Output Feedback Adaptive Robust Learning Control of a Class of Nonlinear Systems with Periodic Disturbances *

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Abstract: In this paper, a discontinuous projection-based output feedback adaptive robust learning control (OARLC) scheme is constructed for a class of nonlinear systems in a semi-strict feedback form by incorporating an observer and a dynamic normalization signal. Since only output signal is available for measurement, an observer is firstly designed to provide exponentially convergent estimates of the unmeasurable states. Using certain known basis functions to capture the characteristics of unknown general periodic disturbances, the discontinuous projection type adaptation law can then be used to tune the amplitudes of those basis functions on-line to recover the unknown general periodic disturbances asymptotically. The estimation errors due to the unknown initial states, uncompensated disturbances, and the uncertain nonlinearities are also effectively dealt with via certain robust feedback at each step of the proposed OARLC backstepping design. The resulting controller achieves a guaranteed transient and a prescribed final tracking accuracy for output tracking performance. In addition, when the general periodic disturbances fall within the approximation ranges of the periodic basis functions, asymptotic output tracking performance is achieved as well.

1. INTRODUCTION

Nonlinear systems are always subjected to various uncertainties and disturbances which affect the performances of controlled systems. Especially, handling the uncertainties and disturbances which can be linearly parameterized in form of unknown constant parameter vectors with vectors of known functions has attracted lots of attentions in Pomet et al. [1992], Krstic et al. [1995], Jiang et al. [1998]. Adaptive control is the major scheme used to deal with such uncertainties and disturbances. In general, adaptive control uses certain parameter adaptation laws and certainty equivalence principle based control law designs to gradually eliminate the impact of unknown constant parameters–asymptotic output tracking can be achieved even in the presence of unknown system parameters.

To the system with unknown frequency periodic disturbances, internal model based adaptive control schemes are effective way to compensate these uncertainties in Ding [2003], Lauuda et al. [2005]. However, unknown parameters in practical systems may be presented in time-variant periodic format with known period in Yao et al. [2001], Sun et al. [2006]. Repetitive learning control schemes have been traditionally developed to deal with such uncertainties, which do not require exact knowledge of the dynamic model of the periodic signals in Yao et al. [2001], Sun et al. [2006]. The basic idea of repetitive controls is to improve the tracking performance from one cycle to the next by adjusting the input based on the error signals between the desired motion and actual motion of the system during previous cycles. Recently, Xu et al. [2000] proposed a robust control scheme. However, transient performance of this controller is unknown and the actual system may have large tracking errors during the initial transient period or have a sluggish response. In addition, it is shown in Yao et al. [2001] that, in continuous time domain, what traditional repetitive learning algorithms do is equivalent to adapting an infinite number of parameters. Such an endeavor not only causes very high dimensions of learning algorithms, which may need huge memory in
Consider a class of single-input-single-output uncertain nonlinear systems described by
\[
\dot{x}_i = x_{i+1} + \omega_i(y, t) + \Delta_i(y, \omega(y, t), u, t), \quad 1 \leq i \leq n-1,
\]
\[
\dot{\Delta}_n = u + \omega_n(y, t) + \Delta_n(y, \omega(y, t), u, t),
\]
\[
y = x_1
\]
where \(x = [x_1, \ldots, x_n]^T \in \mathbb{R}^n\) is the system state, \(u \in \mathbb{R}\) and \(y \in \mathbb{R}\) are the control input and the measured output, respectively. Only output \(y\), i.e., \(x_1\), is assumed measured. \(\omega_i(y, t), i = 1, \ldots, n\), are unknown general periodic disturbances with respect to time \(t\) which could be output dependent as well, i.e., \(\omega_i(y, t) = \omega_i(y, t - T)\), where \(T\) is a known period, \(\Delta_i(y, \omega(y, t), u, t)\) with \(\omega(y, t) = [\omega_1(y, t), \ldots, \omega_n(y, t)]^T\) are lumped unknown nonlinear functions.

