Adaptive Control of MIMO T-S Fuzzy Systems with General Delay Matrices *

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Abstract: This paper proposes a new solution framework for adaptive control of a class of discrete-time nonlinear MIMO systems in an input-output form based on discrete-time MIMO T-S fuzzy models with general delay matrices and unknown parameters. A prediction fuzzy model is derived and its minimum phase property is clarified. Based on the prediction fuzzy model, an adaptive control scheme is presented including parametrization and parameter estimation of the T-S fuzzy model with unknown parameters, parameter adaptation conditions for avoiding the singularity problem and the closed-loop stability proof. An illustrative example and simulation results are presented to demonstrate the studied new concepts and to verify the desired performance of the adaptive MIMO fuzzy control systems.

1. INTRODUCTION

In recent years, fuzzy control methodologies have emerged as promising ways in dealing with nonlinear control problems. Two typical fuzzy models, Mamdani fuzzy models and Takagi-Sugeno (T-S) fuzzy models have been proven to be universal approximators (Zeng and Singh [1995], Ying [1998]). They have been successfully applied to the stabilization control design and tracking control design for nonlinear single-input single-output (SISO) and multiple-input multiple-output (MIMO) nonlinear systems (Ying [1999], Jagannathan et al. [2000], Li and Tong [2003], Chiu [2006], Chen, Liu and Tong [2007], Boukroune et al. [2010]). In those approaches, either Mamdani fuzzy models or T-S fuzzy models are employed to approximate the unknown nonlinear functions (usually scalar functions) of a nonlinear system so that a model-based stabilization controller or tracking controller can be developed.

However, there are some studies concerning with nonlinear control design directly based on MIMO T-S fuzzy models. In this way, a MIMO nonlinear system is approximated by a MIMO T-S fuzzy model consisting of a group of "IF-THEN" fuzzy rules. The "IF" part of a fuzzy rule describes a local operation region of the nonlinear system and the "THEN" part represents a local linear model of the nonlinear system corresponding to the "IF" part. The global MIMO T-S fuzzy model is derived by fuzzily blending all the local MIMO linear models, which makes it a time-varying MIMO nonlinear model. Since in reality many nonlinear systems can be expressed locally in some form of linear mathematical models, it is practically appropriate to approximate those nonlinear systems by this kind of T-S modeling. In Wang et al. [2010], a continuous-time MIMO T-S fuzzy model is applied to approximate a nonlinear system through a group virtual linearized subsystems and an adaptive fuzzy controller is designed based on it to make the system outputs asymptotically track the desired output trajectories. Our work in this paper is concerned with developing an adaptive controller based on a discrete-time MIMO T-S fuzzy model. The discrete-time fuzzy control design is of both theoretical and practical significance since digital computers are used to implement these controllers. Our approach is the extension of the novel model-based adaptive fuzzy control approach rigorously developed in Feng [2010] for SISO single-delay fuzzy systems, to multivariable adaptive control of MIMO fuzzy systems with a general delay matrix.

In this paper, we develop a new solution framework for adaptive discrete-time fuzzy control of MIMO nonlinear systems, by modeling MIMO systems using discrete-time T-S fuzzy systems, parametrizing T-S fuzzy systems with uncertain parameters, designing and analyzing an adaptive control scheme for such systems, and establishing and evaluating desired adaptive control system properties. The new results of this paper include the derivation of a global prediction fuzzy system model for multiple-input multiple-output (MIMO) systems, parametrization and parameter estimation of MIMO fuzzy systems, development of an adaptive control scheme for MIMO fuzzy systems, stability and tracking analysis of such an adaptive control system, and illustration of new features and concepts of adaptive control for MIMO fuzzy systems. The design and analysis of this paper are not only for MIMO fuzzy systems but also applicable to a class of time-varying MIMO dynamic systems with characteristic time-variations extended from the fuzzy membership functions. We will describe the main problems in Section 2, give the solutions including the derivation of a MIMO T-S fuzzy prediction model and the design and analysis of an adaptive control scheme in Section 3, and present a simulation study in Section 4.

