Optimal control of freeway systems based on a linearized prediction model

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Abstract: In this paper a Model Predictive Control scheme is proposed in order to prevent congestion phenomena in freeways by means of variable speed limits. The prediction model to be used in the Model Predictive Control scheme is a linearized version of the first-order dynamic model of traffic flow, and this represents the main novelty of the present work. In particular, we approximate the steady-state speed-density characteristic as a piecewise constant function. The prediction model is then a simplified dynamic model that is obviously less effective to represent the real behaviour of traffic (than the second-order macroscopic model) but its simpler equations makes it suitable to be used on line in real freeway systems. As a matter of fact, in an on-line application of a Model Predictive Control scheme, the finite-horizon optimization problem must be solved in acceptable computational times, allowing a high speed of the regulator computing system; in this work this is achieved thanks to the linearized prediction model. Some numerical results are then reported in the paper, showing the effectiveness of the proposed control scheme.

Keywords: Freeway systems; Traffic control; Model Predictive Control.

1. INTRODUCTION

Traffic congestion phenomena are very common nowadays in freeways and interurban roadways; they increase not only time losses, fuel consumption and pollution, but also the possibility of accidents. These recurrent and non-recurrent congestion phenomena are caused by the fact that the available infrastructure capacity is not in most cases sufficient to face the overall transportation demand. For this reason, it is more and more useful to design suitable modelling, optimization and control methods for the efficient utilization of the available road capacity. Dynamic traffic management measures are generally adopted at this purpose.

In order to prevent and solve congestion phenomena in freeway networks, different traffic control measures have been proposed and implemented, such as ramp metering, variable speed limits, route guidance and vehicle-infrastructure integration systems (Carlson et al. (2010)). In this paper we only focus on variable speed limits as traffic control measures. According to the classification proposed in Hegyi et al. (2005b), it is possible to distinguish between two different views on the use of speed limits. In the first approach, the homogenization effect is emphasized, so that speed limits are used in order to obtain a stabler, and safer, flow. In this way the speed limits (generally above the critical speed) do not limit the traffic flow, but only slightly reduce the average speed and slightly increase the density. The second approach, on the other hand, is focused on preventing traffic breakdowns by reducing flows; the main objective is related to prevent high densities and to limit the inflow to the jammed area, by imposing speed limits that are generally lower than the critical speed.

Several control methodologies are used in the literature for speed control, such as for instance in Alessandri et al. (1998), Di Febbraro et al. (2001), Chiang and Juang (2008). In this work we propose a Model Predictive Control scheme; the effectiveness of this technique for suppressing shock waves has already been proved in Hegyi et al. (2005b) and Hegyi et al. (2005a). As known, Model Predictive Control (MPC) is a control framework in which the current control action, at each sampling instant, is obtained by solving on line a finite-horizon open-loop optimization problem, using the current state of the system as the initial state (Mayne et al. (2000)). Of course, the on-line application of a MPC scheme involves a high computational load for the regulator; if the dynamics of the controlled system is not sufficiently slow, as compared with the speed of the regulator computing system, a practical application of the rolling horizon control mechanism turns out to be unfeasible. For this reason, in this work we are searching for a finite-horizon optimization problem that can be solved in acceptable computational times, allowing a high speed of the regulator computing system.

In order to evaluate the performance of the proposed control scheme in a simulative way, it is necessary to adopt a simulation model representing the dynamic behaviour of the real system. The adopted model is the well-known second-order macroscopic dynamic model of traffic flow, firstly proposed in the Seventies (for major detail, refer to Payne (1971) and Whitham (1974)) and then applied in many real cases, as described in Papageorgiou et al. (1990a) and Papageorgiou et al. (1990b). On the contrary, the prediction model adopted in the proposed MPC scheme is a simplified version of the simulation one. Specifically, the prediction model is based on the first-
order dynamic model, firstly introduced in the Fifties in Lighthill and Whitham (1955), also called LW model. The LW model includes some simplifications with respect to the second-order one and in some cases is not completely suitable for representing specific dynamic phenomena that happen in freeways (Papageorgiou (1998)). Nevertheless, using the first-order model for the prediction in the MPC scheme involves an advantage in computation times (in comparison with the second-order model), due to its simpler form. In this paper, we aim at evaluating whether the use of the first-order model allows acceptable control performances. Specifically, we propose a further simplified version of the first-order model, in which the steady-state speed-density characteristic is discretized and then represented as a piecewise constant function. The obtained prediction model is then linear and it will be used in the MPC scheme.

