An integrated method using intuitionistic fuzzy set and linear programming for supplier selection problem

M. Shahrokhi*, A. Bernard, H. Shidpour*****

**Engineering department, University of Kurdistan, Sanandaj, P.O. Box: 416, Iran (Tel: +98-916-365-6344; e-mail: Shahrokhi292@ Yahoo.com).*
*** Ecole centrale de Nantes, 1, rue de la Noë, 44321 NANTES Cedex 3, France (e-mail: Alain.Bernard@irccyn.ec-nantes.fr)*
****Engineering department, University of Kurdistan, Sanandaj, P.O. Box: 416, Iran (e-mail:H.Shidpour@Yahoo.com).*

Abstract: Supplier selection is one of the most important activities in supply chain management. Aim of this process is to find suppliers which have the most compatible specifications with buyer`s requirements. This study develops an integrated approach by applying intuitionistic fuzzy set and linear programming technique. It uses two indicators to explain supplier's advantages and their flexibility for providing variant orders. It can be used to select appropriate suppliers in a group decision-making environment. A numerical experiment is given to illustrate application of proposed method.

Keywords: Intuitionistic fuzzy set, supplier selection, linear programming, satisfaction, flexibility

1. INTRODUCTION

In the recent years, competitive pressures are forcing enterprises to re-attend on theirs supply chain management (SCM) and use new strategies to design and develop engineering products to quickly and exactly responding the customers' demand. In these new strategies, to ensure the quality and performance of products, suppliers work closely with in-house designers to design some of sub-assemblies and components. Therefore, companies pay particular attention to the identification and selection appropriate suppliers. Supplier selection involves several conflicting criteria, where decision maker's knowledge is usually vague and imprecise. Thus it is a multi-criteria group decision-making problem.

Sonmez (2006) defined supplier selection as "the process of the suppliers where able to provide the buyer with the right quality products and/or services at the right price, at the right quantities and at the right time". Selection of suppliers has a direct impact on the financial, technical and operational performance of an organization. It influences products cost and quality. Therefore it affects directly competitiveness of the organization in the market and end customer satisfaction.

Supplier selection is a multiple criteria decision-making (MCDM) problem affected by qualitative and quantitative criteria (Chan et al. 2007). These factors are defined to measure important aspects of the supplier's business as financial and technical ability, support resources, quality systems and so on.

The overall objective of supplier selection process is to maximize overall value to the buyer, reduce buy risk, and build long term relationships between buyers and suppliers (Chena et.al, 2006).

This article develops a new method for supplier selection problem. Analytical hierarchy process (AHP) is used to evaluate the supplier selection factors.

2. LITERATURE

Researchers have been widely studied multi-criteria techniques to select the best suppliers. They are used many methods such as cluster analysis, case based reasoning systems, statistical models decision support systems ,data envelopment analysis, multi criteria decision making, analytical hierarchy process (AHP), analytical network process(ANP), TOPSIS and SMART. Total cost of ownership models, activity-based costing, artificial intelligence (neural network, fuzzy set theory), mathematical programming and some of hybrid models such as AHP-LP, ANP-GP, FAHP, F-TOPSIS.

A good review of the methods for supporting supplier selection is represented by Aissaoui et.al (2007) and Ho et.al (2010). Akarte et.al (2001) created an AHP system based upon web to evaluate the suppliers. It uses 18 criteria to evaluate the suppliers and related importance weightings are determined by using a pairwise comparison. Gencer et al. (2007) considered supplier selection as a multi-criteria decision problem. They developed a model usage of analytic network process (ANP) in supplier selection .ANP is used for describing relations between supplier selection criteria in a feedback systematic.

A hierarchy model based upon fuzzy set theory is presented by Chen et al. (2006) to select best supplier. They used linguistic variables to assess supplier factors. This model considered both quantitative and qualitative criteria.

Many works integrated several approaches to evaluate the performance of suppliers and to select the best suppliers.

Mendoza et al. (2008) used goal programming to develop an integrated AHP–GP approach to sort best suppliers while determine the optimal order quantity.

For first time, Amid et al. (2006) presented a fuzzy multi-objective linear model to overcome the vagueness of the information and used different weights for various objectives.

