A Performance and Stability Analysis for Cooperative Teleoperation Systems

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Abstract: In this paper the performance, in terms of transparency and stability, of two control architectures for cooperative teleoperation systems are presented and discussed. The cooperative control schemes consider two pairs of teleoperation systems collaborating to carry out operations in a shared remote environment. The information exchange occurs only between the corresponding pairs, and the slave robots may physically interact among themselves either through a common tool or the manipulated object.

Keywords: Robotic Telemanipulation, Cooperative robots, Performance, Transparency, Stability

1. INTRODUCTION

Teleoperation, as widely shown in literature, extends the human capability of operating at a distance. By means of a teleoperation system, a human operator can interact with remote and unknown environments, which could be dangerous or inaccessible, to achieve simple or complex tasks. Such systems are made up of the master device that is manipulated by the operator, and of the slave device which handles the remote environment; both manipulators communicate by means of a communication channel. Typically, in order to execute a task remotely the operator imposes a desired force/velocity to the teleoperation system (acting directly on the master robot) and receives back from the environment a velocity/force feedback (sensed by the slave robot). If the slave robot tracks the master’s motion and the force perceived from master robot tracks the slave’s force then the system is called transparent, Lawrence [1993]. Transparency can be used as an index to evaluate the performance of the teleoperation system. Ideally it would be desirable to have a system with zero inertia in free motion and infinite stiffness in contact with a stiff wall, Hirche [2007]. Moreover, the force feedback reflected on the master robot gives the operator information about the remote environment, yielding a sense of telepresence. The force feedback on one hand increases the operator’s handling ability, but on the other hand compromises the stability of the entire system in presence of time delays, introduced by the physical distance between the master and slave sites or the limited bandwidth of the communication channel.

Therefore, in addition to transparency, the control should also guarantee the stability of the whole system with time delays, keeping into account that, if the network latency increases, the overall performance will worsen. Various control strategies have been developed for classical systems, directed towards resolving these two conflicting problems and overcoming the delays’ effects, see Melchiorri [2003] for an overview. By using the analogy between the mechanical/electric circuits, in Anderson and Spong [1989] a teleoperator has been represented as a network and, with the scattering theory, it has been proved that the instability caused by time delay was due to non-passivity of the communication channel. Successively, the passivity concept has been extended in Niemeyer and Slotine [1991] with the introduction of the wave variables formalism, providing a tool to model the communication media where the stability is guaranteed for any amount of (constant) time delay. Other applications of these concepts have been presented e.g. in Chopra et al. [2003], Leung et al. [1995] and Lee et al. [1997].

The cooperation between multiple robots stems from the need to perform tasks so complex that would otherwise be impossible, and/or to achieve results more efficiently, increasing dexterity and loading capability, and has recently received a noticeable attention from the robotics community. Cooperative teleoperation systems, Fig. 1, are composed of more than one teleoperation device and permit the execution of a task at a distance from one or more operators located in the same or different place. It is necessary to guarantee stability and transparency for them as in the conventional systems. Previous research activity on cooperation proposes multilateral communication frameworks with centralized controllers allowing information flow among all master and slave robots. In Sirouspour [Dec. 2005] and Sirouspour [Aug. 2005] controllers based on μ-synthesis methodology and adaptive techniques are introduced respectively in absence of time delay. In Setoodeh et al. [2006] a LQG algorithm is used considering constant time delays. The authors have proposed two control architectures composed of pairs of wave-based bilateral teleoperators operating in a shared environment, see Bacocco and Melchiorri [2009]. They have applied the force-position and position-position architectures to the cooperative teleoperation systems considering two pairs of single-master/single-slave devices working together in order to carry out operations in a remote environment.

In this paper, the problem of defining proper tools and procedures for an analysis, and possibly a comparison, of the performances of cooperative teleoperation systems is addressed. In particular, a novel generalization of criteria adopted for classical (i.e. one master-one slave) teleoperators is presented and illustrated on the basis of the force-position and the position-position cooperative control schemes Bacocco and Melchiorri [2009], both from a transparency and stability point of view, and by assuming a null time delay in the communication channel.
This paper starts with a general description of the proposed schemes, given in Section 2. In Section 3 the computations of the performance indices and the comparison between the two architectures is presented and discussed. Section 4 examines the stability of the cooperative system, while Section 5 concludes with some final remarks.

2. GENERAL DESCRIPTION OF THE SYSTEM

Figure 1 shows the basic cooperative architecture of the two proposed schemes. It is composed of two cooperating teleoperation systems acting on a remote environment. Both systems are structured as a serial connection of two-port elements including master-slave interconnected devices. Each one is interfaced on one side with an operator (possibly the same) and on the other side with the shared environment. As already mentioned, in the proposed analysis it is assumed a null time delay between the master and slave side. The main features of the control frameworks are:

- In each scheme, the information exchange occurs only between the corresponding pairs;
- The slave robots may influence the behavior of each other through the tool and/or the remote environment;
- The master/slave devices of the same teleoperator are assumed kinematically similar;
- The dynamics of the manipulators and of the environment are supposed to be known or properly estimated.

