Stabilization on A Class of Networked Control Systems with Random Communication Delay

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Abstract: This paper is concerned with the stabilization problem of wireless networked control systems with network-induced delays of time-varying Markovian characterization. The effects of the sample rate changing on the network induced delay is the main topic. At first place, the time delay matrices changing stochastically and arbitrarily are considered. A state feedback controller is designed to guarantee the resulting close-loop system stochastically stable. Following that, a bridge is built to give out a uniform work. A numerical example is presented to illustrate the effectiveness and potential of the developed theoretical results.

1. INTRODUCTION

As wireless network technology is becoming more and more widespread, industrialists and researchers are finding the potentials of wireless network control systems (WNCS)[1, 2, 3]. However, this kind of networks are very vulnerable to environmental disturbances. When a node enters a wireless communication region, the variance of the sending frequency of this node will change other nodes’ delay distribution under the 802.15.4 situation[4]. This kind of frame contention builds up the network induced delay, because frames may remain longer in the buffer bas the other nodes’ competition. Many researchers would like to model this kind of delay as a homogeneous Markov chain, e.g.[5], [6], which is mostly appeared in real communication systems, the current time delay is usually related to the previous time delay[7]. In [5] a controller was designed to stabilize a discrete-time markovian jump linear system via a delayed network.

Efforts like deadbands[8] and model-based feedback[9] are introduced to construct communication control strategies. This kind of control strategies may prevent network from being saturated and buffer queues building up. Sample rate adaption is another form of reduced communication contention, in which the sample rate is varied on the basis of either control performance, network performance or a combination of both[10]. Being independent with controller and protocol makes sample rate adaption more suitable for integration with the current network control technology.

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Consider a communication situation shown in figure 1 using sample rate adaption. The computer takes the part as the controller, plant and the sampler. Two serial ports of the computer connect to two wireless nodes. Node 1 is connected to controller, and node 2 is connected to the sampler. Such kind of WNCS configuration gives prominent to the wireless link part easily. The computer changes the sample rate according to a certain rules. If there is a homogeneous Markov transition matrix(TM) π is used to describe the delay transition from node 2 to node 1 at sample rate f > 0, when the sample rate changes to some other value, i.e., f’ = f/c, c > 0, c ≠ 1, because this will change the frame arrival frequency from node 2 to node 1, which would vary the contention situation at node 1. So it is more reasonable to use different TM to describe the transmission delay of the WNCS under different sample rates.

This paper focuses on the effects brought in the wireless communication contention when the sample rate is changing. In order to describe the vary of the sample rate rules, a homogeneous Markov TM is employed to describe it, firstly. Second, the sample rate changing arbitrarily is taken into consideration. To bridge the gaps between the first and the second case, the elements of the transition matrix in the first case is partially unknown is considered. In the following of this paper, a method to stabilize a plant sampled by sample rate adaption and controlled over a WNCS with different delay TM(DTM) for different sample

Fig. 1. A wireless communication situation
rates is given. The rest of this paper is organized as follows. Problem formulation is presented in section 2. The problem of stability analysis is introduced in section 3. Section 4 gives the controller design. A simulation result verifies the effectiveness of the presented method is acquired in section 5. Section 6 concludes all the work.

2. PROBLEM FORMULATION

It is assumed that there are at most α different sampling rates for the sample rate adaptation mechanism. A index set \( S \triangleq \{1, 2, ..., \alpha\} \), here is used to represent different sampling rate with different number, 1, 2, ..., \( \alpha \). The sequence \( \theta(k) \) is used to indicate the index number of the sampling rate of the plant at time \( k \). For example, if the sampling rate at time \( k \) is the \( i \)th element in \( S \), then we have \( \theta(k) = i \).

Consider a continuous time linear system which is sampled by the sample rate \( \theta(k) \) as
\[
x(k + 1) = \tilde{A}(\theta(k))x(k) + \tilde{B}(\theta(k))u(k)
\]
where, \( x(k) \in \mathbb{R}^{n_x}, u(k) \in \mathbb{R}^{n_u} \). \( \tilde{A}(\theta(k)), \tilde{B}(\theta(k)) \) are constant matrices with appropriate dimensions.