Önüt et al. (2009) developed a supplier evaluation approach based on the ANP and the TOPSIS methods, under the fuzzy environment, to help a telecommunication company. They used Fuzzy ANP to calculate criteria weights and Fuzzy TOPSIS to select a supplier.
Lee (2009) developed a fuzzy analytic hierarchy process (FAHP) model to evaluate suppliers, which incorporates the benefits, opportunities, costs and risks (BOCR) concept and a performance ranking of the suppliers is obtained.
The rest of this paper is structured as follows: section 3 discusses the proposed method for supplier selection. This section is included the overview of Intuitionistic fuzzy set, and developing model. Solution methodology present in section4. Section 5 represents a numerical example to select the best suppliers by suggested method and in final section; conclusion and future research is presented.

3. BACKGROUND

This paper develops a hybrid method by using intuitionistic fuzzy sets (IFS) and linear programming to select suppliers for manufacturing firms. Use of IFSs provides a formal language for explaining lack of information in the human reasoning, to generate decisions. The following sections describe intuitionistic fuzzy sets and applied method for solving multi-objective model.

3.1 *Intuitionistic fuzzy sets*

In some real-life situations, a decision maker (DM) may not be able to accurately express his/her preferences for alternatives due to that DM may not possess a precise level of knowledge and the DM is unable to express the degree to which one alternative are better than others. In such cases, the DM may provide his/her preferences with a degree of doubt. IFSs are suitable for these situations.
Intuitionistic fuzzy set (IFS) is a generalization of fuzzy set theory, introduced by Atanassov (1986). It characterized by a membership function and a non-membership function.
Intuitionistic fuzzy set A is introduced by:

$$A = \{ \mu_A(X), \nu_A(X) | x \in X\}$$

$$0 \le \mu_A(x) + \nu_A(x) \le 1$$

$\mu_A(x)$ and $\nu_A(x)$ are membership and non-membership functions, respectively.
IFSs have a third parameter that usually known as the DM`s hesitation degree. This index expresses lack of knowledge whether x belongs to A or not.

$$\pi_A = 1 - \mu_A(x) - \nu_A(x) \tag{1}$$

It is obvious that $0 \le \pi_A(x) \le 1$, for each $x \in X$.

Smaller $\pi_A(x)$ indicates more certain knowledge about x certain and vice versa. Obviously, when $\pi_A(x) = 0$, fuzzy set concept is resulted (Shu et al. 2006).
If A and B are two intuitionistic fuzzy sets, then:

$$A_1 \cdot A_2 = \{\langle x, \mu_A(x) \cdot \mu_B(x), \nu_A(x) + \nu_B(x) - \nu_A(x) \cdot \nu_B(x) \rangle | x \in X\} \tag{2}$$

$$A_1 + A_2 = \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x), \nu_A(x).\nu_B(x) \rangle | x \in X\} \tag{3}$$

$$\bar{A} = (\nu_A(x), \mu_A(x)) \tag{4}$$

$$\lambda A = \left\{ \left\langle 1-(1-\mu_A(x))^{\lambda}, \nu_A(x)^{\lambda} \right\rangle \middle| x \in X \right\} \tag{5}$$

Score function S of an intuitionistic fuzzy value shown as follows [Xu, 2007]:

$$S(A) = \mu_A(x) - \nu_A(x) \tag{6}$$

The larger S indicates the greater the intuitionistic fuzzy value A.

3.2 *Linear programming model*

This model is proposed to determine suppliers and to calculate the optimum order quantities among the selected suppliers. In order to formulate the model, notations of the model are defined as follow:

Indices:

$i = 1,2,\dots,n$	Index of suppliers
$j = 1,2,\dots,e_1$	Index of satisfaction factors
$k = 1,2,\dots,e_2$	Index of flexibility factors

Parameters:

D	Anticipated demand
C	Supplier capacity
SI	Satisfaction index
FI	Flexibility index
w_{1j}	Relative important of j^{th} element of SI
w_{2k}	Relative important of k^{th} elements FI
S_{ij}	Value of the j^{th} factor of SI for supplier i
F_{ij}	Value of the element j for FI of supplier i

Decision variables:

X_i	Ordered amount to supplier i
Y_i	1 if supplier i is selected, 0 otherwise

The objective functions and the constraints of this model are described as follow:

$$\max f_1 = \sum_{i=1}^{n} SI_i X_i \tag{7}$$

$$\max f_2 = \sum_{i=1}^{n} FI_i Y_i \tag{8}$$

$$SI_i = \sum_{i=1}^{n} \sum_{j=1}^{e_1} w_{1i} s_{ij} \tag{9}$$

$$FI_i = \sum_{i=1}^{n} \sum_{j=1}^{e_2} w_{2i} f_{ij} \tag{10}$$

$$\sum_{i=1}^{n} X_i \ge D \tag{11}$$

$X_i \leq C_i \qquad \forall i$ (12)