It is important to highlight that the possibility of using MPC schemes in which the prediction model results from the approximation of nonlinear dynamics as piecewise linear functions is also adopted in Bemporad and Morari (1999). Actually, the considered systems belong to the class of mixed logical dynamical systems defined in Bemporad and Morari (1999). Nevertheless, the control approach developed in this work has a different objective with respect to Bemporad and Morari (1999). In particular, in Bemporad and Morari (1999) the main objective is the system stabilization on desired reference trajectories, whereas in the present work the minimization of a suitable performance criterion is considered.

The paper is organized as follows. In Section 2 the simulation model of freeway traffic is briefly described. In Section 3 the linear first-order model (to be used as a prediction model) is presented, while the MPC scheme and the finite-horizon optimization problem are described in Section 4. In Section 5 some experimental results are shown and commented on; finally, some conclusions are stated in Section 6.

2. THE SIMULATION MODEL

The adopted simulation model is the well-known second-order macroscopic dynamic model of traffic flow. It is a discrete-time dynamic model based on the discretization in both space and time, in which \( k = 0, \ldots, K - 1 \) denotes the temporal stage and \( i = 1, \ldots, N \) indicates the section of the freeway stretch; \( T \) is the sample time interval and \( \Delta_i \) is the length of section \( i \).

![Fig. 1. The model variables for a generic freeway section \( i \) in a generic time step \( k \).](image)

The main quantities considered in the model (see also Fig. 1 for major detail) are the following:

- \( \rho_i(k) \) is the traffic density (number of vehicles in section \( i \) at time step \( k \) divided by the section length \( \Delta_i \));
- \( v_i(k) \) is the mean traffic speed (mean speed of vehicles in section \( i \) during time interval \([kT, (k+1)T)\));
- \( q_i(k) \) is the traffic volume (number of vehicles leaving section \( i \) during time interval \([kT, (k+1)T)\) divided by \( T \));
- \( r_i(k) \) and \( s_i(k) \) are the on-ramp and off-ramp traffic volumes (number of vehicles entering and exiting section \( i \) during time interval \([kT, (k+1)T)\) divided by \( T \));

The two state variables are the traffic density \( \rho_i(k) \) and the mean traffic speed \( v_i(k) \), for which the following state equations are considered:

\[
\rho_i(k + 1) = \rho_i(k) + \frac{T}{\Delta_i} (q_{i-1}(k) - q_i(k) + r_i(k) - s_i(k))
\quad \text{for } i = 1, \ldots, N, \quad k = 0, \ldots, K - 1 \tag{1}
\]

\[
v_i(k + 1) = v_i(k) + \frac{T}{\tau} \left[ V(\rho_i(k)) - v_i(k) \right] + \frac{T}{\Delta_i} \left[ v_i(k) - v_{i-1}(k) \right] + \frac{\nu T \left( \rho_{i+1}(k) - \rho_i(k) \right)}{\tau \Delta_i (\rho_i(k) + \chi)}
\quad \text{for } i = 1, \ldots, N, \quad k = 0, \ldots, K - 1 \tag{2}
\]

where the traffic volume \( q_i(k) \) is defined as:

\[q_i(k) = \rho_i(k) \cdot v_i(k) \quad \text{for } i = 1, \ldots, N, \quad k = 0, \ldots, K - 1(3)\]

In (2) \( \tau, \nu, \text{ and } \chi \) are parameters to be determined experimentally, whereas \( V(\rho_i(k)) \) is the steady-state speed-density characteristic, which can be expressed as:

\[V(\rho_i(k)) = V_f \left[ 1 - \left( \frac{\rho_i(k)}{\rho_i^{\text{max}}} \right)^m \right] \tag{4}\]

where \( V_f \) denotes the free speed, \( \rho_i^{\text{max}} \) is the so-called jam density for section \( i \), and \( l \) and \( m \) are real-valued positive parameters.

