A Constructive Method for Designing Higher Order Sliding Surfaces for Single-input Nonlinear System

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Abstract: In this paper, we present a constructive method for the design of any order sliding surface using the necessary conditions for the existence of sliding surfaces for single-input affine nonlinear control systems. These conditions are a set of PDEs whose general solution will help in arriving at the sliding surfaces. The constructive design procedure is demonstrated on the cart-beam system to obtain a third-order sliding surface and on the inertia wheel pendulum system to obtain a fourth-order sliding surface. We validate the fourth-order terminal sliding mode control on inertia wheel pendulum with experimental result.

Keywords: Sliding-mode control, cart-beam, inertia wheel pendulum

1. INTRODUCTION

Sliding mode control has a long history and numerous researchers Drakunov et al. [2005], Edwards and Spurgeon [1998], Hirschorn [2006], Lukyanov and Utkin [1981], Utkin [1978] have applied this technique to control unstable systems. The wide applicability of this technique is due to its robust stabilizing property. The design of sliding surface continues to be a challenging field for underactuated systems and systems that are subjected to nonholonomic constraints. Sliding surfaces have been independently constructed in Yang and Kim [1999], Riachy et al. [2008], Sankaranarayanan and Mahindrakar [2009] by exploiting the structure of the state-space equations.

The paper aims to provide constructive design procedure to arrive at sliding surfaces as a solution to a set of homogeneous PDEs, which arise from the conditions that define the notion of relative degree. This school of thought has been pursued by the authors in Hirschorn and Lewis [2003] where, these PDEs reduce to a set of algebraic equations for linear systems. We observe that this approach can be extended to a wide class of underactuated systems. As an application of this constructive procedure we explicitly construct sliding surfaces for two underactuated systems: cart-beam system and inertia wheel pendulum.

The cart-beam system was first introduced in Mahindrakar and Sankaranarayanan [2008]. It is a two degree-of-freedom system with one actuator. The authors in Mahindrakar and Sankaranarayanan [2008] proposed an output feedback nonlinear controller to stabilize the system, while in Sankaranarayanan et al. [2010], a nonlinear second-order sliding mode controller is used to stabilize the same system.

The inertia wheel pendulum is a two degree-of-freedom system with one actuator. It was first introduced in Spong et al. [1999] where a swing-up and stabilize control was used to stabilize the system. The fourth-order sliding surface that we obtain can be found to be same as the output equation for the fourth-order feedback linearization in Spong et al. [1999]. Our main objective in this case is to show that the function is obtained out of a constructive procedure. Also, we present experimental results using a fourth-order finite time stabilizing controller (to the best of our knowledge, this is the first hardware implementation of fourth-order finite time controller using the complete model of a system).

In next section, the necessary and sufficient conditions for the generation of sliding surface is presented.

2. EXISTENCE OF SLIDING SURFACES

Consider a single-input affine in control system

\[ \dot{x} = f(x) + g(x)u \]  

where, \( x \in \mathbb{R}^n \), \( u \in \mathbb{R} \), \( f, g \) are smooth vector fields and let \( x_r \) be the equilibrium point of interest. The control objective is to stabilize the system at the desired equilibrium point \( x_r \). Further, let \( L_x \sigma(x) = (\nabla_x \sigma)f(x) \) denote the Lie derivative of a smooth real valued function \( \sigma(x) \) along a vector field \( f \) where, \( \nabla_x \sigma \) is the gradient of \( \sigma \) with respect to \( x \) and is taken as a row vector. We denote by \( S_i(x) \) the \( i \)th sliding function such that \( S_i : \mathbb{R}^n \rightarrow \mathbb{R} \) with \( S_i(x_r) = 0 \) and the corresponding sliding surface is the set \( \{ x \in \mathbb{R}^n : S_i(x) = 0 \} \).