The two schemes described below differ in the control strategies utilized in each pair of teleoperators. One is force-position (i.e. the slave is under position control), see Fig. 2, the other is position-position control (i.e. both the master and slave of the two teleoperators are under position control), see Fig. 3. In both frameworks the devices under position control are regulated by a local PD controller.

In this paper, the subscripts \( m \) and \( s \) denote the variables of the master and slave manipulators respectively, while the subscript \( i = 1, 2 \) indicates the two teleoperators.

As often considered in the literature, the dynamics of the master/slave devices of each teleoperator is considered as a single-degree-of-freedom (DOF) described as

\[
M_{mi}a_{mi} + B_{mi}v_{mi} = F_{mi} \quad (1)
\]

\[
M_{si}a_{si} + B_{si}v_{si} = F_{si} \quad (2)
\]

where \( a_{mi} \) and \( a_{si} \) are the accelerations for the master and slave devices, \( v_{mi} \) and \( v_{si} \) are the velocities, \( M_{mi} \) and \( M_{si} \) are the masses, \( B_{mi} \) and \( B_{si} \) are the damping coefficients. \( F_{mi} \) and \( F_{si} \) are the forces applied on the master and slave devices respectively and are given as

\[
F_{mi} = F_{hi} - F_{mei} \quad (3)
\]

\[
F_{si} = F_{sc_1} - F_{se_1} \quad (4)
\]

where \( F_{hi} \) and \( F_{se_1} \) denote respectively, the hand/master interaction force and the slave/environment interaction force, and \( F_{sc_1} \) are the control signal computed by the slave controllers. The values of \( F_{mei} \) depend on the type of control architecture: in the force-position it is equal to \( F_{se_1} \), while in the position-position control scheme it is based on the force computed by the master controller (i.e \( F_{mc_1} \), see Fig. 3).
Fig. 4. Forces applied to the tool.

In both Fig. 2 and Fig. 3, the remote side is composed of a shared tool/object $M_0$, through which the slave manipulators apply forces on the environment, characterized by a stiffness $k_e$ and a damping coefficient $b_e$ as depicted in Fig. 4. The dynamics of the tool during interaction with the environment may be described as

$$M_0\ddot{x}_0 = F_{se1} + F_{se2} + F_0$$

with

$$F_{se1} = k_e(x_{s1} - x_0), \quad F_{se2} = k_e(x_{s2} - x_0)$$

and

$$F_0 = \begin{cases} -b_e v_0 - K_e(x_e - x_0), & x_0 \geq x_{e0} \\ 0, & x_0 \leq x_{e0} \end{cases}$$

where:

- $M_0$, $B_e$, $K_e$ are the mass of the tool, and the damping and stiffness coefficients of the environment, respectively;
- $F_{se1}$, $F_{se2}$ are the forces exchanged at the tool by each slave device;
- $F_0$ is the force applied by the tool to the remote environment;
- $x_{s1}$ and $x_{s2}$ are the positions of the two slaves;
- $v_0$ is the velocities of the tool;
- $x_{e0}$ is the initial position of the environment.

The springs $k_1$ and $k_2$ take into account the contact between the slave robots and the common tool (of slave-tool interaction).

3. MATHEMATICAL DESCRIPTION OF PERFORMANCE METRICS

As in classical teleoperation, a cooperative system can be mathematically described by means of the relationships which link the forces and velocities of between each device. Such relationships, represented in the linear case by means of transfer functions, can be expressed in terms of suitable matrices. In the case under study, the Hybrid, Transmission and the Impedance matrices are considered, defined as:

$$\begin{bmatrix} F_{b1} \\ F_{b2} \\ \dot{x}_0 \end{bmatrix} = H \begin{bmatrix} \dot{x}_{m1} \\ \dot{x}_{m2} \\ F_0 \end{bmatrix}$$

Hybrid matrix (6)

$$\begin{bmatrix} F_{b1} \\ F_{b2} \\ \dot{x}_{m1} \\ \dot{x}_{m2} \end{bmatrix} = T \begin{bmatrix} \dot{x}_0 \\ F_0 \end{bmatrix}$$

Transmission matrix (7)

$$\begin{bmatrix} F_{b1} \\ F_{b2} \end{bmatrix} = Z \begin{bmatrix} \dot{x}_{m1} \\ \dot{x}_{m2} \end{bmatrix}$$

Impedance matrix (8)

Given the above matrices, proper indices can be computed to compare the performance of the proposed control architectures. Hannaford, see Hannaford [1989], has shown that the hybrid matrix is the most suitable to characterize the performance of teleoperator systems. For a bilateral system perfect transparency is achieved (Hannaford [1989]) if the hybrid matrix is in the form:

$$H = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

The transmission value of position and force between the master and the slave, while the zeros mean null mass and infinite stiffness for the teleoperator. The position and force tracking, as well known, enable deduction of the performances of a teleoperator since they show both the ability of the operator to act at the remote site and the capability of the operator to perceive the remote environment respectively.