It is worth pointing out that, as opposed to the matched unknown sinusoidal disturbances assumed in Nikiforov [2001], Marino et al. [2003], the uncertain nonlinearities \(\Delta_i(y, \omega(y, t), u, t)\) and the unknown general disturbances \(\omega_i(y, t)\) in (1) are unmatched.

**Assumption 2.1.** The unknown general periodic disturbances \(\omega_i(y, t)\) satisfy
\[
\omega_i(y, t) = \phi_i(y)\mu(t)
\]
where \(\phi_i(y)\) are known smooth vector functions of output \(y\) and \(\mu(t)\) represents unknown but sufficiently smooth time-varying periodic disturbance, i.e., \(\mu(t) = \mu(t - T)\).

The periodic disturbance \(\mu(t)\) can be expressed as follows in Yao et al. [2001], Körner [1988], Liuozuo et al. [2007]:
\[
\mu(t) = \theta^T\varphi(t) + \varepsilon_{\mu}(t)
\]
where \(\theta = [\theta_1, \ldots, \theta_{2p+1}]^T = [A_0/2, A_1, B_1, \ldots, A_p, B_p]\) and \(\varphi(t)\) are the vectors of unknown amplitudes and the known basis functions of the first \(p\) frequencies of the Fourier Series expansion of \(\mu(t)\) respectively; \(\varphi(t)\) is given by
\[
\varphi(t) = [1, \cos(2\pi t/T), \sin(2\pi t/T), \ldots, \cos(2\pi pt/T), \sin(2\pi pt/T)]^T
\]
\(\varepsilon_{\mu}(t)\) represents the truncation error satisfying \(|\varepsilon_{\mu}(t)| \leq \varepsilon_p\) where \(\varepsilon_p\) is a known positive number. When \(\mu(t) \in C^{N-1}\) with \(N > 2p + 1\), \(\varepsilon_p\) can be chosen as in Körner [1988]
\[
\varepsilon_p = B \frac{2^{2N-4}}{N-2} \sqrt{\pi} N > 2
\]
with \(B = 2\sqrt{2}/(2\pi)^{N-1} \sup_{0 \leq t \leq T} |\mu^{(N-1)}(t)|\).

**Remark 2.1.** By choosing \(p\) large enough, the structured approximation part of \(\mu(t)\), \(\theta^T\varphi(t)\), can be very close to \(\mu(t)\), and a very small \(\varepsilon_p\) can be chosen. It is also noted that, unlike in Liuozuo et al. [2007], the expansion form (3) automatically takes care of the unknown phases of sinusoidal terms and is valid for all types of periodic disturbances.

**Assumption 2.2.** The extent of parametric uncertainties \(\theta\) in (3) and uncertain nonlinearities \(\Delta_i(y, \omega(y, t), u, t)\) in (1) are known, i.e., \(\forall i,\)
\[ \theta \in \Omega_{\theta} \triangleq \{ \theta : \theta_{\min} \leq \theta \leq \theta_{\max} \}, \]
\[ \Delta_i \in \Omega_{\Delta_i} \triangleq \{ \Delta_i : |\Delta_i(y, \omega(y, t), u, t)| \leq \delta_i(y) + \sigma_i(\varphi(t)) \} \]

where \( \delta_i(\cdot) \), \( \sigma_i(\cdot) \) are known functions and \( \theta_{\min} \triangleq [\theta_{1, \min}, \cdots, \theta_{2p+1, \min}] \), \( \theta_{\max} \triangleq [\theta_{1, \max}, \cdots, \theta_{2p+1, \max}] \). \( \cdot \) denotes the Euclidean norm.