Let \( S = \{(S_i(x), r_i), i \in J\} \) be a set consisting the ordered pair of scalar function \( S_i \) and its relative degree \( r_i \), \( J \) is an...
index set. The relative degree $r_i$ of the sliding function $S_i$ satisfies the following conditions

$$L_gS_i(x) = L_gL_fS_i(x) = \cdots = L_gL_f^{r_i-2}S_i(x) = 0 \quad (2)$$

$$L_gL_f^{r_i-1}S_i(x) \neq 0. \quad (3)$$

For a given system (1), the set $S$ is not known a priori. This motivates us to aim for a particular order (number of times the function is differentiated) of the sliding surface rather than searching for $(S_i, r_i)$. The function $S_i$ is designed with the goal of obtaining a high order sliding function, denoted as $\mathcal{R}^{th}$-order sliding function. The conditions in (2) are used to obtain the equations leading to the design of $S_i$. Later, the property (3) could be attributed to $S_i$ by the choice of free parameters from the general solution of construction equations.

The sliding surface designed from the constructive procedure should also satisfy the property so that the sliding mode exist for the surface, in other words, apart from being reachable, the surface should also meet the control objective of reaching the desired equilibrium. The sufficient conditions are:

1. The equivalent dynamics on the set $\mathcal{O} = \{x \in \mathbb{R}^n : S_i = S_i = \cdots = S_i^{(n-1)} = 0\}$ is stable about the desired equilibrium.
2. $L_gL_f^{r_i-1}S_i(x) \neq 0$ for all $x$ locally around $x_c$.

Next, we characterize the set $S$ as $\mathcal{R}^{th}$-order differentiable functions, where $\mathcal{R} \leq n$.

3. DESIGN OF SLIDING SURFACES

The $p - th$ derivative of $S_i$ along the trajectories of (1) is given by

$$\frac{dp}{dp}(S_i) = \frac{d}{dt}\left[\frac{dp-1}{dp-1}(S_i)\right]$$

$$= \nabla_x\left[\frac{dp-1}{dp-1}(S_i)\right]\dot{x}$$

$$= \nabla_x\left[\frac{dp-1}{dp-1}(S_i)\right][f(x) + g(x)u] \quad (4)$$

where, $p = 1, \ldots, \mathcal{R} - 1$. From (4), we can see that the $p^{th}$-order derivative of $S_i$ will not have the control appearing if and only if

$$\nabla_x\left[\frac{dp-1}{dp-1}(S_i)\right]g(x) = 0 \quad \text{g(x)^⊥ is the left annihilator of g(x), rank(g(x)^⊥) = (n - 1).} \quad (5)$$

Equation (5) has a solution if and only if the Hessian of $\frac{dp}{dp}(S_i)$ is symmetric (Poincaré’s Lemma¹), which further implies that a vector $v \in \text{span}(g(x)^⊥)$ has a symmetric Jacobian. Hence we obtain the first necessary condition for $S_i$ to be a sliding function as

$$\nabla_xv_p = (\nabla_xv_p)^T \quad (6)$$

where, $v_p \in \text{span}(g(x)^⊥)$. The second condition can be obtained from (5) as a set of PDEs,

$$\nabla_x\left[\frac{dp}{dp}(S_i)\right] = v_p, \ p = 1, 2, \ldots, \mathcal{R} - 1 \quad (7)$$

whose solution gives the general solution of the set $S$.

Equations (6) and (7) are the necessary conditions to be satisfied for the system (1) to obtain $\mathcal{R}^{th}$-order sliding surfaces. In addition to (6) and (7), if the $S_i$’s satisfy (3), they will be finite time reachable.

The general solution of PDEs (5) is in terms of characteristic variables and free function, expressed as

$$S_i(x) = \Phi(z_1, z_2, \ldots, z_a) \quad (8)$$

where, $(z_1, z_2, \ldots, z_a)$ could be nonlinear functions of $x$ which are fixed, $a \leq (n - 1) = \text{rank}(g(x)^⊥)$. The free function $\Phi$ can be chosen independently for each $z_i$, i.e. $\Phi$ itself is a set of infinite functions. But each $\Phi$ has to satisfy $\Phi|_{(x=x_c)} = 0$.

With the general solution in mind, if the necessary or sufficient conditions fail, we propose the following methods to overcome the issue.

Case a: The control does not appear in the $\mathcal{R}^{th}$ derivative of $S_i$, in other words the condition (3) fails. This happens when the characteristic variable(s) $z$ turn out be the characteristic variable(s) of a higher-order sliding surface for the same system. Since we only feed the $(\mathcal{R} - 1)^{th}$ derivative to the PDE solver, its next derivative could still annihilate $g(x)$. This implies that a higher order-sliding surface could be achieved.