Similar considerations can be made for the cooperative teleoperators where transparency is obtained if the hybrid matrix (6) is in the form:

$$H = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Note that in this case there are no negative signs due to the different convention on the sign of $x_e$. For example, in the cooperative systems case at the remote site, besides the remote environment, also the cooperative tool must be considered. Therefore, in the next sections the motion/interaction capabilities will be analyzed both without and with the intervening tool.

3.1 Performances without the cooperative tool

A cooperative system without the common tool can be simply considered as two separated bilateral teleoperators. Therefore, the hybrid and transmission matrices in (6) and (7), used in the sequel to compute the performance indices, are defined by:

$$\begin{bmatrix} F_{b1} \\ F_{b2} \end{bmatrix} = H \begin{bmatrix} \dot{x}_{m1} \\ \dot{x}_{m2} \end{bmatrix}$$

Hybrid matrix (9)

$$\begin{bmatrix} F_{b1} \\ F_{b2} \end{bmatrix} = T \begin{bmatrix} \dot{x}_0 \\ F_0 \end{bmatrix}$$

Transmission matrix (10)
Since we are in a linear case, the transparency indices for each teleoperator are defined by the transfer functions listed below:

\[
\begin{align*}
  & h_{11} = \frac{F_{b1}}{x_{m1}} \bigg|_{x_{se1} = 0} \quad h_{22} = \frac{F_{b2}}{x_{m2}} \bigg|_{x_{se2} = 0} \\
  & h_{31} = \frac{\dot{x}_{m1}}{x_{m1}} \bigg|_{x_{se1} = 0} \quad h_{42} = \frac{\dot{x}_{m2}}{x_{m2}} \bigg|_{x_{se2} = 0} \\
  & t_{13} = \frac{F_{b1}}{x_{se1}} \bigg|_{x_{se1} = 0} \quad t_{24} = \frac{F_{b2}}{x_{se2}} \bigg|_{x_{se2} = 0} \\
  & z_{11} = \frac{F_{b1}}{x_{m1}} \bigg|_{x_{m1} = 0} \quad z_{12} = \frac{F_{b2}}{x_{m2}} \bigg|_{x_{m2} = 0}
\end{align*}
\] (12) (13) (14) (15)

The functions in (12) and (13) refer to the free motion and are determined by imposing equal to zero the forces \(F_{se1}\) and \(F_{se2}\), acting on each slave device. (12) is the impedance in free motion and (13) is the position tracking of each teleoperator. The equations (14) and (15) refer to rigid contact tasks, where the slave devices are considered in a rigid (steady) configuration. (14) gives the tracking of force instead (15) represents the transmissible impedance.

As an example of the application of these definition, Table 1 lists the values of the indices for the force-position and position-position control architectures, where

\[
\begin{align*}
  & C_{mi} = (k_{m1} + k_{pm})/s \quad C_{si} = (k_{s2} + k_{ps})/s \\
  & Z_{mi}(s) = M_{mi}s + B_{mi} \quad Z_{si}(s) = M_{si}s + B_{si} \\
  & Z_{cm} = Z_{m0} + C_{mi} \quad Z_{cs} = Z_{s0} + C_{si}
\end{align*}
\] (16) (17) (18)

(16) are the controllers used at the master and slave side and (17) are the dynamic characteristic of the master and slave robot respectively. By looking at the parameters of Table 1, remarks similar to those for the bilateral architectures can be made. As well known, the tracking performances depend on the choice of the PD controllers and on the relations between them, see Aliaga et al. [2004] for more details.

### 3.2 Performances with the tool

This section analyzes the whole system, considering both the presence of the tool and the interaction with the environment. The slave devices are considered always in contact with the tool, and therefore the indices are now related to a contact or non-contact task of the tool with the environment. The non-contact situation is associated with the free motion of the tool, while the contact of the tool with the environment can be linked to the rigid contact task of the bilateral systems. For the definition of the performance indices, we refer to the hybrid and transmission matrices in the form described by (6) and (7), i.e. considering directly the object velocity \(x_0\) and the interaction force \(F_0\) instead of the slave velocities \(\dot{x}_{m1}, \dot{x}_{m2}\), and the corresponding forces \(F_{se1}\) and \(F_{se2}\).