Let \( y_d \) be the known desired output trajectory, which is bounded with derivatives up to \( n \) th orders. The control objective is to synthesize a control input \( u \) to make the output \( y \) track \( y_d \) with a prescribed accuracy in spite of various uncertainties. In addition, when \( \varepsilon_{\mu}(t) = 0 \) and \( \Delta_i(y, \omega(y, t), u, t) = 0 \), asymptotic output tracking should be achieved.

### 3. STATE ESTIMATION

The purpose of this section is to introduce an observer to provide the exponential convergent estimate of the unmeasurable states, \( x_i, i = 2, \cdots, n \), since the output is the only signal available for measurement. 

#### 3.1 State Estimation

The system model (1) can be rewritten as
\[
\dot{x} = A_0x + e_nu + ky + (\phi(y)\mu(t)) + \Delta(y, \omega(y, t), u, t) \\
y = Cx
\]

where \( e_n = [0, \cdots, 0, 1]^T \), \( k = [k_1, \cdots, k_n]^T \), \( \phi(y) = [\phi_1(y), \cdots, \phi_n(y)]^T \), \( \Delta(\cdot) \triangleq [\Delta_1(\cdot), \cdots, \Delta_n(\cdot)]^T \), \( C = [1, 0, \cdots, 0] \), and
\[
A_0 = \begin{bmatrix}
-k_1 & 1 & 0 & \cdots & 0 \\
-k_2 & 0 & 1 & \cdots & 0 \\
& \vdots & \vdots & \ddots & \vdots \\
-k_{n-1} & 0 & 0 & \cdots & 1 \\
-k_n & 0 & 0 & \cdots & 0
\end{bmatrix}
\]

By suitably choosing a constant vector \( k \), the matrix \( A_0 \) can be made Hurwitz. Thus, there exists a symmetric positive definite matrix \( P \) and identity matrix \( I \) such that
\[ PA_0 + A_0^T P = -I \] (9)

Since \( \mu(t) \) is an unknown general periodic disturbance, the observer can not be built using the same structure as
\[
\hat{x} = A_0\hat{x} + e_nu + ky + (\phi(y)\mu(t))\]

Following the procedure in Krstic et al. [1995] to estimate the unmeasured states, we define the following filters
\[
\dot{\zeta} = A_0\zeta + ky, \\
\dot{\nu} = A_0\nu + e_nu, \\
\dot{\lambda}_i = A_0\lambda_i + (\phi(y)\varphi_i(t)), 1 \leq i \leq 2p + 1
\]

Then the state estimates can be expressed as follows
\[
\hat{x} = \zeta + \nu + \lambda \theta
\]

where \( \lambda = [\lambda_1, \cdots, \lambda_{2p+1}] \). Let \( \hat{x} = x - \hat{x} \) be the estimation error. From (7), (11) and (12), it can be verified the the observer error dynamics is given by
\[
\dot{\hat{x}} = A_0\hat{x} + \Delta(y, \omega(y, t), u, t)
\]

where \( \Delta(y, \omega(y, t), u, t) = \phi(y)\varepsilon_{\mu}(t) + \Delta(y, \omega(y, t), u, t) \).

Since \( A_0 \) is a Hurwitz matrix, the unperturbed system of (13) is exponentially stable, i.e., \( \Delta_i = 0 \) and \( \varepsilon_{\mu}(t) = 0 \).

Define a positive semi-definition (p.s.d.) function as
\[
V_\varphi(\hat{x}) = \hat{x}^T P_2 \hat{x}
\]

where \( \varphi(t) \) is the estimate of \( \omega(t) \).