Case b: In existing literature Bhat and Bernstein [1998], Hong [2002], Levant [2001], the control input(s) is extracted from

$$\frac{d\mathcal{R}}{dt}(S_i) = u_f = \nabla_x\left[\frac{d^{\mathcal{R}-1}}{d\mathcal{R}-1}(S_i)\right][f(x) + g(x)u]$$

by choosing a desired finite time controller $u_f$ and then extracting the control law $u$. But if $\nabla_x\left[\frac{d^{\mathcal{R}-1}}{d\mathcal{R}-1}(S_i)\right]g(x) = 0$, $u$ is undefined.

This problem is due to nature of the characteristic variables $z$ and the choice of $\Phi$. The freedom in the choice of $\Phi$ and $z$ could be used to mitigate the problem of control being undefined. If for all choices of $\Phi$ and $z$, $u$ cannot be extracted then the sliding surface is not reachable. Hence we work with a reduced order sliding surface. If for every reduced order sliding surface $u$ cannot be extracted, it can be inferred that the original set $S$ is not reachable.

Case c: The equivalent dynamics could be unstable for the desired equilibrium. It is again influenced by the choice of $\Phi$ and $z$. By a proper choice of $\Phi$ and $z$, the equivalent dynamics can be rendered stable. If for all choices of $\Phi$ and $z$, the equivalent dynamics is unstable then we work with a reduced order sliding surface. If the equivalent dynamics of

¹ Given a smooth vector field $v(x), v : \mathbb{R}^n \to \mathbb{R}^n$, there exists a smooth function $h(x), h : \mathbb{R}^n \to \mathbb{R}$ such that $\nabla_xh = v(x)$ if and only if $\nabla_xv = (\nabla_xv)^\top$. 3945
every reduced order sliding surface is unstable then sliding mode based stabilization of the desired equilibrium is not feasible.

The characteristic z and free function $\Phi$ arising out of the solution of PDEs are the key design variables in obtaining a $R^{th}$ order sliding mode controller.

In the ensuing section, we demonstrate the applicability of the proposed constructive method to obtain sliding surfaces for two underactuated examples: the cart-beam and the inertia wheel pendulum.

4. EXAMPLES

4.1 Example 1: Cart-Beam system

The cart-beam system consists of an actuated cart on a beam which is free to rotate about its pivoted point (see Figure 1). The control objective is to stabilize the beam at the horizontal position using the cart motion. The beam angle is taken as $q_1$, the cart position from center of the beam is taken as $q_2$ and the force applied on the cart is $F$.

The system also has practical constraints on the configuration variables $q_1 \in (-\pi/2, \pi/2)$, $q_2 \in (-\pi/2, \pi/2)$, where $L$ is the total length of beam, $0 \leq l_c < (L/2)$ is the distance of the center-of-gravity of the beam from pivot point, $m$ is mass of the beam, $M$ is mass of the cart and $J$ is the moment of inertia of the beam about the pivot. It is assumed that the pivot is either a rotary joint or a non-slip contact. With the states defined as $x = [x_1 \ x_2 \ x_3 \ x_4]^T = [q_1 \ (q_2 + \frac{ML}{M}) \ q_2 \ q_1]^T$, the state-space model (refer Sankaranarayanan et al. [2010]) is given by

$$\dot{x} = f(x) + g(x)u$$

where,

$$f(x) = \begin{pmatrix} x_3 \\ x_2 \frac{(-2Mx_3x_4\alpha(x_2) - Mgx_2\cos(x_1))}{\beta(x_2)} \\ x_2^2\alpha(x_2) - g\sin(x_1) \end{pmatrix}, \quad g(x) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix},$$

$u \triangleq \frac{F}{M}, \alpha(x_2) \triangleq (x_2 - ml_c/M)$ and $\beta(x_2) \triangleq J + Ma^2(x_2) > 0 \ \forall x_2$. The control objective is to stabilize the system at the equilibrium $x_e = [0 \ 0 \ 0 \ 0]^T$. In particular, we seek a third-order ($R = 3$ since we get no solution for $R = 4$) sliding surface for the system. Following the necessary conditions expressed in earlier section, the PDEs (5) for the cart-beam system are

$$\frac{\partial \Phi}{\partial x_1}(L_1 S(x)) = 0. \quad (10)$$

The general solution of the PDEs (10) using MAPLE\textsuperscript{TM} is given by

$$S_i(x) = \Phi(z_1, z_2)$$

where, the characteristic variables are

$$z_1 = x_1, \quad z_2 = x_3M\beta(x_2).$$

Then $S = \{(S_i, 3), i = 1, 2, \ldots\}$. A few choices of sliding functions are:

**Choice 1**: $S_1(x) = \Phi(z_1) \equiv x_1$

$$\dot{S}_1 = x_3$$

$$\tilde{S}_1 = (-2Mx_3x_4\alpha(x_2) - Mgx_2\cos(x_1))/\beta(x_2)$$

$$\bar{S}_1 = h_1^1(x) + h_2^1(x)u$$

where, $h_2^1(x) = \frac{-2Mx_3\alpha(x_2)}{\beta(x_2)}$. It can be seen that for any choice of $\Phi$, the term $h_1^1(x)$ is zero at the desired equilibrium. This corresponds to the issue discussed in Case b. In Voytsekhovsky and Hirschorn [2005], the authors use the notion of approximate feedback linearization to mitigate the problem of the control being undefined at the desired equilibrium. We change the characteristic variable as follows:

**Choice 2**: $S_2(x) = \Phi(z_2) \equiv x_3M\beta(x_2)$

Note that $x_3\beta(x_2)$ represents momenta of the beam and it is not intuitive that momenta is a higher order sliding surface.

$$\dot{S}_2 = -M^2gx_2\cos(x_1)$$

$$\bar{S}_2 = M^2g(x_2x_3\sin(x_1) - x_4\cos(x_1))$$

$$\bar{S}_2 = h_1^2(x) + h_2^2(x)u$$

where, $h_2^2(x) \equiv -M^2g\cos(x_1) \neq 0 \ \forall x_1 \in (-\pi/2, \pi/2)$. But the equivalent dynamics converges to the set $x = [k_1 \ 0 \ 0 \ 0]^T$, $k_1$ is dependant on initial condition. This corresponds to the issue discussed in Case c. So we change both the characteristic variable and the free function $\Phi$.

**Choice 3**: $S_3(x) = \Phi(z_1, z_2) \equiv x_1 + x_3M\beta(x_2)$

$$\dot{S}_3 = x_3 - M^2gx_2\cos(x_1)$$

$$\bar{S}_3 = (-2Mx_3x_4\alpha(x_2) - Mgx_2\cos(x_1))\beta(x_2) + M^2g(x_2x_3\sin(x_1) - x_4\cos(x_1))$$

$$\bar{S}_3 = h_1^3(x) + h_2^3(x)u$$

where, $h_2^3(x) \equiv -M^2g\cos(x_1) + \frac{-2Mx_3\alpha(x_2)}{\beta(x_2)}$, which is non-zero at the desired equilibrium.
One choice for $u_f$ is the controller introduced in Hong [2002]

$$\ddot{S}_3 = u_f = -l_3(l_2(S_5^3 + \dot{S}_4^7)^{\frac{1}{3}} + \dot{S}_3)^{\frac{1}{2}}$$

where $l_2, l_3$ are positive constants.

Remark 4.1. Another choice of $u_f$ is the relay controller introduced in Levant [2001] which is robust to disturbances that preserve the relative degree property.

The proof of stability of the equivalent dynamics follows from the following Lemma.

**Lemma 4.1.** The equivalent dynamics, defined on the set $S = \{ x \in \mathbb{R}^4 : S_3 = \dot{S}_3 = \ddot{S}_3 = 0 \}$ is asymptotically stable.

**Proof:**
Since $h^2_3(x)$ is non-zero locally around $x_e$, the surfaces $S_3, \dot{S}_3, \ddot{S}_3$ are finite-time reachable through a finite time controller.

Consider a candidate Lyapunov function $V(x) = \frac{1}{2} x_1^2$ defined on $S$ and its derivative along the trajectories of the system (12).

