Decentralized stabilization of continuous time large scale switched descriptors

Dalel Jabri*,**, Kevin Guelton*, Noureddine Manamanni*

* CReSTIC, University of Reims Champagne Ardenne, Moulin de la Housse, 51000 Reims, France
e-mail: {kevin.guelton; noureddine.manamanni; dalel.jabri}@univ-reims.fr, Tel: +33 3 26 91 83 86;
** MACS, University of Gabès, Route Médenine, 6029 Gabès, Tunisia

Abstract: Decentralized stabilization of large scale hybrid descriptors under arbitrary switching law is proposed in this paper. At first, one uses the propriety that a global large scale descriptor can be split into a set of small interconnected switched descriptors subsystems. Then, to ensure the stability of the overall closed-loop system, a set of switched controllers is employed. The stability conditions are then derived into Linear Matrix Inequalities (LMI) using a multiple switched Lyapunov candidate function. Finally, a numerical example is proposed to illustrate the effectiveness of the suggested decentralized approach.

1. INTRODUCTION

In the recent years, considerable research has been devoted to stability analysis and stabilisation issue for switched systems (Branicky 1997; Hespanha et al., 1999; Liberzon et al., 1999; Sun et al., 2001; Decarlo et al., 2002; Wang et al., 2004; Mansouri et al., 2008; Lin et al., 2009; Chesi et al., 2010). Switched systems are a subclass of hybrid dynamic systems composed of a family of time-invariant systems and a switching law specifying the actuation behaviour. Their wide area of applications includes computer networks, embedded control systems, traffic control systems, automatic highway systems, chemical process, see e.g. (Mourik et al., 2010; Corona et al., 2007).

In several primary results, dealing with the stability and stabilization issue of switched systems under arbitrary switched law, the basic problem is to find a common quadratic Lyapunov function able to verify some stability linear matrix inequality (LMI) conditions (Shorten et al., 2006). Despite the simplicity of LMI formulation, finding a common Lyapunov function can be a tricky challenge to rise and may leads to conservatism. To reduce the conservatism, some relaxed approaches are proposed using piecewise quadratic Lyapunov functions (Johanson, 2003) or switched Lyapunov functions (Daafouz et al., 2002; Mahmoud et al., 2009). In this study, we are interested in the design of decentralised switched controllers to insure the stability of continuous time large scale hybrid switched descriptors.

Switched descriptors, also known as switched singular or implicit systems, have the ability to represent a wider class of dynamic system. For instance, they allow preserving some structural or modelling information of physical systems in descriptor matrices as well as describing, in a same state space representation, both dynamic and static equations (Cobb, 1983). However, stability analysis and stabilisation issue of such switched systems has been seldom treated in the literature (Zhai et al., 2009a, b, c). This lack may be due to the fact that switching between several descriptors may increase the complexity of the design problem. Recently, new quadratic based approaches for both continuous-time and discrete-time switched autonomous linear descriptor switching under arbitrary law have been proposed (Zhai et al., 2010). However, these studies were only focusing on stability analysis of autonomous switched descriptors. Moreover, these stability conditions were obtained by assuming some modelling restrictions. Indeed, they consider the class of switched descriptors with common or same rank descriptor matrices $E_i$ between subsystems as well as each pair of matrices $A_i$ and $E_i$, describing subsystems dynamics, are pairwise commutative.

In other hand, the attention of some researchers has been focussed on stability and stabilization of large scale dynamical systems; see e.g. (Sandell et al., 1987, Mukaidani, 2006, Siljak et al. 2005, Jabri et al., 2009a, b, Barcelli et al. 2010). Nevertheless, few investigations can be found in the literature dealing with the problem of large scale switched systems stabilization (Mahmoud et al., 2010; Jabri et al., 2010). In (Mahmoud et al., 2010), LMI based controller design conditions for interconnected continuous time switched systems with bounded uncertainties and bounded time-varying state-delay have been proposed. Unfortunately, they don’t take into account some necessary conditions (at the switched time) to ensure the stabilization of the overall switched system. In our preliminary works, one has proposed LMI based stabilization for discrete-time large scale switched systems (Jabri et al., 2010). Moreover, to the best of the authors’ knowledge, the stabilization issue of interconnected switched descriptors hasn’t been investigated in literature.

Motivated by this situation, the objective of this paper is to propose a methodology for the design of decentralized switched controller able to stabilize large scale switched descriptors for arbitrary switching law.