The dynamic model given by (1)-(4) is the macroscopic second-order model of traffic flow, generally applied in order to represent the dynamic of traffic flow in road stretches. In this work we are considering a road stretch in which variable speed limits are applied as a control measure in order to reduce congestion phenomena. Therefore, the dynamic model to be used must take into account the effect of speed limits on the dynamic behaviour of traffic. In particular, we apply the approach proposed in Hegyi et al. (2005b) and Hegyi et al. (2005a), where the speed-density diagram is defined as the minimum between its typical form based on the experienced density and the speed caused by the limit displayed on the variable message sign:

\[V(\rho_i(k), v_i^{\text{tri}}(k)) = \min \left( (1 + \alpha) v_i^{\text{tri}}(k), V_f \left[ 1 - \left( \frac{\rho_i(k)}{\rho_i^{\text{max}}} \right)^m \right] \right) \quad \text{for } i \in \mathcal{I}_v, \quad k = 0, \ldots, K - 1 \tag{5}\]

where \( \mathcal{I}_v \) is the set of indices of freeway sections equipped with a variable message sign, \( v_i^{\text{tri}}(k) \) is the imposed speed
limit and \((1 + \alpha)\) is a factor expressing the driver noncompliance. Of course, in freeway sections not equipped with variable message signs, the steady-state speed–density characteristic maintains the expression (4). Therefore, the second-order dynamic model adopted as a simulation model of freeway traffic in presence of variable speed limits is given by equations (1)-(3), in which the speed-density diagram is given by (4) for sections \(i \notin I_v\) and by (5) for sections \(i \in I_v\).

This second-order model could be applied as a prediction model in a MPC scheme, as already done in many research papers. For instance, in Hegyi et al. (2005b) a MPC approach is proposed in order to optimally coordinate vehicle classes are explicitly modelled. Anyway, as already pointed out in the Introduction, in this work we try to test the use of a simpler model in the finite-horizon optimization problem.

### 3. THE PREDICTION MODEL

The prediction model adopted in this work originates from the macroscopic traffic theory but is based on the first-order dynamic model of traffic flow (LW model). The LW model obviously includes some simplifications with respect to the second-order one and in some cases it is not completely suitable for representing specific dynamic phenomena that characterize the traffic in freeways. Anyway, the simple form of its equations makes it suitable for the purposes of this work.

In the LW model, the system dynamics is represented by the dynamic equation (1) for density, coupled with (3), in which the mean speed \(v_i(k)\) is given by (4). In this way it is assumed that the mean speed adjusts instantaneously according to the steady-state speed-density characteristic. This is the most crucial assumption which differentiates this model from the second-order one. Therefore, the dynamic equation for the first-order model is the following:

\[
\rho_i(k + 1) = \rho_i(k) + \frac{T}{\Delta T} (\rho_{i-1}(k)v_{i-1}(k) - \rho_i(k)v_i(k) + r_i(k) - s_i(k)) \quad i = 1, \ldots, N k = 0, \ldots, K - 1 \quad (6)
\]

with

\[
v_i(k) = V_f \left[ 1 - \left( \frac{\rho_i(k)}{\rho_{i,\text{max}}} \right)^m \right]^{i-1} \quad i = 1, \ldots, N k = 0, \ldots, K - 1 \quad (7)
\]

Note that in equation (6), when considering the first section of the freeway stretch, corresponding to \(i = 1\), the term \(\rho_{i-1}(k)v_{i-1}(k)\) represents the measured value of the traffic volume entering the freeway stretch.