We have $\dot{V} = x_1 \dot{x}_1 = x_1 x_2$. On the sliding surface, $S_3 = 0$ implies $x_3 = -x_1^2 M^2(x_2)$ which yields $\dot{V} = -x_1^2 M^2(x_2) < 0$ for $x_1 \neq 0$ since $\beta(x_2) > 0 \forall x_2$.

\[ \Box \]

4.2 Example 2: Inertia Wheel Pendulum

![Fig. 2. Schematic of the inertia wheel pendulum system](image)

The inertia wheel pendulum consists of an unactuated pendulum to which an inertia wheel is attached at the free end (see Figure 2). The pendulum angle is $\theta_1$ and the inertia wheel angle is $\theta_2$ (both angles are measured clockwise). With the states defined as $x_1 = [x_1, x_2, x_3, x_4]^T = [\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2]^T$ and $u = \tau$, the state-space model (refer Spong et al. [1999]) is given by

$$\dot{x} = f(x) + g(x)u$$

where,

$$f(x) = \begin{pmatrix} x_3 \\ x_4 \\ c_1 c_3 \sin x_1 - c_1 c_3 \sin x_1 \\ 0 \\ 0 \\ -c_3 \\ \frac{c_2}{I_w} \end{pmatrix}, \quad g(x) = \begin{pmatrix} 0 \\ 0 \\ -c_3 \\ \frac{c_2}{I_w} \end{pmatrix},$$

where, $c_1 \triangleq (m_p l_2 + (m_\theta + m_w) l_2) g, \quad c_2 \triangleq I_1 + I_w + (m_\theta + m_w) l_2^2, \quad c_3 \triangleq \frac{l_3}{I_2}, \quad m_\theta$ is mass of the pendulum, $l_2$ is center of mass of the pendulum, $l_\theta$ is length of the pendulum, $m_a$ is mass of the actuator, $m_w$ is mass of the inertia wheel, $I_1$ is inertia of the pendulum about its end and $I_w$ is inertia of the inertia wheel about the wheel center. The control objective is to stabilize the system at the equilibrium $x_e = [2\pi 2m\pi 0 0]^T, (n, m) \in \mathbb{Z}$. We seek a fourth-order ($K = 4$) sliding surface for the system. Following the necessary conditions expressed in earlier section, the PDEs (5) for inertia wheel pendulum are

$$\begin{align*}
-c_3 \frac{\partial S}{\partial x_3} + \frac{c_2}{I_w} \frac{\partial S_2}{\partial x_4} & = 0 \\
-c_3 \frac{\partial S}{\partial x_3} (L_f S(x)) + \frac{c_2}{I_w} \frac{\partial S_2}{\partial x_4} (L_f S(x)) & = 0 \\
-c_3 \frac{\partial S}{\partial x_3} (L_2 f S(x)) + \frac{c_2}{I_w} \frac{\partial S_2}{\partial x_4} (L_2 f S(x)) & = 0.
\end{align*}$$

The general solution of PDEs (13) is given by

$$S_i(x) = \Phi(z)$$

where, $z = x_2 + x_1 \frac{c_2}{I_w}$.

Then $S = \{(S_i, 4), i = 1, 2, ...\}$. We choose $S(x) = z$

$$S = x_2 + x_1 \frac{c_2}{I_w} \quad \frac{\dot{S}}{I_w} = c_1 c_3 (c_2 - I_w) \sin x_1 \quad \frac{\ddot{S}}{I_w} = h_1(x) + h_2(x) u$$

where, $h_1(x) = c_1 c_3 (c_2 - I_w) \sin x_1 (-x_3^2 + c_1 c_3 \cos x_1)$, $h_2(x) = c_1 c_3 (c_2 - I_w) \cos x_1$ and $h_2(x) \neq 0$ locally around $x_e$ hence $x_e$ is finite time stabilizable.

Consider a particular fourth order finite time controller from the generalized controllers introduced in Hong [2002]

$$\ddot{x} = -l_4 \left\{ \dddot{x}^5 + \frac{l_3}{l_1} \left[ \dddot{x}^5 + \frac{l_2}{l_1} \left( \dddot{x}^2 + \frac{l_4}{l_1} \dddot{x}^2 \right)^{\frac{3}{2}} \right]^{\frac{3}{2}} \right\}^{\frac{1}{2}}$$

where, the power operator is defined as $a^b = ([a])^b \text{sign}(a)$ for $b \in (0, 1)$ and $a$ is any real valued function and $l_1, l_2, l_3, l_4$ are positive constants.