Next we assume the network induced delay takes values in set \( T \triangleq \{0, 1, 2, ..., d\} \). Under sampling rate \( \theta(k) \), the delay term is described by
\[
\tau_{\sigma(k)}^{(k)} = \sigma(k) - 1
\]
where \( \sigma(k) \) is the \( \sigma(k) \)th element in set \( T \). So \( \sigma(k) \) takes values in \( T' \triangleq \{1, 2, ..., d+1\} \). We assume that the random communication delay under sampling rate \( \theta(k) + 1 \) in this work obeys the following homogeneous DTM,
\[
\pi^{\theta(k+1)} = \begin{bmatrix}
\pi_{1,1}^{\theta(k+1)} & \cdots & \pi_{1,d+1}^{\theta(k+1)} \\
\vdots & \ddots & \vdots \\
\pi_{d+1,1}^{\theta(k+1)} & \cdots & \pi_{d+1,d+1}^{\theta(k+1)}
\end{bmatrix}
\]
(2)

In order to obtain a better control result, \( \theta(k) \) and \( x(k) \) are wrapped in one packet and sent to the controller over the delayed communication network. Then the delayed state feedback controller can be described as
\[
u(k) = \hat{K}(\theta(k) - 1)\pi^{\theta(k)}x(k - 1)\pi^{\theta(k)}
\]
so (1) can be equivalently rewritten as
\[
x(k + 1) = \tilde{A}(\theta(k))x(k) + \tilde{B}(\theta(k))\hat{K}(\theta(k))x(k - 1)\pi^{\theta(k)}
\]
(4) is equivalent to
\[
\xi(k + 1) = [A(\theta(k)) + B(\theta(k))R_1(\sigma(k))K(\theta(k))R_2(\sigma(k))]\xi(k)
\]
(6)
with,
\[
A(\theta(k)) = \begin{bmatrix}
\tilde{A}(\theta(k)) & 0 & 0 \\
I & 0 & 0 \\
0 & I_{n-2} & 0
\end{bmatrix}, B(\theta(k)) = \begin{bmatrix}
\tilde{B}(\theta(k)) \\
0 \\
0
\end{bmatrix}
\]
(7)

First, the changing rule of \( \theta(k) \) governed by a homogeneous system \( TM(STM) \) is considered, where, \( \Pr[\theta(k + 1) = n|\theta(k) = m] = \lambda_{mn} \). After that, the following two cases are considered as deductions, \( \theta(k) \) changes arbitrarily and \( \theta(k) \) changes stochastically with partial elements unknown.

\[\Lambda = \begin{bmatrix}
\lambda_1 & \cdots & \lambda_{\alpha} \\
\vdots & \ddots & \vdots \\
\lambda_{\alpha 1} & \cdots & \lambda_{\alpha \alpha}
\end{bmatrix}\]

(8)

Now we are in a position to give the definition of stochastic stability. For more details, we refer the readers to [11] and the references therein.

Definition 1. The closed-loop system (6) is said to be stochastically stable if \( \mathbb{E}(\sum_{k=0}^{\infty} \|\xi(k)\|^2 \xi_0) < \infty \) for the initial condition \( \xi_0 \triangleq \xi(0) \in \mathbb{R}^{(\tau+1)xn} \).

3. STOCHASTIC STABILITY ANALYSIS

Lemma 2. [7] The closed-loop system (6) possesses \((d + 1)\alpha^{d+1}\) modes. If \( \theta(k) \) varies stochastically, the transition probability for \( \theta(k) \) is \( \gamma_{s,t} \triangleq \lambda_{i,j} \delta(i-1,j-2) \delta(i,j-1,\cdots, j-2) \delta(i-2,\cdots, j-2) \delta(i-j+1,\cdots, j-1) \delta(j-1,\cdots, j-1) \). Where, \( \theta_{k+1} = [i, i+1, \cdots, i+d], \theta_{k} = [i-j, \cdots, j], \theta_{k+1} = [i, i+1, \cdots, i+d], \theta_{k} = [i-j, \cdots, j], s = \Phi(\theta_{k+1}) = i + j - 1 - \alpha \delta + \cdots + j - \alpha \delta^d, \gamma_{s,t} = \Pr\{\Phi(\theta_{k+1}) = i + j - 1 - \alpha \delta + \cdots + j - \alpha \delta^d, \gamma_{s,t} \}

Case 1: Stochastic variation

The following theorem solves the stabilization problem for (6) when the STM changes in the sense of stochastic variation.