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2. PROBLEM STATEMENT

Consider a multiple-input multiple-output (MIMO) nonlinear system in its discrete-time input-output form

\[ y(t) = f(y(t-1), \ldots, y(t-n), u(t-d_0), \ldots, u(t-n)), (1) \]

where \( f(\cdots) \in R^r \) is a vector of nonlinear functions, \( y(\cdot) \in R^r \) is the system output signal, \( u(\cdot) \in R^r \) is the system input signal, \( t = 0, 1, 2, \ldots \), is the discrete-time variable, \( d_0 \) is the number of nominal system input-output delays. In this paper, we will address the general case with \( d_0 \geq 1 \), for the MIMO case characterized by a general system delay structure.

A prediction model of (1), in the form

\[
\begin{align*}
  y(t + d_0) &= f_d(y(t), y(t-1), \ldots, y(t-n+1), \nonumber \\
  u(t), u(t-1), \ldots, u(t-n+1)), (2)
\end{align*}
\]

for some function \( f_d(\cdots) \), is useful, as shown in the literature, for feedback control. If the function \( f \) is known, such a prediction model may be directly derived by iterations for the single-input single-output (SISO) case. For the MIMO case, the situation is more complicated, because of the interactions between system inputs and outputs. In Section 3.1, using a fuzzy system modeling method, we derive one such model and demonstrate that such a model has a sophisticated form for MIMO systems.

To handle the nonlinearity and uncertainty of \( f \) in deriving a parametrized model of (1), an effective method is to approximate the nonlinear plant (1) by some well-defined approximation functions. In this paper, we employ the fuzzy system theory, to develop such an approximate system model, using the following MIMO local T-S fuzzy model with the \( ith \) rule:

\[ R^i : IF \ \xi_1 \ \text{is} \ \mathcal{F}_{1}^i \ \text{and} \ \ldots \ \text{and} \ \xi_L \ \text{is} \ \mathcal{F}_{L}^i, \]

THEN \[ A^i(z^{-1})y(t) = B^i(z^{-1})u(t), (3) \]

\( i = 1, \ldots, N, \) where \( N \) is the number of fuzzy rules, \( \mathcal{F}_{j}^i \) being typically an interval of real numbers, called a fuzzy set associated with which there is a membership function \( F_j(\xi_j(t)) \) to indicate the degree of membership of \( \xi_j(t) \) in \( \mathcal{F}_{j}^i \), and the system dynamics matrices are

\[
\begin{align*}
  A^i(z^{-1}) &= I + A_1^i z^{-1} + \cdots + A_n^i z^{-n} \\
  B^i(z^{-1}) &= z^{-d_0^i} (B_{0}^i + B_1^i z^{-1} + \cdots + B_{n-1}^i z^{-n-1} + B_n^i z^{-n}) (4)
\end{align*}
\]

\[ d_0^i \leq n_0 \leq n \]

with \( A_j^i \in R^{r \times r} \) and \( B_j^i \in R^{r \times d_0} \) being constant matrices, and \( d_0^i \geq 1 \) being the nominal delay of the system (3). Note that in this expression, the matrices \( B_0^i \) are non-zero but may be singular (that is, \( \det[B_0^i] = 0 \)), which is a main feature of MIMO systems. Hence the delay \( d_0^i \) is only a nominal delay, and the essential delay of a MIMO system is characterized by a delay structure called an interactor matrix to be introduced in Section 3.1.

In this paper we will solve the following two problems.

**Problem I:** Derive a MIMO fuzzy system prediction model with a general input-output delay structure, based on the fuzzy rules (3), for a MIMO nonlinear system.

**Problem II:** Design an adaptive control scheme for the MIMO fuzzy prediction model, to achieve the control objective: closed-loop stability and asymptotic tracking of a desired output \( y_m(t) \) by the plant output \( y(t) \).

3. ADAPTIVE CONTROL SCHEME

In this section we solve the stated system modeling and adaptive control problems, by first deriving a global MIMO fuzzy prediction model, and then developing an adaptive control scheme to meet the control objective.

3.1 A MIMO Fuzzy System Prediction Model

Consider the transfer matrix \( T_i(z) = (A^i(z^{-1}))^{-1} B^i(z^{-1}) \) of the local system model in (3). Since \( d_0^i \geq 1 \), the transfer matrices \( T_i(z) \), \( i = 1, \ldots, N \), are all strictly proper. For output tracking control, we assume

\[ (A.1a): T_i(z), \ i = 1, \ldots, N, \ \text{all have full rank}. \]

The above condition means that there is a delay of at least one unit between each input and each output, and, by Assumption (A.1a), that the output function controllability of each local model is ensured.