It can be shown in Yao et al. [1996] that for any adaptation assumption
\[
\dot{\gamma}(\hat{x}) \leq \gamma(\hat{x}) + d, \\
\gamma(\hat{x}) \geq \gamma(x) \quad \text{and} \quad \gamma(\hat{x}) \geq \gamma(x), \quad \text{let} \quad r = \text{dynamic signal generated by}
\]

Both the case with \( \hat{x} = x \) and \( \hat{x} = \hat{x} \) are considered. It follows that
\[
V_{\hat{x}}(\hat{x}) \leq \gamma(x) + \gamma(\hat{x}) + d
\] (18)

For all \( t \geq 0 \) where the solutions are defined. It follows that
\[
|\hat{x}(t)| \leq \gamma^{-1}(2r(t)) + \rho, \rho = \gamma^{-1}(2D(t))
\] (19)

4. DISCONTINUOUS PROJECTION-BASED OARLC BACKSTEPPING DESIGN

#### 4.1 Parameter projection

In this paper, discontinuous projection type adaptation law in Yao [1997] will be used to estimate the unknown constant parameters in (3). Specifically, let \( \vartheta \) denote the estimate of \( \theta \) and \( \theta \) the estimation error (i.e. \( \vartheta = \theta - \theta \)). Under Assumption 2.2, the following discontinuous projection type adaptation law can be used
\[
\dot{\vartheta} = \text{Proj}_{\theta_{\max}}(\Gamma \tau)
\] (20)

where \( \Gamma > 0 \) is a diagonal matrix, \( \tau \) is an adaption function to be synthesized later. The projection mapping
\[
\text{Proj}_{\theta_{\max}}(\cdot) = [\text{Proj}_{\theta_1}(\cdot), \cdots, \text{Proj}_{\theta_{2p+1}}(\cdot)]^T
\]

is defined as
\[
\text{Proj}_{\theta_{\max}}(\cdot) = \begin{cases}
0 & \text{if } \hat{\theta}_i = \theta_{\max} \text{ and } \theta_i > 0 \\
0 & \text{if } \hat{\theta}_i = \theta_{\min} \text{ and } \theta_i < 0 \\
\theta_i & \text{otherwise}
\end{cases}
\] (21)

It can be shown in Yao et al. [1996] that for any adaption function \( \tau \), the projection mapping used in (21) guarantees
\[
P1 \quad \theta \in \Omega_{\theta} \triangleq \{ \theta : \theta_{\min} \leq \theta \leq \theta_{\max} \} \\
P2 \quad \Gamma \text{[} \Gamma^{-1} \text{Proj}_{\theta_{\max}}(\Gamma \tau) - \tau \text{]} \leq 0, \quad \forall \tau
\] (22)

1 See Jiang et al. [1998], Lemma 3.1
4.2 OARLC Controller design

The design combines the adaptive backstepping technique in Kristic et al. [1995] with the ARC design procedure in Yao [1997].

Step 1 For the control objective, we define output tracking error as \( z_1 = y - y_d \). In view of the first equation in (1) and (3), the derivative of \( z_1 \) is

\[
\dot{z}_1 = x_2 + \phi_1(y) [\theta^T \varphi(t) + \epsilon_\mu(t)] + \Delta_1(y, \omega(y(t), u(t), t) - y_d
\]

(23)

From (12), the unmeasurable state \( x_2 \) can be expressed as

\[
x_2 = \zeta_2 + \nu_2 + \lambda_2 \theta + \bar{x}_2
\]

(24)

where \( \lambda_2 = [\lambda_{12}, \ldots, \lambda_{2p+1,2}] \), since \( \theta \) is unknown.

In (23), by viewing \( \nu_2 = z_2 + \alpha_1 \) as a virtual control, we choose the desired control law as

\[
\alpha_1(y, \zeta_2, \lambda_2, \theta, r, t) = \alpha_{1a} + \alpha_{1s}, \quad \alpha_{1s} = \alpha_{1s1} + \alpha_{1s2}
\]

(25)

where \( \alpha_{1a} \) is the adjustable model compensation given by

\[
\alpha_{1a} = \alpha_{1a1} + \alpha_{1a2}, \quad \alpha_{1a1} = -k_{1a} \zeta_1 \geq g_1 + |C_0 \Gamma \hat{\phi}_1(y(t), t)|^2
\]