Theorem 3. System (6) with piecewise homogeneous DTM changing in the sense of stochastic variation is stochastically stable if there exist matrices \( P(s, m) > 0 \) such that the following matrix inequalities are satisfied,
\[
\begin{bmatrix}
\hat{P}^0_m & \hat{P}^0_m \xi(s) + B(s)K_{s,m}R_2(m) - P(s, m) \end{bmatrix} < 0
\]
(9)
where, \( A(s) = A(\theta(k)), B(s) = B(\theta(k)), K_{s,m} = K(i-m), \hat{P}^0_m = \sum_{j \in S} \lambda_{s,j} \hat{P}^0_m, \hat{P}^0_m \triangleq \sum_{m \in T} \hat{P}^0_m \xi(s) + B(s)K_{s,m}R_2(m), \eta = (i-1) \alpha + (i-j) \alpha^2 + \cdots + (i-d+1) \alpha^d.

Proof. Consider a Lyapunov candidate as
\[
V_i(\xi(k), \theta(k), \sigma(k)) = \xi^T(k)P(\Phi(\theta(k), \sigma(k))\xi(k)
\]
(10)
We set \( \theta(k) = [i, i-1, \cdots, i-d], \Phi(\theta(k)) = s, \sigma(k) = m \) as the starting point. At \( k + 1, \) we have \( \theta_{k+1} = [i+j-1, \cdots, j], \Phi(\theta_{k+1}) = t, \sigma(k + 1) = n. \)

Then with the transition probability matrices (2), (8), we have (see the equation on the top of next page).
\[
E[\Delta V(\xi(k), \theta_k, \sigma(k))] = E[V(\xi(k+1), \theta_{k+1}, \sigma(k)) - V(\xi(k), \theta_k, \sigma(k))]
= \xi^T(k+1) P(\eta+j, n) Pr(\theta_{k+1} = j, \sigma(k+1) = n|\theta_k = i, \sigma(k) = m) \xi(k) - V(\xi(k), \theta_k, \sigma(k))
\]

For all \(\xi \in \mathcal{T},\forall j \in S\), \(\eta = n\theta_k = i, \sigma(k) = m\), where \(P(\eta+j, n)\) is the transition probability from state \(\eta\) to \(\eta+j\) in \(n\) steps. The above development can be summarized as follows.

Theorem 1. The closed-loop system (6) is stochastically stable under the completely known matrix \(A\). If \(\lambda^*_m = 0\), the left side of (9) can be rewritten as

\[
\exists \Omega = \Upsilon_{jm}, \forall j \in Q^\mathbb{UK}_m \Omega = \Upsilon_{im}, \text{otherwise}
\]

with \(\Upsilon_{jm} = \frac{1}{\lambda^*_m} \hat{P}^m_\Omega[A(s) + B(s)K_{i-m}R_2(m)] - \hat{P}^m_\Omega[A(s) + B(s)K_{i-m}R_2(m)] - P(s, m)\). The proof is as follows.

Proof. This is omitted here, because it is similar to the proof of Theorem 1.

Case 2: \(\theta(k)\) Changing arbitrarily

Corollary 4. Consider the system (6) with the piecewise homogeneous DTM matrix in the sense of arbitrary variation. If there exist matrices \(P(s, m) > 0\), such that

\[
\begin{bmatrix}
\hat{P}^m_\Omega \dot{P}^m_\Omega [A(s) + (B(s)K_{i-m}R_2(m))] - P(s, m)
\end{bmatrix} < 0
\]

(11)

where \(\hat{P}\) is defined in Theorem 1, the system (6) is stochastically stable.

Proof. According to Theorem 1, if (9) holds, the system (6) is stochastically stable under the completely known matrix. If \(\lambda^*_m = 0\), the left side of (9) can be written as

\[
\exists \Omega = \Upsilon_{jm}, \forall j \in Q^\mathbb{UK}_m \Omega = \Upsilon_{im}, \text{otherwise}
\]

with \(\Upsilon_{jm} = \frac{1}{\lambda^*_m} \hat{P}^m_\Omega[A(s) + B(s)K_{i-m}R_2(m)] - \hat{P}^m_\Omega[A(s) + B(s)K_{i-m}R_2(m)] - P(s, m)\). Theorem 4.