$X_i \leq MY_i \qquad \forall i$ (13)

$Y_i \leq X_i \qquad \forall i$ (14)

$X_i \geq 0 \qquad \forall i$ (15)

$y_i \in \{0,1\} \qquad \forall i$ (16)

Objective 7 maximizes satisfaction index of the suppliers and objective 8 maximizes flexibility index.
Constraints 9 and 10 show satisfaction and flexibility indices of supplier i, respectively. Constraint 11 says that ordered values to suppliers have to support buyer demand. Constraint 12 shows that ordered values to each supplier have to be less than its capacity.
Constraints 13 and 14 relate two variables X and Y, and if Xi>0 then Yi=1. M in constraint 13 is a large number.

3.3. To solve multi-objective model

Zimmermann (1978) used the following steps to solve multi-objective linear programming (MOLP) model, by applying fuzzy logic approach:
Step1. Solve the MOLP of as a single-objective linear programming model by using only one objective at a time and ignoring the others.

Step2. By using optimal solutions calculated from previous step, values for other functions are obtained and pay-off matrix is developed as follows:

$$\begin{array}{cc} & f_1 \quad \cdots \\ x_1^* & \left(f_{11}^* \quad \cdots \right. \\ \vdots & \vdots \quad \ddots \quad \vdots \\ x_n^* & \left. f_{n1} \quad \cdots \right. \end{array}$$

Here $x_1^*, \ldots$ are the optimal solutions of the objective functions $f_1(x), \ldots$.

Step3. Obtain lower bound (L) and upper bound (U) for each objective:

$.\, L_r = \min\{f_r(x_1), \ldots \} \quad r = 1,2,..,n$

$.\, U_r = \max\{f_r(x_1), \ldots \} \quad r = 1,2,..,n$

Step4. Define membership function for each objective.

$$\mu_{f_r(x)} = \begin{cases} 1 & f_r(x) \leq L_r \\ \dfrac{U_r - f_r(x)}{U_r - L_r} & L_r \leq f_r(x) \leq U_r \\ 0 & f_r(x) \leq L_r \end{cases} \qquad (17)$$

Step5. Convert multi-objective problem into a single objective problem.

$max\ \lambda$ (18)

$\lambda(U_r - L_r) \leq U_r - f_r(x) \qquad r = 1, \ldots, n$ (19)

$g(x) \leq b$ (20)

$x \geq 0\,, \lambda \in [0,1]$ (21)

4. THE PROPOSED APPROACH

This method uses IFSs to explain two indices, satisfaction and flexibility. Satisfaction Index (SI) is a measure of the extent to which a buyer is satisfied by a supplier capability and it is calculated by 3 importance factors of the quality, price, and lead-time. Flexibility Index (FI) shows the additional capability of the supplier to respond when buyer requirement change and is calculated by 2 factors: extra production volume and product variety.
FI in the product volume (FIvo) shows extra capacity percent what supplier can allocate buyer contrast changes in demand. FI in the product variety (FIva) show the ability to create different products.

The proposed approach includes the following steps:

Step1- Evaluate SI and FI.
Satisfaction functions are defined for satisfaction factors. Satisfaction functions for any factor show the buyer satisfaction`s measurement for the related factor.

Step 2- Determine relative weight of each element of index.
Relative score these indexes are determined by IFSs which is described as follows:
2-1- Determine a group of the decision makers and their weights $(w_D = w_{D1}, w_{D2}, \ldots, w_{D_v})$ which is expressed by linguistic variables. These linguistic variables are shown in table 1.
2-2- Construct intuitionistic fuzzy preference relations to determine score for each factor of each index by each DM.
Preference relations are expressed by linguistic variables (table 1). The DMs provides his/her intuitionistic preference for each pair of criteria. To calculate score, the following steps are implemented:

Table1. Linguistic variables for importance of each criterion and DMs

Linguistic values	IFNs
Very low (VL)	(0.1,0.9)
Low (L)	(0.15,0.25)
Medium low(ML)	(0.25,0.35)
Medium (M)	(0.5,0.4)
Medium high (MH)	(0.55,0.25)
High (H)	(0.85,0.1)
Very high (VH)	(0.9,0.1)

2-2-1- Calculate score of each factor, by each DM:

$$w_{c_i}^{v} = \frac{1}{n} \sum_{j=1}^{n} w_{ij}^{v}, \quad i = 1,2,\ldots,n \qquad (22)$$

$w_{c_i}^{v}$ is Averaged intuitionistic fuzzy value i of the criterion over all the other criteria which is concluded from vth DM. w_{ij}^{v} is intuitionistic preference relation of the criterion i on j that is determined by vth DM. Sum of the intuitionistic fuzzy numbers obtain by equation 3. Also, multiply a constant number in intuitionistic fuzzy number compute equation 5.