**MIMO system interactor matrix.** We first present the following MIMO system characterization which plays an important role in MIMO system parametrization and control and will be used for MIMO fuzzy systems.

**Proposition 1.** (Goodwin and Sin [1984]) For any \( r \times r \) strictly proper full rank rational transfer matrix \( T(z) \) there exists a unique lower triangular polynomial matrix \( \xi_d(z) \), defined as the interactor matrix of \( T(z) \), of the form

\[
\xi_d(z) = \begin{bmatrix}
  z^{d_1} & 0 & \cdots & 0 \\
  z^{d_1} h_{21}(z) & z^{d_2} & \cdots & 0 \\
  \vdots & \vdots & \ddots & \vdots \\
  z^{d_1} h_{r1}(z) & z^{d_2} h_{r2}(z) & \cdots & z^{d_{r-1}} h_{r,r-1}(z) z^{d_r}
\end{bmatrix}, (6)
\]

where \( h_{ij}(z) \) are polynomials divisible by \( z \), and \( d_i \geq 1 \) are integers, \( j = 1, \ldots, r-1, \ i = 2, \ldots, r \), such that

\[
\lim_{z \to \infty} \xi_d(z) T(z) = K_p (7)
\]

is finite and nonsingular.

An important property of \( \xi_d(z) \) is that \( \xi_d^{-1}(z) \) is a strictly proper and stable transfer matrix. In the above MIMO characterization, \( K_p \) is called the high frequency gain matrix of \( T(z) \), and the inverse of the interactor matrix \( \xi_d(z) \) can be seen as the delay structure of \( T(z) \). For example, when \( \xi_d(z) \) is diagonal:

\[
\xi_d(z) = \text{diag}\{z^{d_1}, z^{d_2}, \ldots, z^{d_r}\} (8)
\]

we have \( \xi_d^{-1}(z) = \text{diag}\{z^{-d_1}, z^{-d_2}, \ldots, z^{-d_r}\} \), which represents the \( r \) delays in the \( r \) input-output channels.

In this paper, we consider the local prediction models (3) with a general (non-diagonal) interactor matrix (input-output delay structure), and assume:

\[ (A.1b): \text{the local model transfer matrices} \ T_i(z), \ i = 1, \ldots, N, \ \text{have a common known interactor matrix} \ \xi_d(z). \]
The associated high frequency gain matrices are
\[
\lim_{z \to \infty} \xi d(z)T_i(z) = K^i_p (9)
\]
which are nonsingular, \(i = 1, 2, \ldots, N\).

**MIMO fuzzy system prediction model.** We now derive a MIMO fuzzy system prediction model based on a general plant interactor matrix (delay structure). Following the procedure in deriving Theorem 5.2.4 of Goodwin and Sin [1984], with the interactor matrix \(\xi_d(z)\), we define the new variable
\[
\bar{y}(t) = \xi_d(z)[y](t),
\]
and express \(\bar{y}(t)\) in the predictor form:
\[
\bar{y}(t) = \alpha^i(z^{-1})[y](t) + \beta^i(z^{-1})[u](t),
\]
where \(\alpha^i(z^{-1}) = \alpha_0^i + \alpha_1^i z^{-1} + \cdots + \alpha_n^i z^{-(n-1)}\) and \(\beta^i(z^{-1}) = \beta_0^i + \beta_1^i z^{-1} + \cdots + \beta_m^i z^{-(m-1)}\), with \(\beta_0^i = K_p^i\) being nonsingular. In this expression,
\[
\alpha^i(z^{-1}) = G^i(z^{-1}), \quad \beta^i(z^{-1}) = F^i(z)z^{-d_0^i}B^i(z^{-1}),
\]
where \(F^i(z)\) and \(G^i(z)\) are the unique polynomial matrices satisfying
\[
\xi_d(z) = F^i(z)A^i(z^{-1}) + G^i(z^{-1}).
\]

In our study, we will use the local models (11) to form a global prediction fuzzy system model. To proceed, based on (11), using the standard technique of singleton fuzzification, product inference and center-average defuzzification, we obtain the following global MIMO prediction fuzzy (nonlinear) dynamic system model.