Starting from the LW model, our objective stands in further simplifying it in order to obtain a piecewise linear model. To do that, the form of the steady-state speed-density characteristic is simplified to become a piecewise constant function \(V(\bar{\rho}_i)\):

\[
V(\bar{\rho}_i) = V_f \left[ 1 - \left( \frac{\bar{\rho}_i}{\bar{\rho}_{\text{max}}} \right)^m \right] \approx \bar{V}(\bar{\rho}_i) \quad i = 1, \ldots, N \quad (8)
\]

Fig. 2. The typical form of \(V(\bar{\rho}_i)\) and its piecewise constant approximation \(\bar{V}(\bar{\rho}_i)\).

Note that in (8) the reference to section \(i\) is explicit since different road sections are in general characterized by different speed-density diagrams, thus leading to different approximations. As shown in Fig. 2, for each section \(i\) the relation \(V(\bar{\rho}_i)\) is approximated by \(M_i\) segments of equal length as follows:

\[
\bar{V}(\bar{\rho}_i) = \bar{v}_{i,l} \quad \text{ with } \quad \bar{v}_{i,l} \leq \bar{\rho}_i \leq \bar{v}_{i,l+1} \quad i = 1, \ldots, N \quad l = 1, \ldots, M_i \quad (9)
\]

where \(\bar{\rho}_{i,1} = 0\) and \(\bar{\rho}_{i,M_i+1} = \bar{\rho}_{i,\text{max}}\). The values \(\bar{v}_{i,l}\) are obtained by dividing the range of variation of \(\bar{\rho}_i\), i.e. \([0, \bar{\rho}_{i,\text{max}}]\), in \(M\) equal parts, then leading to the following:

\[
\bar{v}_{i,l} = \frac{\bar{\rho}_{i,\text{max}}}{M_i} (l - \frac{1}{2}) \quad i = 1, \ldots, N \quad l = 1, \ldots, M_i \quad (10)
\]

The quantities \(\bar{v}_{i,l}\) are then obtained as the values of the speed-density characteristic in the average point of each discretization segment, as:

\[
\bar{v}_{i,l} = V \left( \frac{\bar{\rho}_{i,\text{max}}}{M_i} (l - \frac{1}{2}) \right) \quad i = 1, \ldots, N \quad l = 1, \ldots, M_i \quad (11)
\]

Once the approximation of the speed-density characteristic is determined for each section \(i\), the dynamic equation of the first-order linear model for traffic flow can be rewritten as:

\[
\dot{\rho}_i(k + 1) = \rho_i(k) + \frac{T}{\Delta T} \left( \rho_{i-1}(k) \cdot \bar{v}_{i-1}(k) - \rho_i(k) \cdot \bar{v}_i(k) + r_i(k) - s_i(k) \right) \quad i = 1, \ldots, N \quad k = 0, \ldots, K - 1 \quad (12)
\]

in which \(\bar{v}_{i-1}(k)\) and \(\bar{v}_i(k)\) are determined at each time step \(k\) on the basis of the actual value of the corresponding densities, i.e. \(\rho_{i-1}(k)\) and \(\rho_i(k)\).

As already pointed out in Section 2, the adoption of variable speed limits as control actions must be taken into account in the model, as done with expression (5) for the second-order model. Therefore, the prediction model to be considered in the finite-horizon problem is given by equations (12), in which, at each time step \(k = 0, \ldots, K - 1\), the values of \(\bar{v}_i(k)\) are obtained as follows:

\[
\bar{v}_i(k) = \begin{cases} \min \left( (1 + \alpha)\bar{v}^{\text{ctrl}}_i(k), V(\bar{\rho}_i(k)) \right) & i \notin I_v \quad (13) \\
V(\bar{\rho}_i(k)) & i \in I_v 
\end{cases}
\]

where \(V(\bar{\rho}_i(k))\) is given by (9).
We have compared the two models (the second-order model and the linearized first-order one) with different traffic patterns and, of course, the two models present different behaviours (an extensive numerical campaign is reported in Sacone and Siri (2010)). By assuming that the second-order model is actually more effective in representing the traffic behaviour (since the model represents also the dynamics of the traffic mean speed and because the second-order model has been validated in many case studies that can be found in the literature), the numerical comparisons show that the linearized first-order model overestimates the traffic densities whenever a congestion occurs. As a matter of fact, in all cases the traffic density determined with the first-order model keeps higher than the corresponding density obtained with the second-order model. Nevertheless, the first-order model is used in this work for determining the control measure. In this context, it can be expected that the derived control action is precautionary in congested conditions: this turns out to be acceptable (or, even, suitable for safety reasons) for the applicative field considered in this work.