2-2-2- Obtain final score by using equations 2 and 3.

$$w_{c_i} = \sum_{v=1}^{m} w_{D_v} w_{c_i}^{v} \quad , \quad i = 1,2,\ldots,n \qquad (23)$$

Where w_{c_i} is intuitionistic fuzzy weight criterion i and w_{D_k} is weight of v[th] DM.

3-2- Obtain score by using equation 6.

Step 3- Putting the results of the previous step in the linear programming problem.

4. NUMERICAL EXAMPLE

A hair drier manufacturer wants to produce a new model. He needs by purchasing engines with power 2000w from external suppliers. After primary evaluations, 5 suppliers are selected as the qualified suppliers. Suppliers' data is represented in the table 2.

Table 2. Suppliers` information

supplier	price	leadtime	quality	variety	volume
1	15-.5	11-.8	3-.78	3/10	.2
2	12-.8	8-1	1.5-.94	2/10	.15
3	14-.6	6-.5	4-.67	4/10	.24
4	16-.4	10-1	3.5-.72	6/10	.12
5	11-.9	12-.6	2.5-.83	5/10	.09

Supplier selection process is shown below:

Step1- Evaluate SI and FI.

1-1-Determine satisfaction functions for any factor. Satisfaction function, indeed, shows desirable level of customers for each factor. These functions are shown in the table 3. Amount of the satisfaction for each factor is show in table 4.

2-1- Determine flexibility index.

FI in the product volume (FI-vo), shows the capacity of suppliers for answering extra demand. Proportion of this potential capability is shown in the 6th column of table 4. FI in the product variety (FI-va) shows the ability to create different products. We assume that the ten versions of the desired product are produced by different companies. Suppliers Ability to produce different products is shown in Table 4.

Step 2- Determine relative weight of each element of index.

2-1- Determine decision makers and their weights by linguistic variables. Intuitionistic weights of DMs are: (0.85, 0.1), (0.9, 0.1) and (0.85, 0.1).

Table 4. Suppliers` information

supplier	price	leadtime	quality	variety	Volume (%)
1	0.5	0.8	0.78	3/10	0.2
2	0.8	1	0.94	2/10	0.15
3	0.6	0.5	0.67	4/10	0.24
4	0.4	1	0.72	6/10	0.12
5	0.9	0.6	0.83	5/10	0.09

1- 2-2- Construct intuitionistic fuzzy preference relations to determine score of each factor index by each DM. Score of each factor of the index by each DM is calculated by equations 13. The results are presented in Table 5 and 6 for SI and FI.

3-2- Obtain intuitionistic fuzzy weights (IFWs).

To calculate IFWs, equations 22 and 23 are used. The results are presented in Table 7.

4-2-Final score

Final score for factors of SI and FI are calculated by using equation 6, and the results are shown in tables 7 and8, respectively.

4- Put weights in the linear programming model and solve it.

Weights are put in the linear programming model and order quantities to each supplier identified that are shown in table 9.

As you can see from table suppliers 1, 4 and 5, with values 300, 500 and 700, respectively, are selected.

6- CONCLUSION AND FUTURE RESEARCH

This article outlined a new method by using intuitionistic fuzzy set (IFS) and linear programming for supplier selection problem. To order select suppliers, two indices were introduced: satisfaction index (SI) and flexibility index (FI).

For SI, 3 factors quality, price and leadtime are defined. In contrast to the previous works, in this paper flexibility is determined as a factor with a detailed definition. Two factors, production volume and product variety, is developed for FI. The relative importance of the factors of SI and FI are calculated by IFS method.

By Using IFSs, decision-making process is more realistic.

In IFS, DM may provide his/her preferences with a degree of doubt in a more realistic form.

A linear programming model uses the relative weights of each factor to determine the most suitable suppliers.

There are a number of opportunities for expanding the research, including defining further factors or other indices or considering the inter-dependency between the evaluation factors.