**Proposition 2.** Following a standard fuzzy modeling procedure, a nonlinear dynamic system (1), via the local fuzzy system model (3), can be approximated by a global fuzzy fuzzy system prediction model:
\[
\bar{y}(t) = \sum_{i=1}^N \mu_i \alpha^i(z^{-1})[y](t) + \sum_{i=1}^N \mu_i \beta^i(z^{-1})[u](t),
\]
where \(\mu_i\) is the normalized membership function: \(\mu_i(\xi) = \frac{\lambda^i(\xi)}{\sum_{i=1}^N \lambda^i(\xi)}\), \(\lambda^i(\xi) = \prod_{j=1}^L F^i_j(\xi_j), \ \xi = [\xi_1, \ldots, \xi_L]^T\) and \(\lambda^i(\xi) \geq 0, \ i = 1, \ldots, N\).

**Remark 1.** There are naturally approximation and modeling errors \(\Delta(y)(\cdot), u(\cdot), t)\) in representing the original nonlinear system (1) by the fuzzy system model (14):
\[
\bar{y}(t) = \sum_{i=1}^N \mu_i \alpha^i(z^{-1})[y](t) + \sum_{i=1}^N \mu_i \beta^i(z^{-1})[u](t) + \Delta(y)(t), u(t-1), \ldots, u(t-1), u(t-2), \ldots, t).
\]
Robust adaptive control and nonlinear damping/bounding design tools can be used to deal with approximation and modeling errors. In this study, the focus is on the design and analysis of some baseline adaptive control scheme for discrete-time MIMO fuzzy systems with general delay matrices in the form (14).

Next we define the control objective for such a system.

### 3.2 Control Objective

In this study, the control objective is to find an adaptive control law to generate the input signal \(u(t)\) for the system (14) with unknown parameters \(a_0^i, a_1^i, \ldots, a_{n-1}^i, \beta_0^i, \beta_1^i, \ldots, \beta^i_{m-1}, i = 1, \ldots, N\), to ensure closed-loop signal boundedness and asymptotic tracking of a given reference output signal \(y_{\text{ref}}(t)\) by the system output signal \(y(t)\), under the following assumptions:

(A.2): the system (14) is minimum phase.

(A.3): \(\sum_{i=1}^N \mu_i \beta_0^i\) is nonsingular, for all possible \(\mu_i\).

(A.4): the system order \(n\) is known.

Note that an upper bound of \(n\) would be sufficient, and for simplicity we assume the knowledge of \(n\).

**Minimum phase property.** We now clarify Assumption (A.2). For a regular multiple-input multiple-output (MIMO) linear time-invariant system
\[
A(z^{-1})[y](t) = z^{-d_0}B(z^{-1})[u](t), \ B(0) \neq 0
\]
with \(A(z^{-1})\) and \(B(z^{-1})\) similar to \(A^i(z^{-1})\) and \(B^i(z^{-1})\) in (5) and with an interactor matrix \(\xi_d(z)\) for \(T(z) = A^i(z^{-1})z^{-d_0}B^i(z^{-1})\), then \(\lim_{z \to \infty} \xi_d(T(z)) = K_p\) is finite and non-singular. Under the condition that \(T(z)\) has full rank (and so does \(B(z^{-1})\)), the values of \(z\) such that \(\det[B(z^{-1})] = 0\) defines the zeros of \(T(z)\).

Under the assumption that all zeros of \(T(z)\) are in \(|z| < 1\) and with \(d\) being the maximum degree of \(\xi_d(z)\), we can derive some relationship between \(u(t)\) and \(y(t)\), based on the expression
\[
\|u(t-d)\| \leq c_1 \|\bar{y}(t-d)\| + c_2 \sum_{\tau=0}^{t-1} \lambda^{t-\tau-1} \|\bar{y}(\tau-d)\|, \ (17)
\]
for all \(t \geq d\), for some constants \(c_1 > 0, c_2 > 0\) and \(\lambda \in (0, 1)\), where \(|\cdot|\) is the \(L^2\) vector norm. Since \(z^{-d}\xi_d(z)\) is a proper transfer matrix, the inequality (17) implies that
\[
\|u(t-d)\| \leq c_3 \|y(t)\| + c_4 \sum_{\tau=0}^{t-1} \lambda^{t-\tau-1} \|y(\tau)\|, \ \forall t \geq d, \ (18)
\]
for some constants \(c_3 > 0\) and \(c_4 > 0\).