(27)

where \( g_1 > 0, C_0 > 0 \) is a constant diagonal matrix to be specified later and \( \hat{\phi}_1(y(t), t) \) is another robust performance control term satisfying

\[
(\dot{z}_1 = z_2 - k_{1a} z_1 + \alpha_{1a2} + \bar{x}_2 - \theta^T \phi_1(y, t) + \phi_1(y) \mu(y(t), u(t), t) + \Delta_1(y, \omega(y(t), u(t), t)) \leq \varepsilon_1 (1 + \rho^2),
\]

(28)

where \( \varepsilon_1 > 0 \) is designed according to desired performance. Noting (25), the derivative of \( z_1 \) is

\[
\dot{z}_1 = z_2 - k_{1a} z_1 + \alpha_{1a2} + \bar{x}_2 - \theta^T \phi_1(y, t) + \phi_1(y) \mu(y(t), u(t), t) + \Delta_1(y, \omega(y(t), u(t), t)) \leq \varepsilon_1 (1 + \rho^2)
\]

(29)

Let \( V_1 = \frac{1}{2} z_1^2 \), the derivative of \( V_1 \) is

\[
\dot{V}_1 = z_1 \dot{z}_1 + z_1 \alpha_{1a2} + \frac{\partial \alpha_{1a2}}{\partial \mu} \mu + \frac{\partial \alpha_{1a2}}{\partial \mu} \mu + \alpha_{1a2} \mu + \Delta_1(y, \omega(y(t), u(t), t)) \leq \varepsilon_1 (1 + \rho^2)
\]

(30)

Remark 4.1. Noting Assumption 2.1 and 2.2, P1 of (22) and the bounding function of dynamic uncertainties \( e(z) \) in (3), we have the following inequality

\[
\dot{z}_1 \alpha_{1a2} - \lambda_2 \theta^T - \phi_1(y) \theta^T \varphi(t) + \phi_1(y) \varphi(t) + \Delta_1(y, \omega(y(t), u(t), t)) \leq \varepsilon_1 (1 + \rho^2)
\]

(31)

where \( \varepsilon_1 > 0 \) is designed according to desired performance.

Noting (35), the derivative of \( z_1 \) is

\[
\dot{z}_1 = z_1 - z_1 \alpha_{1a2} - k_{1a} z_1 + \alpha_{1a2} + \bar{x}_2 - \theta^T \phi_1(y, t) + \phi_1(y) \mu(y(t), u(t), t) + \Delta_1(y, \omega(y(t), u(t), t)) \leq \varepsilon_1 (1 + \rho^2)
\]

(36)

where \( \varepsilon_1 > 0 \) is designed according to desired performance.
\[ \dot{V}_i = z_i z_{i+1} + \sum_{j=1}^{i} \left\{-k_{j2} z_j^2 + z_j (\alpha_{j2} - \frac{\partial \alpha_{i-1}}{\partial y} z_2 - \bar{\theta}^T \phi_2(y, t) - \frac{\partial \alpha_{i-1}}{\partial y} \phi_1 \varepsilon_p(t) - \frac{\partial \alpha_{i-1}}{\partial y} \Delta_1(y, \omega(t), u, t) - \frac{\partial \alpha_{i-1}}{\partial \theta} \hat{\theta} z_j \right\} \]

(38)

A smooth or continuous example of \( \alpha_{i2} \) can be worked out in the same way in Remark 4.1, where \( h_i \) satisfies

\[ h_i \geq \frac{\partial \alpha_{i-1}}{\partial y} \left\{ \left[ \theta M \right] \left( \bar{\lambda}_2 + |\phi_1| |\phi(\cdot)| \right) + |\phi_1| \varepsilon_p + \gamma_1 \left( 2T(r) + \delta_1(y) + \sigma_1(\phi(t)) \right) \right\} \]

(39)