Table 5. Determined score by DM for SI

Element SI	DM_1	DM_2	DM_3
Price	0.42,0.31	0.42,0.31	0.65,0.23
Leadtime	0.43,0.31	0.43,0.3	0.32,0.27
Quality	0.39,0.52	0.3,0.4	0.3,0.41

Table 6. Determined score by DM for FI

Element FI	DM_1	DM_2	DM_3
variety	0.73,0.22	0.39,0.42	0.53,0.35
volume	0.33,0.65	0.43,0.35	0.32,0.27

Table 3. Satisfaction functions

$U_p = \begin{cases} 1 & c \le 10 \\ \frac{20-c}{20-10} & c \ge 10 \end{cases}$	Satisfaction function for price: satisfaction measurement for buyer is desirable when goods price is lower or equal than10$.
$U_l = \begin{cases} \frac{r-5}{7-5} & 5 \le r \le 7 \\ 1 & 7 < r \le 10 \\ \frac{15-r}{15-10} & 10 < r \le 15 \end{cases}$	Satisfaction function for lead time: The product is desirable when its lead-time is between 7 and 10 days. Also, for lead-time less than a predefined limit (i.e. 7 days), due to problems related to inventory capacity, satisfaction of buyer reduced.
$U_q = \begin{cases} 1 & r \le 1 \\ \frac{10-r}{10-1} & 1 \le r \le 10 \\ 0 & r \succ \end{cases}$	Satisfaction function for quality: An order is desirable when defect rate of the product (r, as the defect percentage) is lower or equal than 1%.

Table 8. Final score FI

H,VH,H	variety	volume
IFW	0.86,0.06	0.68,0.1
score	0.8	0.58
Crisp weight	0.58	0.42

Table 9. Allocated values each supplier

Supplier	Ordered value
1	300
2	0
3	0
4	500
5	700

REFERENCES

Aissaoui, N., Haouari, M., Hassini,E.(2007).Supplier selection and order lotsizing modeling: Areview. *Computers and Operations Research*, volume (34-12), pp 3516–3540.

Akarte, M.M., Surendra, N.V., Ravi, B., Rangaraj, N. (2001). Web based casting supplier evaluation using analytical hierarchy process. *Journal of the Operational Research Society,* volume (52-5), pp 511–522.

Amid, A., Ghodsypour, S.H., O'Brien, C. (2006). Fuzzy multi objective linear model for supplier selection in a supply chain. *International Journal of Production Economics*, volume (104), pp 394 – 407.

Atanassov, K. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, volume (20), pp 87–96.

Chan, F.T.S., Kumar, N. (2007). Global supplier development considering risk factors using fuzzy extended AHP-based approach. *Omega*, volume (35), pp 417–431.

Chen, C.T., Lin, C.T., Huang, S.F. (2006). A fuzzy approach for supplier evaluation and selection in supply chain management. *International Journal of Production Economics*, volume (102-2), pp 289–301.

Gencer, C., Gurpinar, D. (2007). Analytic network process in supplier selection: A case study in an electronic firm. *Applied Mathematical Modelling*, volume (31), pp 2475–2486.

Ho, W., Xu, X., Dey, P.K. (2010). Multi-criteria decision making approaches for supplier evaluation and selection: A literature review. *European Journal of Operational Research*, volume (202), pp16–24.

Lee, A.H.I. (2009). A fuzzy supplier selection model with the consideration of benefits, opportunities, costs and risks. *Expert Systems with Applications*, volume (36), pp 2879–2893

Mendoza, A., Ventura, J.A. (2008). An effective method to supplier selection and order quantity allocation. *International Journal of Business and Systems Research,* volume (2-1), 1–15.

Onüt, S, Kara, S.S, Isik, E. (2009). Long term supplier selection using a combined fuzzy MCDM approach: A case study for a telecommunication company. *Expert Systems with Applications,* volume (36), pp 3887–3895

Shu, M. S., Cheng, C. H., Chang, J. R. (2006). Using intuitionistic fuzzy sets for fault tree analysis on printed circuit board assembly. *Microelectronics Reliability*, volume (46-12), pp 2139–2148.

Sonmez, M. (2006). A review and critique of supplier selection process and practices. Lough borough University Business School occasional papers series, No.2006:1.

Xu, Z. (2007). Intuitionistic preference relations and their application in group decision making. *Information Sciences*, volume (177), pp 2363–2379.

Zimmermann, H. J. (1978). Fuzzy programming and linear programming with several objective functions, *Fuzzy Sets and Systems,* volume (1), pp 45-55.