Analogous to this nature of minimum phase regular MIMO LTI systems, we use the following minimum phase definition for the global fuzzy system model (14).

**Definition 1.** The fuzzy system (14) is minimum phase if the condition (17) is satisfied.

While the minimum phase property of a regular LTI system (15) can be checked using the knowledge of the zeros of \(B(z^{-1})\), the fuzzy system (14) is nonlinear and time-varying in nature and the zeros of \(B^i(z^{-1})\) can not completely determine its minimum phase property which also largely depends on the membership functions \(\mu_i(t)\).

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3.3 Nominal Control Law

If all the parameters in (14) are known, the feedback control law can be derived from the following equation:

\[ \sum_{i=1}^{N} \mu_i \beta_i (z^{-1}) [u(t) = y_m(t) - \sum_{i=1}^{N} \mu_i \alpha_i (z^{-1}) [y(t)] \tag{19} \]

where \( y_m(t) = \xi_d(z) [y_m](t) \) for a bounded reference signal \( y_m(t) \). Solvability of the above equation for \( u(t) \) is guaranteed by Assumption (A.3), in which the coefficient matrix of \( u(t) \) is \( \sum_{i=1}^{N} \mu_i \beta_0 \) nonsingular under Assumption (A.3).

The resulting closed-loop system with (19) is

\[ \ddot{y}(t) = y_m(t). \tag{20} \]

With Assumption (A.2) and \( \xi_d^{-1}(z) \) being an stable operator, the closed-loop characterized by (19) and (20) is stable in the sense that all signals are bounded, and \( \lim_{t \to \infty} (\ddot{y}(t) - y_m(t)) = 0 \), with a transient response produced by the dynamics of \( \xi_d(z) \) and the initial conditions.

For the case of unknown system parameters, we shall employ a parameter estimation algorithm to generate adaptive estimates of the parameters in (19), based on which an adaptive controller is implemented.

3.4 Parameter Estimation

To design an adaptive parameter algorithm, we express the global fuzzy prediction model (14) as

\[ \ddot{y}(t) = \Theta \dot{\phi}(t) \tag{21} \]

where \( \dot{\phi}(t) = [\phi_1^T(t), \ldots, \phi_N^T(t)]^T \), \( \Theta = [\Theta_1^T, \ldots, \Theta_N^T]^T \), \( \Theta_i = [\alpha_0^i, \alpha_1^i, \ldots, \alpha_{n-1}^i, \beta_0^i, \beta_1^i, \ldots, \beta_{n-1}^i]^T \), and \( \phi_i(t) = [\mu_i y(t)^T, \mu_i y(t-1)^T, \ldots, \mu_i y(t-n+1)^T, \mu_i u(t)^T, \mu_i u(t-1)^T, \ldots, \mu_i u(t-n+1)^T]^T \).

Let \( \Theta(t-1) \) be the estimate of the unknown parameter matrix \( \Theta \) at \( t-1 \), and define the estimation error

\[ \varepsilon(t) = \ddot{y}(t) - \dot{\Theta}(t-1) \dot{\phi}(t-1). \tag{22} \]

Recall that for \( \xi_d(z) = K_d z^d + \cdots + K_1 z \),

\[ \ddot{y}(t) = \xi_d(z) [y(t)] = K_d y(t + d) + \cdots + K_1 y(t + 1), \tag{23} \]

so that \( \ddot{y}(t-1) \) and all the components in the regressor \( \dot{\phi}(t-1) \) are available at time \( t \). Hence, we can estimate \( \Theta \) from the following recursive algorithm:

\[ \dot{\Theta}(t) = \Theta(t-1) + \frac{\gamma(t) \dot{\phi}(t-1) \varepsilon(t)}{\dot{\phi}(t-1) \dot{\phi}(t-1)}, \tag{24} \]

where \( \gamma(t) \in (\gamma_0, 2 - \gamma_0) \) is an adaptation gain, for some constant \( \gamma_0 \in (0, 1) \), \( c > 0 \) is a small design parameter, and an initial estimate \( \Theta(0) \) is chosen to make the matrix \( \sum_{i=1}^{N} \mu_i(\Theta(0)) \beta_0(0) \) nonsingular.