4. THE MODEL PREDICTIVE CONTROL SCHEME

The control approach adopted in this paper for dealing with the problem of regulating traffic behaviour on freeway stretches is Model Predictive Control. In a MPC scheme, at the k-th time interval, a finite-horizon optimization problem is solved over a prediction horizon $K_p$, by optimizing a suitable objective function subject to constraints on control variables and on state variables. A sequence of optimal control variables from the k-th time interval to the $(k + K_p - 1)$-th are derived; the first element of the sequence becomes the control action at time $k$. This scheme is then applied at time interval $k + 1$, by updating the data of the finite-horizon problem using new measurements; this procedure is iterated for all the following time intervals.

In a MPC scheme, among other constraints, the finite-horizon optimization problem to be solved at each iteration also includes state equations. In this way, the state variables are constrained to fulfill the system state equations by realizing a prediction of the system behaviour over the prediction horizon $K_p$. In order to express equation (13), and in particular equation (9), in the optimization problem, it is necessary to introduce some binary variables indicating the relevant discretization segment for each section $i$ at time $k$ (on the basis of the value of the state variable $\rho_i(k)$), as follows:

$$ y_{i,l}(k) = \begin{cases} 1 & \text{if } \bar{\rho}_{i,l} \leq \rho_i(k) \leq \bar{\rho}_{i,l+1} \\ 0 & \text{otherwise} \end{cases} \quad i = 1, \ldots, N \quad l = 1, \ldots, M_i \quad k = 0, \ldots, K - 1 \quad (14) $$

Before stating the finite-horizon optimization problem, it is still necessary to introduce $V_i$, that is the set of discrete speed limit values for section $i$; as a matter of fact, the speed limits to be shown on variable message signs must be chosen among some (discrete) possible alternative values. Moreover, let $v_i^{ctrl}$ represent the minimum acceptable difference in speed limits that a driver can encounter.

In the finite-horizon optimization problem to be solved at time interval $k$, the traffic densities $\rho_i(h), \quad i = 1, \ldots, N, \quad h = k, \ldots, k + K_p - 1$, are the state variables, whereas the imposed speed limits $v_i^{ctrl}(h), \quad i \in I_v, \quad h = k, \ldots, k + K_p - 1$, are the control variables. This finite-horizon optimization problem can be stated as follows.

**Problem 1.** Given the initial conditions $\rho_i(k), \quad i = 1, \ldots, N$, find variables $y_{i,l}(h), \quad i = 1, \ldots, N, \quad l = 1, \ldots, M_i, \quad h = k, \ldots, k + K_p - 1$, $\rho_i(h), \quad i = 1, \ldots, N, \quad h = k + 1, \ldots, k + K_p - 1$ and the optimal control variables $v_i^{ctrl}(h), \quad i \in I_v, \quad h = k, \ldots, k + K_p - 1$ that minimize the cost function