One example of \( \alpha_{i2} \) can be

\[ \alpha_{i2} = -\frac{1}{4 \epsilon_i} \left( h_i^2 + 1 \right) z_i. \]

**Step n**

Let \( u = \nu_{n+1} \) and noting (1), (3) and (11), the derivative of \( z_n = \nu_n - \alpha_n - 1 \) is

\[ \dot{z}_n = -k_n \nu_n + u - \sum_{j=1}^{n-1} \frac{\partial \alpha_{j-1}}{\partial y} \zeta_j - \sum_{j=1}^{n-1} \frac{\partial \alpha_{j-1}}{\partial y} \dot{\nu}_j - \sum_{j=1}^{n-1} \frac{\partial \alpha_{j-1}}{\partial y} \lambda_{j,2} + \frac{\partial \alpha_{n-1}}{\partial y} \bar{\theta} z_2 - \frac{\partial \alpha_{n-1}}{\partial \theta} \dot{\theta} - \frac{\partial \alpha_{n-1}}{\partial \theta} \theta \]

(40)

Let \( u = \alpha_n \) and \( \alpha_n \) is given as

\[ \alpha_n(y, \tilde{\alpha}_n, \nu_n, \bar{\lambda}_2, \bar{\theta}, r, t) = \alpha_{na} + \alpha_{ns}, \]

\[ \alpha_{na} = -z_{n-1} - k_n \nu_n + \sum_{j=1}^{n-1} \frac{\partial \alpha_{j-1}}{\partial y} \zeta_j + \sum_{j=1}^{n-1} \frac{\partial \alpha_{j-1}}{\partial y} \dot{\nu}_j + \sum_{j=1}^{n-1} \frac{\partial \alpha_{j-1}}{\partial y} \lambda_{j,2} + \frac{\partial \alpha_{n-1}}{\partial y} \bar{\theta} z_2 - \frac{\partial \alpha_{n-1}}{\partial \theta} \dot{\theta} - \frac{\partial \alpha_{n-1}}{\partial \theta} \theta \]

\[ \alpha_{ns} = \alpha_{ns1} + \alpha_{ns2}, \alpha_{ns1} = -k_n z_n, \]

\[ k_n \geq \frac{g_0}{\lambda_0} + C_{\phi_2} \bar{\phi}_n(y, t)^2 + \frac{\left| \frac{\partial \alpha_{n-1}}{\partial \theta} \right|^2 C_{\phi_n}^2}{\lambda_0} \]

(41)

where \( \tilde{\alpha}_n = [\alpha_1, \ldots, \alpha_n]^T, \nu_n = [\nu_1, \ldots, \nu_n]^T, \bar{\lambda}_2 = [\lambda_2^1, \ldots, \lambda_2^n]^T, \bar{\theta} = [\theta_1, \ldots, \theta_n]^T, g_n \geq 0, C_{\phi_2} \) and \( C_{\phi_n} \) are positive definite constant diagonal matrices to be specified later and \( \bar{\phi}_n(y, t) = -\frac{\partial \alpha_{n-1}}{\partial y} [\lambda_2^1 + \phi_1(\cdot)|\phi(\cdot)|]. \)

\( \alpha_{ns2} \) satisfies the following conditions:

(i) \( z_n \{ \alpha_{ns2} + \frac{\partial \alpha_{n-1}}{\partial y} \bar{\lambda}_2 \bar{\theta} - \frac{\partial \alpha_{n-1}}{\partial y} \bar{\theta} z_2 + \frac{\partial \alpha_{n-1}}{\partial y} \phi_1 \theta \phi(\cdot) - \frac{\partial \alpha_{n-1}}{\partial y} \phi_1 \varepsilon_p(t) - \frac{\partial \alpha_{n-1}}{\partial \theta} \Delta_1(y, \omega(t), u, t) \} \leq \epsilon_n(1 + \rho^2), \)

(42)

(ii) \( z_n \alpha_{ns2} \leq 0 \)

where \( \epsilon_n > 0 \) is designed according to desired performance.