This adaptive algorithm has the desired properties:

Lemma 1. The parameter adaptation law (24), when applied to the fuzzy system (21), has the properties:

(i) \( ||\dot{\Theta}(t) - \Theta|| \leq ||\dot{\Theta}(t) - \Theta|| \leq ||\Theta(t) - \Theta|| \), for the matrix norm \( ||\dot{\Theta}(t) - \Theta|| = \sqrt{\text{tr}[(\Theta(t) - \dot{\Theta}(t-1) \dot{\phi}(t-1)]}. \]

(ii) \( \sqrt{\varepsilon^2(t) \dot{\phi}(t-1) \dot{\phi}(t-1)} \leq L^2; \)

(iii) \( \lim_{t \to \infty} \sqrt{\varepsilon^2(t) \dot{\phi}(t-1) \dot{\phi}(t-1)} = 0; \)

(iv) \( ||\dot{\Theta}(t) - \Theta(t-1)|| \leq L^2; \) and

(v) \( \lim_{t \to \infty} ||\dot{\Theta}(t) - \Theta(t-1)|| = 0, \) for any finite \( t_1 > 0 \).

The proof of this lemma is standard (Goodwin and Sin [1984]), based on the positive definite function

\[ V(\dot{\Theta}) = \text{tr}(\dot{\Theta}^2 \dot{\Theta}), \] \( \dot{\Theta} = \dot{\Theta} - \Theta. \tag{25} \]

While the adaptive law generates online estimates \( \dot{\Theta}(t) \) of the unknown parameter \( \Theta \), with desired stability and \( L^2 \) properties, the choice of the adaptation gain \( \gamma(t) \) needs to be further specified to ensure certain nonsingularity condition for adaptive control (see Section 3.4 below).

3.5 Adaptive Control Law

With the parameter estimation algorithm (24), we use the following adaptive feedback control law:

\[ \sum_{i=1}^{N} \mu_i \beta_i (z^{-1}) [u(t)] = y_m(t) - \sum_{i=1}^{N} \mu_i \alpha_i (z^{-1}) [y(t)], \tag{26} \]

or equivalently, in terms of \( \dot{\Theta}(t) \),

\[ \dot{\Theta}(t) \dot{\phi}(t) = y_m(t), \] \( y_m(t) = \xi_d(z) [y_m](t). \tag{27} \]

The coefficient matrix of \( u(t) \) in (26) is \( \sum_{i=1}^{N} \mu_i(t) \beta_i(t) \). Solvability of the control equation (26) requires that the matrix \( \sum_{i=1}^{N} \mu_i(t) \beta_i(t) \) is nonsingular for all \( t \).

A similar issue was encountered in Goodwin and Sin [1984] and Tao and Ioannou [1989] for the case of adaptive control of a regular MIMO system with \( N = 1 \) and \( \mu_1(t) = 1 \) so that \( \sum_{i=1}^{N} \mu_i(t) \beta_i(t) = \beta_0(t) \) as the coefficient matrix of \( u(t) \). For this case, some suitable choice of \( \gamma(t) \) can be made for the parameter estimation adaptive law to ensure that \( \beta_0(t) \) is nonsingular for any \( t \geq 0 \), under the condition that \( \beta_0(0) \) is nonsingular. The basic idea of selecting \( \gamma(t) \) is that, with a nonsingular \( \beta_0(0) \) as a part of the parameter matrix \( \Theta(0) \), a \( \gamma(1) \) can be chosen for the adaptive law to make \( \beta_0(1) \) as a part of the parameter matrix \( \Theta(1) \) nonsingular, and then a \( \gamma(2) \) can be chosen to make \( \beta_0(2) \) nonsingular, and so on.

In our current new problem of dealing with MIMO fuzzy systems, the solvability issue for \( u(t) \) in (27) is more complicated than that in Goodwin and Sin [1984] and Tao and Ioannou [1989] for regular MIMO systems, due to the presence of \( \mu_i(\xi(t)) \) in the system parametrization and estimation, for \( i = 1, 2, \ldots, N \), that is, the coefficient matrix of \( u(t) \) is \( \sum_{i=1}^{N} \mu_i(t) \beta_i(t) \) not just a simple \( \beta_0(t) \). The main difficulty is the time-variation of \( \mu_i(\xi(t)) \) in determining the parameter estimates \( \beta_i(t) \), \( i = 1, 2, \ldots, N \), to make \( \sum_{i=1}^{N} \mu_i(t) \beta_i(t) \) nonsingular.