The controller (15) achieves finite time stabilization of $x_e \forall |x_1| < \frac{c_2}{2}$ since the order of the sliding surface is the same as state-space dimension of the system. This is known as terminal sliding mode in literature.

It is evident from above analysis that by a proper choice of $\Phi$ and $z$, a sliding surface of a feasible order can be
obtained from the proposed procedure. With a certain amount of intuition, \( \Phi \) and \( z \) can be used to achieve the control objective. In ensuing sections, we present the simulation and experimental results.

5. SIMULATIONS

5.1 Cart-beam system

The cart-beam system is simulated with the following parameters, \( M = 1 \text{kg}, \ m = 2 \text{kg}, \ J = 2 \text{kg m}, \ L = 1 \text{m}, \ l_c = 0.1 \text{m}, \ g = 9.81 \text{m/s}^2 \) and the control parameters used in the finite-time controller are \( l_2 = l_3 = 4 \). The plots for the initial condition \( x(0) = \left[ \frac{\pi}{4}, -0.3, -0.1, 0.1 \right] \) are shown in Figure 3. It can be observed that the magnitude of control input is small when compared to the results in Mahindrakar and Sankaranarayanan [2008], Sankaranarayanan et al. [2010]. The bandwidth requirement of the control input can also be observed to be less due to the use of a continuous finite-time controller.

5.2 Inertia wheel pendulum

The inertia wheel pendulum is simulated with the following nominal system parameters of the experimental setup, \( m_p = 0.0274 \text{kg}, \ m_a = 0.19 \text{kg}, \ m_w = 0.074 \text{kg}, \ l = 0.1265 \text{m}, \ l_c = l/2, \ l_1 = 1.4615 \times 10^{-4} \text{kg m}, \ l_2 = 2.3125 \times 10^{-2} \text{kg m}, \ g = 9.81 \text{m/s}^2 \) and the control parameters used in the finite-time controller are \( l_1 = 4, l_2 = 1, l_3 = l_4 = 250 \). The plots for the initial condition \( x(0) = [\pi/4, 0.5 - 0.5] \) are shown in Figure 4.

6. EXPERIMENTAL RESULT FOR INERTIA WHEEL PENDULUM

The experimental setup consists of the “Mechatronics Control Kit” from Quanser™ (shown in Figure 6) which has a 24V dc brush motor with 1024 counts/rev optical encoder and PWM amplifier, another optical encoder with 1024 counts/rev resolution. Both the optical encoders are interfaced in \( \times 4 \) mode to get four times the resolution. The motor has a resistance \( R = 12.1 \Omega \) and torque constant of \( k_t = 0.0274 \text{Nm/A} \) which is used to get the voltage \( (V_m) \) corresponding to the torque as \( V_m = \frac{R}{k_t}u + k_t x_4 \) (neglecting the inductance of the motor).

The unwinding feature introduced in Bhat and Bernstein [1998] is used to stabilize \( x_2 \) at any integral multiple of \( 2\pi \). With \( l_1 = 1, l_2 = 1, l_3 = l_4 = 1000 \), the experimental plots are shown in Figure 5. It can be seen from the plots that the inertia wheel velocity is almost zero. Practically it oscillates around the zero velocity due to error in system parameters and cable disturbance. A modified version of an energy based swingup (Spong et al. [1999]) is used. The switch from swing-up phase to the stabilizing phase occurs at \( x = [5.5668, 975.3755, 6.5140, 82.0822] \) and the fourth-order controller (15) stabilizes it in a large domain (experimentally observed).

A demonstration video of the experiment is available at http://www.youtube.com/watch?v=cz2UEhb66gk.

7. CONCLUSIONS

We have presented a constructive procedure for designing sliding surfaces for a single-input affine in control system and also discussed its shortcomings and methods to overcome it. The procedure was successfully applied to generate a third-order sliding mode controller for the cart-beam system and a fourth-order sliding mode controller for inertia wheel pendulum. We also successfully implemented the fourth-order sliding mode controller on inertia wheel pendulum experimental setup. There is large scope for additional conditions to be added to existing PDEs to
take care of reachability and the stability of equivalent dynamics.

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