This paper is organized as follows: first, the studied class of continuous-time interconnected switched descriptors will be
described. Then, a switched state feedback decentralized controller is proposed. Next, the problem statement and the main result are presented. Finally, a simulation example is given to illustrate the efficiency of the designed approach.

2. SYSTEM DESCRIPTION

Consider the class of hybrid systems $S$ composed of $n$ continuous time interconnected switched descriptor subsystems $S_i$ represented in figure 1.

The state equation of the interconnected switched descriptor $S$ is given as follows:

For $i = 1, ..., n$,

$$
\sum_{j=1}^{n} \xi_{i,j}(t) E_{i,j} \dot{x}_i(t) = \sum_{j=1}^{n} \xi_{i,j}(t) \left[ A_{i,j} x_i(t) + B_{i,j} u_i(t) + \sum_{a=1}^{m} F_{a,i,j} x_a(t) \right]
$$

(1)

where $x_i(t) \in \mathbb{R}^{n_i}$, $u_i(t) \in \mathbb{R}^{m_i}$ represent respectively the state and the input vectors associated to the $i$th subsystem. $x_a(t) \in \mathbb{R}^{n_a}$ denotes the state vector of the $a$th model with $a = 1, ..., n$ and $a \neq i$, $m_i$ is the number of modes of the $i$th model. $E_{i,j} \in \mathbb{R}^{n_i \times n_j}$ (if necessary singular), $A_{i,j} \in \mathbb{R}^{n_i \times n_j}$, $B_{i,j} \in \mathbb{R}^{n_i \times m_j}$ are constant matrices describing the local dynamics, $F_{a,i,j} \in \mathbb{R}^{n_a \times n_j}$ are matrices representing the interconnections expressing the influence of the $a$th subsystem on the $i$th one, $\xi_{i,j}(t)$ are the switching rules.

The latter are considered unknown but assumed to be real time available. These are defined such that the $i$th subsystem is active in the $j$th mode as follow:

$$
\begin{align*}
\xi_{i,j}(t) &= 1 & \text{if } j = l \\
\xi_{i,j}(t) &= 0 & \text{if } j \neq l
\end{align*}
$$

(2)

In order to ensure the stabilization of the overall closed-loop interconnected switched descriptor $S$, a decentralized state feedback switched control law is proposed. The basic idea is to synthesize a global controller composed of $n$ local switched controllers assuming that each local controller is able to ensure the stability of the subsystem $S_i$ regarding to the interconnections among the others subsystems. Therefore, this set of decentralized controllers is proposed as:

For $i = 1, ..., n$,

$$
\sum_{j=1}^{n} \xi_{i,j}(t) E_{i,j} \dot{x}_i(t) = \sum_{j=1}^{n} \xi_{i,j}(t) \left[ A_{i,j} x_i(t) + B_{i,j} u_i(t) + \sum_{a=1}^{m} F_{a,i,j} x_a(t) \right]
$$

(3)

where $K_{i,j}$ are the gain matrices to be synthesized.

Substituting (3) into (1), one obtains the overall closed-loop system $S$ described as:

$$
\sum_{j=1}^{n} \xi_{i,j}(t) E_{i,j} \dot{x}_i(t) = \sum_{j=1}^{n} \xi_{i,j}(t) \left[ (A_{i,j} + B_{i,j} K_{i,j}) x_i(t) + \sum_{a=1}^{m} F_{a,i,j} x_a(t) \right]
$$

(4)

The goal is now to design the matrices $K_{i,j}$, for $i = 1, ..., n$, $j = 1, ..., m_i$, in order to guarantee the stability of the whole closed-loop interconnected switched descriptor system (4). To do so, the following lemma will be useful in the sequel.

Lemma 1 (Zhou and Khargonedkar, 1988): Let us consider two matrices $A$ and $B$ with appropriate dimensions, the following inequality is always satisfied:

$$
A^T B + B^T A \leq \tau A^T A + \tau^{-1} B^T B
$$

(5)

with $\tau$ a positive scalar.

As usual a star (*) indicates a transpose quantity in a matrix. For more readability of the mathematical expressions, the time $t$ will be omitted in the sequel when there is no ambiguity.

3. DECENTRALIZED CONTROLLER DESIGN

In this section, the attempt is to synthesise a decentralized switched controller able to stabilize the closed-loop interconnected switched descriptor system (4). The main result is given in the following theorem.