The time-variation of \( \mu_i(\xi(t)) \) may invalidate a desired solvability property. On the other hand, when the time-variation of \( \mu_i(\xi(t)) \) is small, the solvability property may
be retained, that is, the nonsingularity of $\sum_{i=1}^{N} \mu_i(0) \hat{\beta}_i(1)$ may imply the nonsingularity of $\sum_{i=1}^{N} \mu_i(1) \hat{\beta}_i(0)$, or the nonsingularity of $\sum_{i=1}^{N} \mu_i(t-1) \hat{\beta}_i(t)$ may imply the nonsingularity of $\sum_{i=1}^{N} \mu_i(t) \hat{\beta}_i(0)$ which can be then used for the adaptive law (24) with a proper choice of $\gamma(1)$ to make $\sum_{i=1}^{N} \mu_i(1) \hat{\beta}_i(0)$ nonsingular.

Motivated by this observation, we need to assume:

(A.5): the signals $\mu_i(\xi(t)), i = 1, \ldots, N$, are such that either the nonsingularity of $\sum_{i=1}^{N} \mu_i(t-1) \hat{\beta}_i(t)$ implies the nonsingularity of $\sum_{i=1}^{N} \mu_i(t) \hat{\beta}_i(0)$, or the nonsingularity of $\sum_{i=1}^{N} \mu_i(t-1) \hat{\beta}_i(t-1)$ implies the nonsingularity of $\sum_{i=1}^{N} \mu_i(t) \hat{\beta}_i(1)$.

Clearly, this assumption is satisfied if $\mu_i(\xi(t)) - \mu_i(\xi(t-1))$ is sufficiently small, for all $i = 1, 2, \ldots, N$. It should be noted here that although $\mu_i(\xi(t))$ is time-varying, it is varying within $[0,1]$, by its definition.

We now present the following lemma which is similar to that in Goodwin and Sin [1984] and Tao and Ioannou [1989] but with the consideration of the time-variation of $\mu_i(\xi(t))$, to show how to select a proper adaptation gain $\gamma(t)$ in the adaptive law (24) to ensure the solvability of the adaptive control equation (27).

Lemma 2. Under Assumption (A.5) and with $\hat{\Theta}(0)$ is chosen such that the coefficient matrix of $u(0)$ is nonsingular (that is, $\sum_{i=1}^{N} \mu_i(0) \hat{\beta}_i(0)$ is nonsingular), then the adaptation gain $\gamma(t)$ for the adaptive law (24) can be chosen in the range $\gamma_0 < \gamma < 2 - \gamma_0$, $0 < \gamma_0 < 1$, to ensure that the coefficient matrix $\sum_{i=1}^{N} \mu_i(t) \hat{\beta}_i(t)$ of $u(t)$ in the adaptive control law (27) is nonsingular for all $t$.\[80]

Remark 2. It is the time-varying nature of the MIMO fuzzy system model (14), which requires, for the adaptive control system stability, Assumption (A.5) that the time-variations of the membership functions $\mu_i(\xi(t))$ are small. We should note that such an assumption is not needed for the SISO system case and some special MIMO cases, such as all local systems (3) whose transfer matrices are $T_i(z)$ having a common gain matrix $K_p = K_p(0)$ for which case, $\hat{\beta}_0 = K_p$, $i = 1, 2, \ldots, N$, such that $\sum_{i=1}^{N} \mu_i \hat{\beta}_i = K_p \beta_0$ and its estimate is $\sum_{i=1}^{N} \mu_i \hat{\beta}_i(t) = \hat{\beta}_0(t)$ with $\hat{\beta}_0 = \beta_0$, $i = 1, 2, \ldots, N$, such that $\hat{\beta}_0(t)$ can be ensured to be nonsingular for all $t > 0$, or $K_p$ may be different but are all lower triangular (or upper triangular) (for which case parameter projection can be employed to ensure $\sum_{i=1}^{N} \mu_i(t) \hat{\beta}_0(t)$ is nonsingular for all $t > 0$).