$$ J = T \sum_{i=1}^{N} \sum_{h=k+1}^{k+K_p} \rho_i(h) \Delta_i \quad (15) $$

subject to

\begin{align*}
\rho_i(h + 1) &= \rho_i(h) + \frac{T}{\Delta_i} (\rho_{i-1}(h) \cdot \bar{v}_{i-1}(h) + \\
&- \rho_{i+1}(h) \cdot \bar{v}_{i+1}(h) + v_{i}(h) - s_i(h)) \\
&\quad \quad \quad i = 1, \ldots, N \quad h = k, \ldots, k + K_p - 1 \quad (16) \\
\bar{v}_i(h) &= \max_{l=1,\ldots,M_i} \left(\bar{v}_{i,l} \cdot y_{i,l}(h)\right) \\
&\quad \quad \quad \quad \quad \quad \quad \quad \quad i \in I_v \quad h = k, \ldots, k + K_p - 1 \quad (17) \\
\bar{v}_i(h) &= \min \left((1 + \alpha) v_i^{ctrl}(h), \bar{v}_i(h)\right) \\
&\quad \quad \quad \quad \quad \quad \quad \quad \quad i \in I_v \quad h = k, \ldots, k + K_p - 1 \quad (18) \\
\bar{v}_i(h) &= \max_{l=1,\ldots,M_i} \left(\bar{v}_{i,l} \cdot y_{i,l}(h)\right) \\
&\quad \quad \quad \quad \quad \quad \quad \quad \quad i \notin I_v \quad h = k, \ldots, k + K_p - 1 \quad (19) \\
\rho_i(h) - \bar{\rho}_{i,l} + M (1 - y_{i,l}(h)) &> 0 \\
&\quad \quad \quad \quad \quad \quad \quad \quad \quad i = 1, \ldots, N \quad l = 1, \ldots, M_i \quad h = k, \ldots, k + K_p - 1 \quad (20) \\
\rho_{i,l} - \rho_i(h) + M y_{i,l}(h) &> 0 \\
&\quad \quad \quad \quad \quad \quad \quad \quad \quad i = 1, \ldots, N \quad l = 1, \ldots, M_i \quad h = k, \ldots, k + K_p - 1 \quad (21) \\
\rho_i(h) - \rho_{i,l+1} + M y_{i,l}(h) &> 0 \\
&\quad \quad \quad \quad \quad \quad \quad \quad \quad i = 1, \ldots, N \quad l = 1, \ldots, M_i \quad h = k, \ldots, k + K_p - 1 \quad (22) \\
\rho_{i,l+1} - \rho_i(h) + M (1 - y_{i,l}(h)) &> 0 \\
&\quad \quad \quad \quad \quad \quad \quad \quad \quad i = 1, \ldots, N \quad l = 1, \ldots, M_i \quad h = k, \ldots, k + K_p - 1 \quad (23) \\
v_i^{ctrl}(h) - v_i^{ctrl}(h) &\leq v_i^{diff} \\
&\quad \quad \quad \quad \quad \quad \quad \quad \quad i \in I_v \quad h = k + 1, \ldots, k + K_p - 1 \quad (24) \\
v_i^{ctrl}(h) - v_{i,l+1}^{ctrl}(h) &\leq v_i^{diff} \\
&\quad \quad \quad \quad \quad \quad \quad \quad \quad i, l + 1 \in I_v \quad h = k, \ldots, k + K_p - 1 \quad (25) \\
v_i^{ctrl}(h) - v_{i,l}^{ctrl}(h) &\leq v_i^{diff} \\
&\quad \quad \quad \quad \quad \quad \quad \quad \quad i, l + 1 \in I_v \quad h = k + 1, \ldots, k + K_p - 1 \quad (26) \\
v_i^{ctrl}(h) &\in V_i \\
&\quad \quad \quad \quad \quad \quad \quad \quad \quad i \in I_v \quad h = k, \ldots, k + K_p - 1 \quad (27)
\end{align*}
\[ y_{i,l}(h) \in \{0, 1\} \quad i = 1, \ldots, N \quad l = 1, \ldots, M_i \quad h = k, \ldots, k + K_p - 1 \]  

(28)

where \( M \) is a sufficiently large number.

The control objective adopted in Problem 1 is the minimization of the Total Time Spent, as usually done in optimal control problems for avoiding congestion in freeways. Constraints (16)-(19) constitute the prediction model with the assumptions already described. Constraints (20)-(23) define the values of the binary variables \( y_{i,l}(h) \), as in (14). Constraints (24)-(26) limit the difference of speed limits that a driver can encounter. Finally, constraints (27)-(28) impose, respectively, that \( v_{\text{tri}}(h) \) can assume only discrete values belonging to the set \( V_i \) and variables \( y_{i,l}(h) \) are binary.