Theorem 1: Let us consider the closed-loop system $S$ composed of $n$ continuous-time interconnected switched descriptors $S_i$ described by (1). Assuming that, for each system $i$, the active mode is denoted by $j_i$. The overall closed-loop system $S$ is stabilized by the set of $n$ decentralized switched state feedback control laws described
in (3), if there exists, for all combinations of \( i = 1, \ldots, n \) and \( j = 1, \ldots, m \), the matrices \( X_{i,j} = (X_{i,j}^T)^T > 0 \), \( X_{i,j}^T, X_{i,j}^* \) and \( Y_{i,j} \), and the real positive scalars \( \delta_{i,j}, \delta_{2,j}, \ldots, \delta_{i-1,j} \), \( \delta_{i+1,j}, \ldots, \delta_{i,j} \), except \( \delta_{i,j} \) such that the following LMIs are satisfied:

\[
\begin{pmatrix}
\Psi_{i,j} & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
X_{i,j} & -\delta_{i,j} I & 0 & \cdots & \cdots & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
X_{i,j} & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\
\end{pmatrix} < 0 \quad (6)
\]

with \( X_{i,j} = \begin{bmatrix} X_{i,j}^T & 0 \\ X_{i,j}^* & X_{i,j}^* \end{bmatrix} \) and

\[
\Psi_{i,j} = \begin{pmatrix}
\left(X_{i,j}^T + X_{i,j}^*\right) & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\left(X_{i,j}^T + X_{i,j}^*\right) & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\left(X_{i,j}^T + X_{i,j}^*\right) & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\end{pmatrix} > 0
\]

Replacing \( K_{i,j} = Y_{i,j} (X_{i,j}^* - X_{i,j})^{-1}. \)

**Proof:** Let, for \( i = 1, \ldots, n \), \( \tilde{x}_i = [x_{i_1} \ldots x_{i_m}] \) be the extended state vectors. The overall closed-loop interconnected descriptor system described in (1) can be rewritten, with the proposed notations, as:

\[
V(x_{i_1}, \ldots, x_n) = \sum_{i=1}^{n} v_i(x_i) \quad (8)
\]

with \( v_i(x_i) = \sum_{j=1}^{n} \xi_{i,j}(\tilde{x}_i^T \tilde{E}(X_{i,j}^*)^{-1} \tilde{x}_i) > 0 \)

and \( \tilde{E}(X_{i,j}^*)^{-1} = (X_{i,j}^*)^{-1} \tilde{E}. \)

The previous symmetric condition is verified with \( X_{i,j} = \begin{bmatrix} X_{i,j}^T & 0 \\ X_{i,j}^* & X_{i,j}^* \end{bmatrix} \) and \( X_{i,j}^* = (X_{i,j}^* )^T > 0. \)

Then, assuming that the current mode (at time \( t \)) is \( j_i \) and the upcoming one (at time \( t' \)) is \( j_i' \), one has:

\[
\begin{aligned}
\xi_{i,j}(t) &= 1 \\
\xi_{i,j'}(t') &= 0 \\
\xi_{i,j'}(t') &= 1
\end{aligned}
\quad (9)
\]

and the interconnected closed-loop switched descriptor (7) is stable if the condition (10) and (11) are satisfied.

\[
V(x_{i_1}, \ldots, x_n) = \sum_{i=1}^{n} \tilde{x}_i^T \tilde{E}(X_{i,j}^*)^{-1} \tilde{x}_i + \tilde{x}_i^T (X_{i,j}^*)^{-1} \tilde{E} \tilde{x}_i < 0 \quad (10)
\]

where \( t' \) are the switching instants.

First, one deals with (10). Substituting (8) into (10), it yields:

\[
\sum_{i=1}^{n} \sum_{a=1}^{m} \left( \frac{n-1}{(n-1)} \tilde{x}_i^T (\tilde{A}_{i,j}(X_{i,j})^{-1} + (X_{i,j})^{-1} \tilde{A}_{i,j}) \tilde{x}_i \right) < 0 \quad (11)
\]

Using Lemma 1, the inequality (12) can be bounded by:

\[
\sum_{i=1}^{n} \sum_{a=1}^{m} \left( \frac{n-1}{(n-1)} \tilde{x}_i^T (\tilde{A}_{i,j}(X_{i,j})^{-1} + (X_{i,j})^{-1} \tilde{A}_{i,j}) \tilde{x}_i \right) < 0 \quad (13)
\]

with \( \tau_{i,a} > 0 \).