4. CONTROLLER DESIGN

Theorem 7. The closed-loop system (6) is stochastically stable with a state feedback control law (3) and stochastically varying \(\theta(k)\), if there exist matrices \(\hat{P}(i, m) > 0\) and \(K_{i-m}\) with appropriate dimensions, and the following LMI holds.

\[
\begin{bmatrix}
-\Omega [A(s) + (B(s)K_{i-m}R_2(m))] - P(s, m)
\end{bmatrix} < 0
\]

(20)

Proof. The LMI (20) can be obtained from (9) by multiplying

\[
\begin{bmatrix}
\hat{P}^m_\Omega^{-1} & 0 \\
0 & I
\end{bmatrix}
\]

at both leftsides and rightsides. Then using the fact that \(-X^{-1} \leq X - 2I\), we can get (20), where \(X\) is any symmetric positive matrix.

Corollary 8. The closed-loop system (6) is stochastically stable under the state feedback control law (3) and arbitrarily varying \(\theta(k)\), if there exist matrices \(\hat{P}(i, m) > 0\) and \(K_{i-m}\) with appropriate dimensions, and the following LMI holds.

\[
\begin{bmatrix}
-\hat{P}^m_\Omega^{-1} (A(s) + (B(s)K_{i-m}R_2(m))) - P(s, m)
\end{bmatrix} < 0
\]

(21)
And, the closed-loop system (6) is stochastically stable with a state feedback control law (3) and a $a_\theta(k)$ is governed by a partially known STM $\Lambda$, if there exist matrices $P(i,m) > 0$ and $K_{1-m}$, with appropriate dimensions, and the following LMI holds.

$$\begin{bmatrix}
\Omega - 2I & A + BK_{1-m}R_{2}(m) \\
* & -P(s,m)
\end{bmatrix} < 0 \quad (22)$$

5. NUMERICAL EXAMPLE

In this section, we present a numerical example to show the application of the developed theory on the STM changes stochastically situation. The other two cases presented in this paper can also be verified in the same way. Three discrete systems and two delay terms considered are described as follows, $A_1 = \begin{bmatrix} 0.6 & 0.1 \\ 0 & 0.8 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $A_2 = \begin{bmatrix} 1.2 & 0 \\ 0 & 0.2 \end{bmatrix}$, $B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $A_3 = \begin{bmatrix} 0.3 & 0 \\ 0.1 & 0.7 \end{bmatrix}$, $B_3 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\theta_\sigma(k) \in \{1,2\}$. In this situation, the transition matrices are taken as $\pi^1 = \begin{bmatrix} 0.2 & 0.8 \\ 0.4 & 0.6 \end{bmatrix}$, $\pi^2 = \begin{bmatrix} 0.3 & 0.7 \\ 0.4 & 0.6 \end{bmatrix}$, $\pi^3 = \begin{bmatrix} 0.5 & 0.5 \\ 0.2 & 0.8 \end{bmatrix}$, $\Lambda = \begin{bmatrix} 0.2 & 0.1 & 0.7 \\ 0.3 & 0.1 & 0.6 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$.

Assume the initial condition of the system (1) to be $x(-2) = x(-1) = x(0) = [3,4]^T$. By Theorem 3, we can obtain a mode dependent controller $K_1 = [-0.1393 - 0.1481], K_2 = [-0.2867 -0.2333], K_3 = [-0.1063 -0.1184]$. Some simulation results of the resulting closed-loop system are introduced. Fig.2, Fig.3 shows one of the possible realizations of the Markovian jumping modes $\theta(k)$ and $\tau_\theta(k)$. Under these model sequences the corresponding state trajectories of the closed-loop system(6) is shown in Fig.4. It is shown that the closed-loop system is stochastically stable.

6. CONCLUSION

In this paper, the stochastically stabilization problem for a class of WNCS is considered. The method employed here can give out effective controllers with the changed delay distribution due to the sample rate adaption adopted taken into consideration to stabilize the target plant. The simulation result shows the effectiveness of the algorithm given above.

REFERENCES


