be designed (Goupil, 2010). The following nonlinear model of the hydraulic actuator is considered (Alcorta-Garcia, et al., 2011; Goupil, 2010):
\[ \dot{y} = k_i (u - y) \sqrt{k_2 / \{k_3 + k_4 (u - y)^2\}}, \quad (15) \]
where \( y \in R \) is the actuator position, \( u \in R \) is the actuator input, \( k_i, \ i = 1, 4 \) are some known positive parameters. Assuming that (2) and (3) hold it is required to design an observer of the faulty signal \( f \). In (Alcorta-Garcia, et al., 2011; Goupil, 2010) the case of harmonic signal \( f \) is considered. The time of fault detection should be proportional to its period and, thus, it depends on the signal \( f \) frequency. The approaches of (Alcorta-Garcia, et al., 2011; Goupil, 2010) are based on observer or Kalman filter fault detection.

To apply the proposed approach, note that the system (15) is in the form (1), \( n = 1 \) and the super-twisting differentiator (5) provides the derivative \( \dot{y} \) estimation with an accuracy proportional to \( v \). In (11)
\[ \dot{\beta} (t, f) = k_1 \sqrt{k_2 k_3} \left\{ z_1(t) - k_4 (u_0 + f - \Psi) \right\} \times \sqrt{k_2 / \{k_3 + k_4 (u_0 + f - \Psi)^2\}^2} \} \right] \right]^{1/5} \]
and during simulation \( \varphi(\beta) = \text{sat}(\beta) \). In this example
\[ \nabla_y F(t, y, u) = k_1 \sqrt{k_2 k_3} \left\{ k_4 (u - y)^2 + k_3 \right\}^{1/5} \]
and assumption 3 is satisfied for any \( y \in R, u \in R \).

Next consider the problem of the faults \( f_1 \) and \( f_2 \) compensation. For this purpose we take \( \Phi = 1 \) and \( \gamma = 5 \) in (14). The results of simulation for the faults \( f_1 \) and \( f_2 \) are shown in figures 3 and 4 correspondingly (the error \( \varepsilon_1 \) denotes the output deviation without compensation and the symbol \( \varepsilon_2 \) states for the output error with compensation).

6. CONCLUSION
The problem of input observer design for nonlinear actuated systems is solved. The obtained hybrid estimation algorithm has two parts: the continuous-time HOSM differentiator and the discrete-time numerical optimization algorithm. Accuracy of the estimates is established as a function of the measurement noise. Applicability conditions for the fault compensation scheme are established. The proposed estimator is applied to the problem of oscillatory failure detection and compensation for an airplane. The results of the simulation confirm robustness and performance of the proposed solutions.
Fig. 3. Results of compensation for $f(t) = f_1(t)$.

Fig. 4. Results of compensation for $f(t) = f_2(t)$.

For FDI problem a key feature of the presented approach is that, the finite time of detection and the smallest detectable fault amplitude can be evaluated as a function of the measurement noise.

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