Since the sequence of optimal control variables computed at each time interval is found by on-line solving the finite-horizon optimization problem, attention must be posed on the computational complexity of the solution algorithm for such a problem. This aspect is crucial in real applications and it represents the main motivation that has inspired the present approach. A MPC scheme for freeway systems using the macroscopic second-order model as prediction model was already present in Caligaris et al. (2009). Such a control scheme showed its effectiveness, but its main weakness was related to the high computational burden for solving the finite-horizon optimization problem. Instead, the strength of the present work stands in the definition of a simplified prediction model. Consequently, the optimization problem to be solved at each time instant is simpler (even though it is still a mixed-integer nonlinear mathematical programming problem), both because of the lower number of state variables (and constraints) and thanks to the simplified form of the state equations. Moreover, as shown in the following section, preliminary experimental results show that this new approach can lead to good performances of the control scheme.

5. EXPERIMENTAL RESULTS

In order to evaluate the effectiveness of the proposed MPC scheme, some numerical experiments have been performed. We have considered as example case a two-lane freeway stretch composed of \( N = 10 \) road sections of equal length \( \Delta_i = 2 \) (expressed in kilometers), \( i = 1, \ldots, 10 \), and a time horizon of \( K = 30 \) time instants, each one of one minute (\( T = 1/60 \), in hours); the prediction horizon \( K_p \) is equal to 10. Moreover, we assume the free speed \( V_f = 120 \) and the jam density \( \rho_i^{\text{max}} = 300 \), \( i = 1, \ldots, 10 \). The values of the parameters included in the second-order model are standard values taken from the literature (see Papageorgiou et al. (1990a), Papageorgiou (1998)).

Moreover, note that macroscopic traffic flow models derive from a discretization of continuous models. To ensure that the discretization is correctly realized, some constraints on the values of \( T \) and \( \Delta_i \), \( i = 1, \ldots, N \), must be fulfilled, as detailed in Caligaris et al. (2010) for the adoption of a second-order model. In this case we also consider a discretization of the steady-state speed-density characteristic in \( M_i = 8 \) segments, \( i = 1, \ldots, 8 \).

The control scheme has been implemented in Matlab by using Tomlab/MINLP solver for the solution of Problem 1. We have performed an extensive simulative campaign in order to test the proposed MPC approach. An example is shown in Figs. 3 ÷ 6 where the traffic density and the traffic mean speed are plotted, in case of no control action and in case the MPC regulator is adopted. The considered traffic conditions refer to a congestion that is cleared very effectively by the proposed control scheme; in particular, the application of the proposed control technique drives the system state to a regular traffic condition faster than in the no-control case. In Fig. 7 the control actions (i.e. speed limits) obtained by applying the MPC scheme are displayed. Note that in the considered case we suppose...
that each road section is provided with a variable message sign, i.e. \( T_i \) = \{1, \ldots, N\}. Moreover, the set of discrete speed limit values is \( V_i = \{60, 70, \ldots, 120\} \), \( \forall i \in T_i \).

As pointed out throughout the paper, the computational complexity of the finite-horizon problem is very important if the control scheme is applied on line in real traffic networks. The finite-horizon optimization problems resulting from the considered example case are generally solved in some seconds and this proves the applicability of the MPC framework to real cases.

6. CONCLUSIONS

In this work we have proposed a Model Predictive Control scheme for avoiding congestion in freeways by means of variable speed limits. The finite-horizon optimization problem adopted within the MPC scheme is based on a linearized first-order dynamic model of traffic (obtained by approximating the speed-density characteristic as a piecewise constant function). This simplification in the prediction model is motivated by the fact that the relevant optimization problem can be solved in acceptable times also for applications in large freeway systems.

Present and future research is devoted to compare the proposed optimal control approach with other approaches already present in the literature, in terms of system performances and computational times. Moreover, we are trying to test the proposed MPC scheme on a real freeway stretch that is being equipped with variable message signs for speed limitations.

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