Before establishing the stability and tracking properties of the developed adaptive control system, we first present the following bounding property for the regressor vector $\phi(t)$.

Lemma 3. Under Assumption (A.2) and with $\bar{\ell}(t) = \xi_2(z) \bar{c}(t)$ for $\bar{c}(t) = y(t) - y_m(t)$ and $\xi_2(z)$ having degree $d$, the regressor $\phi(t)$ defined in (21) satisfies

$$||\phi(t-d)|| \leq \rho_1 + \rho_2 \max_{\tau=0,1,\ldots,t} ||\bar{c}(\tau-d)||$$

for some positive constants $\rho_1$ and $\rho_2$.

The proof of this lemma is given in (Qi and Tao [2011]).

We now establish the stability and asymptotic tracking properties of the closed-loop control system.

Theorem 1. The adaptive feedback control law (26), updated by the parameter estimation algorithm (24) and applied to the system (14) subject to Assumptions (A.1)–(A.4), ensures that all the closed-loop signals are bounded and $\lim_{t \to \infty} (y(t) - y_m(t)) = 0$.

Based on Lemma 1, 2 and 3, the above theorem can be proven (Qi and Tao [2011]).

Thus far, we have solved Problem II, the adaptive control problem formulated in Section 2, for the system (14) with uncertain parameters. Next, we present an illustrative example with simulation results to show the key design steps and the desired system performance.

4. SIMULATION STUDY

In this section we present a numerical example to show the fuzzy system modeling and parametrization and the feedback control designs, and to demonstrate the effectiveness of the proposed adaptive control scheme.

Considering the two-input-two-output T-S fuzzy model:

$$R^i : IF \quad y_1(t) \in \mathcal{F}_1^i \quad \text{and} \quad y_2(t) \in \mathcal{F}_2^i,$$

THEN $A^i(z^{-1})[y(t) = B^i(z^{-1})]u(t), \quad (29)$

where the parameter matrices are

$$A^i(z^{-1}) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 0.5 \end{bmatrix} z^{-1} + \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} z^{-2},$$

$$B^i(z^{-1}) = z^{-1} \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0.5 & 2 \\ 0 & 1 \end{bmatrix} z^{-1},$$

$$A^2(z^{-1}) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + 4 \begin{bmatrix} 0 & 0 \end{bmatrix} z^{-1} + \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} z^{-2},$$

$$B^2(z^{-1}) = z^{-1} \begin{bmatrix} 2 & 3 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} z^{-1}. \quad (30)$$

and the membership functions for $\mathcal{F}_1^i$ and $\mathcal{F}_2^i$ are shown in Fig. 1.

From (12) and (30), we can specify $\alpha^i(z^{-1}), i = 1, 2,$ and

$$\beta^1(z^{-1}) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0.5 & 2 \\ 0 & -0.5 \end{bmatrix} z^{-1},$$

$$\beta^2(z^{-1}) = \begin{bmatrix} 2 & 3 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ 0 & -0.2 \end{bmatrix} z^{-1}. \quad (31)$$

From (14), we can obtain the global fuzzy system with $\xi = [\xi_1, \xi_2]^T = [y_1(t), y_2(t)]^T$. From (31), the zeros of $\beta^1(z^{-1})$ and $\beta^2(z^{-1})$ are: $z_1 = 0.5, z_2 = 0.5, z_1 = 0.5, z_2 = 0.2$. Since in this case, $\beta^2(z^{-1}), i = 1, 2$, are first-order and triangular, it can be verified that the global prediction fuzzy model (14) is minimum phase, given that $\mu_i(\xi) \in [0,1], i = 1, 2.$

If the parameters of $\alpha^i(z^{-1})$ and $\beta^i(z^{-1})$ are unknown, the adaptive control scheme (26) is applied with the
In this paper, we have studied an adaptive fuzzy control scheme for nonlinear MIMO systems in the input-output form based on discrete-time MIMO T-S fuzzy models with general delay matrices. Similar to that for a regular adaptive control problem, a prediction model is crucial for solving a fuzzy adaptive control problem, which has been derived as a fuzzy prediction model with a general delay structure. A model-based adaptive controller is then designed and closed-loop stability and tracking performance analysis have been given. Simulation results demonstrated the desired stability and tracking performance of the developed adaptive fuzzy control systems.

REFERENCES