Assuming that, for \( i = 1, \ldots, n \), \( \tau_{i,a} = 0 \), one can write

\[
\sum_{a=1}^{m} \tau_{i,a}^T \tilde{x}_a = \sum_{p=1}^{n} \tau_{i,p}^T \tilde{x}_p \text{, therefore (13) becomes:}
\]

\[
\sum_{i=1}^{n} \sum_{a=1}^{m} \left( \tilde{x}_i^T (X_{i,j})^{-1} + (X_{i,j})^{-1} \tilde{A}_{i,j} \right) \tilde{x}_i + \sum_{p=1}^{n} \tau_{i,p}^T \tilde{x}_p < 0
\]

which is equivalent to:
\[
\sum_{i=1}^{n} \dot{x}_i = \left( \bar{A}_{i,i} \right) (X_{i,i})^{-1} + \left( X_{i,i} \right)^{-1} \bar{A}_{i,i} + \left( X_{i,i} \right)^{-1} \sum_{a=1}^{n} \left( \tau_{i,a} \bar{F}_{i,a,i} \right) (X_{i,i})^{-1} \dot{x}_i < 0
\]

Inequality (15) is verified \( \forall x_i \) if

For \( i = 1, \ldots, n \) and \( j_i = 1, \ldots, m_i \),

\[
\sum_{i=1}^{n} \dot{x}_i = \left( \bar{A}_{i,i} \right) (X_{i,i})^{-1} + \left( X_{i,i} \right)^{-1} \bar{A}_{i,i} + \sum_{a=1}^{n} \tau_{i,a} \bar{F}_{i,a,i} (X_{i,i})^{-1} \dot{x}_i < 0
\]

Then, left and right multiplying (16) by \( X_{i,i} \), it yields,

For \( i = 1, \ldots, n \) and \( j_i = 1, \ldots, m_i \),

\[
X_{i,i} \bar{A}_{i,i} + \bar{A}_{i,i} X_{i,i} + \sum_{a=1}^{n} \tau_{i,a} \bar{F}_{i,a,i} \bar{F}_{i,a,i}^T + \sum_{p=1}^{n} \tau_{i,p} X_{i,i} X_{i,i} < 0
\]

Let us recall that \( \tau_{i,i} = 0 \), applying the Schur complement and using the extended matrices defined in (7), (17) is equivalent to:

For \( i = 1, \ldots, n \) and \( j_i = 1, \ldots, m_i \)

\[
\begin{pmatrix}
\Psi_{i,i} & \Psi_{i,j} \\
\Psi_{j,i} & X_{i,i}^{-1} I - \tau_{i,j}^2 J \\
\end{pmatrix}
< 0
\]

(18)

with \( \Psi_{i,j} = \left( \begin{array}{cc} X_{i,i}^T & X_{i,j} \\
\Psi_{i,j} & \Psi_{j,j} \end{array} \right) \),\n
\[
\Psi_{i,j} = \left( X_{i,i} \right)^T A_{i,j} X_{i,j} + B_{i,j} K_{i,j} X_{i,j} - E_{i,j} X_{i,i}^T \]

and

Now, let us focus on the inequality (11). Let us recall that \( \bar{E} (X_{i,i})^{-1} = (X_{i,i})^{-1} \bar{E} \) and the continuity of the Lyapunov function (8) (at the switching instant \( t' \)) must be guaranteed by (11). Thus, one can write \( X_{i,i} \bar{E} = \bar{E} X_{i,i} \), which is verified \( \forall x_i \) if:

For \( i = 1, \ldots, n \), \( j_i = 1, \ldots, m_i \), and \( j_i' = 1, \ldots, m_i' \),

\[
X_{i,i} = X_{i,i}^T
\]

(19)

Substituting (19) into (18) and then using the changes of variable \( \tau_{i,a} = \tau_{i,a}' \), one obtains the LMI conditions (6). That ends the proof.

4. NUMERICAL EXAMPLE

In order to illustrate the efficiency of switched decentralized controller design approach, we consider the following hybrid systems composed of two interconnected switched descriptors with different dimension given by:

Subsystem 1:

\[
\sum_{j=1}^{2} \dot{x}_{1,j}(t) E_{1,j} \dot{x}_1(t) = \sum_{j=1}^{2} \dot{x}_{2,j}(t) \bar{A}_{1,j} x_1(t) + B_{1,j} u_1(t) + F_{1,j} x_1(t)
\]

with \( E_{1,1} = \begin{bmatrix} 1 & 0.02 \\ 0 & 1 \end{bmatrix} \), \( A_{1,1} = \begin{bmatrix} -2 & 1 \\ 0 & -0.1 \end{bmatrix} \), \( B_{1,1} = \begin{bmatrix} -0.2 \\ 0 \end{bmatrix} \), \( F_{1,1} = \begin{bmatrix} 0.01 & 0.1 \\ 0.01 & 0.1 \end{bmatrix} \), \( E_{1,2} = \begin{bmatrix} 1 & 0 \\ -0.03 & 1 \end{bmatrix} \), \( A_{1,2} = \begin{bmatrix} -1 & 1 \\ -0.1 & -2 \end{bmatrix} \), \( B_{1,2} = \begin{bmatrix} -1.5 \\ -0.5 \end{bmatrix} \), \( F_{1,2,2} = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.2 \end{bmatrix} \).

Subsystem 2:

\[
\sum_{j=1}^{2} \dot{x}_{2,j}(t) E_{2,j} \dot{x}_2(t) = \sum_{j=1}^{2} \dot{x}_{2,j}(t) \bar{A}_{2,j} x_2(t) + B_{2,j} u_2(t) + F_{2,j} x_2(t)
\]

with \( E_{2,1} = \begin{bmatrix} 1 & 0.01 \\ 0 & 1 \end{bmatrix} \), \( A_{2,1} = \begin{bmatrix} -2 & 0 \\ 0 & -0.1 \end{bmatrix} \), \( B_{2,1} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} \), \( F_{2,1,1} = \begin{bmatrix} 0.01 & 0.6 \\ 0.2 & 0.1 \end{bmatrix} \), \( E_{2,2} = \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix} \), \( A_{2,2} = \begin{bmatrix} 0.2 & 1 \\ 0 & 0.1 \end{bmatrix} \), \( B_{2,2} = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} \), \( F_{2,2,2} = \begin{bmatrix} 0.01 & 0.5 \\ 0.2 & 0.2 \end{bmatrix} \).

For simulation purpose, let us assume that each subsystems switched under the law defined by the surfaces:

\[
S_{11} = 0.9 x_{11} + x_{12}, \quad S_{12} = -0.2 x_{11} + 9 x_{12}, \quad S_{21} = -x_{21} + x_{22} \quad \text{and} \quad S_{22} = x_{21} - 2 x_{22}.
\]
A set of a decentralized switched controller (3) can be synthesized using theorem 1. The Matlab LMI toolbox is used to solve the LMI conditions (6) and one obtains the following gain matrices:

\[
K_{1,1} = \begin{bmatrix} 1.365 & -8.013 \end{bmatrix}, \quad K_{1,2} = \begin{bmatrix} 0.428 & 0.636 \end{bmatrix}, \\
K_{2,1} = \begin{bmatrix} -27.88 & -33.24 & -15.75 \end{bmatrix}, \quad K_{2,2} = \begin{bmatrix} -30.67 & -28.9 & -18.52 \end{bmatrix}.
\]

The close-loop subsystem dynamics are shown in Figure 2 for the initial states \( x_1(0) = [2 \ 2]^T \) and \( x_2(0) = [-1 \ 1.5 \ -1]^T \).

![Fig. 2. States dynamics of the overall closed loop continuous time interconnected switched descriptor.](image)

Figure 3 shows the control signals as well as the switching modes’ evolution. As expected, the synthesized decentralized switched controller stabilize the overall large scale switched descriptor \( S \).

![Fig. 3. Control signals and switched laws evolution.](image)

Figure 4 shows that Lyapunov functions \( v_i(x_i) \) and \( v(x_2) \) of each subsystem, as well as the global Lyapunov function \( V(x_1, x_2, \ldots, x_n) \), are monotonously decreasing along the systems’ trajectories.

![Fig. 4. Lyapunov functions evolutions.](image)

5. CONCLUSIONS

In order to stabilize a wider class of descriptors enlarging the results found in the literature (Zhai et al., 2010), this paper have established LMI based stabilisation for large scale switched descriptors under arbitrary switching law. The considered class is represented by a set of interconnected switched descriptors. Thus, a set of decentralized switched controllers are proposed. Hence, using multiple Lyapunov functions, LMI stability conditions allowing the design of such decentralized controllers are derived. Finally, to show the efficiency of the proposed decentralized control approach, a numerical example has been provided.

ACKNOWLEDGEMENT

This work has been done under the support of the GIS 3SGS within the framework of the project COSMOS, the RCA and the FEDER within the CPER MOSYP. The authors would like to thank Ms Shannon Finipa for her valuable comments.

REFERENCES