Noting (41), the derivative of \( z_n \) is

\[ \dot{z}_n = -z_{n-1} - k_n z_n + \alpha_{na} = -k_n z_n + \frac{\partial \alpha_{n-1}}{\partial y} z_2 - \frac{\partial \alpha_{n-1}}{\partial y} \phi_1 \varepsilon_p(t) - \frac{\partial \alpha_{n-1}}{\partial \theta} \Delta_1(y, \omega(t), u, t) - \frac{\partial \alpha_{n-1}}{\partial \theta} \dot{\theta} \]

(43)

Let \( V_n = V_{n-1} + \frac{1}{2} z_n^2 \), the derivative of \( V_n \) is

\[ \dot{V}_n = \sum_{j=1}^{n} \left\{ -k_j z_j^2 + z_j (\alpha_{j2} - \frac{\partial \alpha_{j-1}}{\partial y} z_2 + \frac{\partial \alpha_{j-1}}{\partial y} \phi_1 \varepsilon_p(t) - \frac{\partial \alpha_{j-1}}{\partial \theta} \Delta_1(y, \omega(t), u, t) - \frac{\partial \alpha_{j-1}}{\partial \theta} \dot{\theta} z_j \right\} \]

(44)

A smooth or continuous example of \( \alpha_{ns2} \) can be worked out in the same way in Remark 4.1, where \( h_n \) satisfies

\[ h_n \geq \frac{\partial \alpha_{n-1}}{\partial y} \left\{ \left[ \theta M \right] \left( \bar{\lambda}_2 + |\phi_1| |\phi(\cdot)| \right) + |\phi_1| \varepsilon_p + \gamma_1 \left( 2T(r) + \delta_1(y) + \sigma_1(\phi(t)) \right) \right\} \]

(45)

One example of \( \alpha_{ns2} \) can be

\[ \alpha_{ns2} = -\frac{1}{4 \epsilon_n} \left( h_n^2 + 1 \right) z_n. \]

**Theorem 4.1** Consider the system (1) subjected to the Assumptions 2.1 and 2.2, OARLC, which consists of the robust control law (41) in which \( c_{\phi k} \) and \( c_{\phi j} \), the ith diagonal elements of the diagonal matrices \( C_{\phi j} \) and \( C_{\phi j} \) are chosen such that \( c_{\phi k} \geq \frac{b}{4} \sum_{j=1}^{n} c_{\phi j}^2 \) and the learning control law (20) in which \( \tau = \sum_{j=1}^{n} \phi_j(y, t) z_j \) guarantees

A. The control input and all internal signals are bounded, with \( V_n \) bounded above by

\[ V_n \leq e^{-\lambda_n t} V_n(0) + \frac{\varepsilon}{\lambda_n} (1 - e^{-\lambda_n t}) + \frac{\varepsilon}{\lambda_n} \int_{0}^{t} e^{-\lambda_n(t-v)} \rho^2(v) dv \]

(46)

where \( \lambda_n = 2 \min \{g_1, \ldots, g_n\} \) and \( \varepsilon = \sum_{j=1}^{n} \varepsilon_j \).

B. If after a finite time \( t_f, \varepsilon_p(t) = 0 \), i.e. \( \mu(t) = \bar{\theta} \phi(t) \) and \( \Delta_1 = 0 \), i.e. in the presence of unknown periodic disturbance which can be described by \( \mu(t) = \bar{\theta} \phi(t) \) only, then, in addition to results in A, asymptotic output tracking (or zero final tracking error) is also achieved. \( \hat{\theta} \)

**Remark 4.2** The proof is similar to that of Theorem 1 in Yao [1997]. It is omitted.

5. SIMULATION

Consider the following specific system

\[ \begin{align*}
\dot{x}_1 &= x_2 + y^2 \mu(t) + \Delta_1, \\
\dot{x}_2 &= a, \\
y &= x_1, \Delta_1 &= 0.6 \sin(2t).
\end{align*} \]

(47)

To avoid unnecessary complexity in the simulation, \( \mu(t) = 0.5 + \sin(2nt/T) + 0.2 \cos(2nt/T) - 1.5 \sin(4nt/T) + 1.7 \cos(4nt/T) + 2.3 \sin(6nt/T) - 0.8 \cos(6nt/T), \)

(48)
Then (3) is satisfied with unknown parameter \( \theta = [0.5, 1, 0.2, -1.5, 1.7, 2.3, -0.8]^T \) and \( \varepsilon_p = 0 \). Thus \( \varepsilon_p \) can be any positive value, and by choosing the parameter bounding \( \theta_{\text{min}} = [0, 0.5, 0.1, -2, 1, 2, -1]^T \), \( \theta_{\text{max}} = [1, 1.5, 0.4, -1, 2, 3, 0]^T \), and \( \delta_1 = 1 \), Assumption (2.2) is satisfied.

According to the method in Section 3 to estimate the unmeasurable states, we select \( k = [2, 5]^T \), and the filters are designed as (11), where \( \phi(y) = [y^T, 0]^T \).

Let \( V_2 = \tilde{x}^T \left[ \begin{array}{cc} 3/2 & -1/2 \\ -1/2 & 1/2 \end{array} \right] \tilde{x} \) and \( \gamma_1(|\tilde{x}|) = (1 - \sqrt{2}/2)|\tilde{x}|^2 \), \( \gamma_2(|\tilde{x}|) = (1 + \sqrt{2}/2)|\tilde{x}|^2 \). The derivative of \( V_2 \) is given as

\[
\dot{V}_2 \leq -0.5V_2 + 10
\]

Let \( \dot{r} = -0.4r + 10 \), then \( |\tilde{x}| \leq 1.85(2r)^{0.5} + \rho \).

According to the procedure in Section 4, we design control law as (41) and adaptation law as (20) in Theorem 1.

The design parameters and initial conditions for the proposed OALRC are given as follows,

\[
\begin{align*}
\tau(0) &= 0.3, x_1(0) = 0.2, x_2(0) = 0, \xi_1(0) = \xi_2(0) = 0, \\
\nu_1(0) &= \nu_2(0) = 0, \lambda_i(0) = 0, i \in \{1, \cdots, 7\}, \\
\delta_1(0) &= 0.3, \delta_2(0) = 0.7, \delta_3(0) = 0.4, \delta_4(0) = -1.75, \\
\delta_5(0) &= 1.6, \delta_6(0) = 2.4, \delta_7(0) = -0.3, g_1 = g_2 = 10, \\
C_{g12i} &= C_{g22i} = 0.25, C_{g22i} = 2, \Gamma = I, \epsilon_p = 0.5, \\
\varepsilon_1 &= 30, \varepsilon_2 = 300.
\end{align*}
\]

Let \( y_d = 0.5(1 - \cos(1.4\pi t)) \). The simulation is run for the actual system subjected to the general periodic disturbances and uncertain nonlinearities as well. It is observed from the simulation results shown in left figure of Fig. 1 that the proposed extended OALRC scheme has good transient performance and final tracking accuracy – output tracking error is small during the entire transient period. The unknown sinusoidal disturbance estimate shown in right figure of Fig. 1 gradually converges to the neighborhood of its true value as well.

6. CONCLUSIONS

In this paper, an observer is incorporated into the discontinuous projection-based adaptation law to synthesize performance oriented controllers for a class of uncertain nonlinear systems in semi-strict feedback form with general periodic disturbances. The proposed OALRC is designed to achieve a guaranteed transient and a prescribed final tracking accuracy for system’s performance. Simulation results illustrate the effectiveness of the proposed OALRC.

REFERENCES