The elements (transfer functions) of the hybrid and transmission matrices have been computed on the basis of the equations of the entire system reported below:

\[
\begin{align*}
  & F_{b1} = Z_{m1}\dot{x}_{m1} + F_{mc1} \\
  & \dot{x}_{s1} = \frac{C_{s1}}{Z_{s1}} \dot{x}_{m1} - \frac{F_{se1}}{Z_{s1}} \\
  & F_{b2} = Z_{m2}\dot{x}_{m2} + F_{mc2} \\
  & \dot{x}_{s2} = \frac{C_{s2}}{Z_{s2}} \dot{x}_{m2} - \frac{F_{se2}}{Z_{s2}}
\end{align*}
\] (19)

note that the last equation describes the physical interconnection between the two teleoperators.

**Performances with the tool in free motion** These indices describe the behavior of the system when the tool is grasped by each slave device and is not in contact with the environment. This is expressed by imposing the force applied on the environment equal to zero \(F_0 = 0\).

\[
\begin{align*}
  & h_{11} = \dot{x}_0 \bigg|_{\dot{x}_{m1} = 0, F_0 = 0} \quad h_{22} = \dot{x}_0 \bigg|_{\dot{x}_{m2} = 0, F_0 = 0} \quad h_{31} = \dot{x}_0 \bigg|_{\dot{x}_{se1} = 0, F_0 = 0} \quad h_{42} = \dot{x}_0 \bigg|_{\dot{x}_{se2} = 0, F_0 = 0}
\end{align*}
\] (20)

Eq. (20) represents the position tracking capabilities of the tool. It permits to understand and evaluate the capabilities of the tool to follow the motion of each master device. The impedance in free motion can be added as a metric to describe the unconstrained motion. This parameter represents the impedance felt by each operator: its value should be as low as possible and is given as

\[
\begin{align*}
  & h_{11} = \frac{F_{m1}}{x_{m1}} \bigg|_{x_{m2} = 0, F_0 = 0} \quad h_{22} = \frac{F_{m2}}{x_{m2}} \bigg|_{x_{m1} = 0, F_0 = 0}
\end{align*}
\] (21)

**Performances with the tool in contact** The case of interaction between the common tool and the remote environment is now considered. In ‘classical’ bilateral systems, this case is studied...
Force - Position

Position - Tracking
\[ h_{31} = \left( \frac{Z_{cs2} s + k_2}{d_f p} \right) (k_1 C_{s1}) \]
\[ h_{32} = \left( \frac{Z_{cs1} s + k_1}{d_f p} \right) \frac{k_2 C_{s2}}{C_{s2}} \]

Force - Tracking
\[ t_{12} = \frac{Z_{cm1} Z_{cm2} s + k_1 (Z_{cm2} - C_{s2})}{C_{s1}} \]
\[ t_{22} = \left( \frac{Z_{cm1} Z_{cm2} s + k_2 (Z_{cm2} - C_{s2})}{C_{s2}} \right) \]

Impedance - Free motion
\[ h_{11} = Z_{m1} + \frac{k_1 Z_{cm1}}{Z_{cm1} s + k_1} + h f_{11} \]
\[ h_{22} = Z_{m2} + \frac{k_2 Z_{cm2}}{Z_{cm2} s + k_2} + h f_{22} \]

Trasmittable - Impedance
\[ z_{11} = Z_{m1} + \frac{k_1 Z_{cm1}}{Z_{cm1} s + k_1} \]
\[ z_{22} = Z_{m2} + \frac{k_2 Z_{cm2}}{Z_{cm2} s + k_2} \]

Table 2. Set of parameters for force-position architecture with the tool.

Position - Position

Position - Tracking
\[ h_{31} = \left( \frac{Z_{cs2} s + k_2}{d_f p} \right) (k_1 C_{s1}) \]
\[ h_{32} = \left( \frac{Z_{cs1} s + k_1}{d_f p} \right) \frac{k_2 C_{s2}}{C_{s2}} \]

Force - Tracking
\[ t_{12} = \frac{Z_{cm1} Z_{cm2} s + k_1 (Z_{cm2} - C_{s2})}{C_{cm1}} \]
\[ t_{22} = \left( \frac{Z_{cm1} Z_{cm2} s + k_2 (Z_{cm2} - C_{s2})}{C_{cm2}} \right) \]

Impedance - Free motion
\[ h_{11} = Z_{m1} + \frac{\zeta_{m1} C_{m1}}{Z_{m1} s + \zeta_{m1}} - h p_{11} \]
\[ h_{22} = Z_{m2} + \frac{\zeta_{m2} C_{m2}}{Z_{m2} s + \zeta_{m2}} - h p_{22} \]

Trasmittable - Impedance
\[ z_{11} = Z_{m1} + \frac{\zeta_{m1} C_{m1}}{Z_{m1} s + \zeta_{m1}} \]
\[ z_{22} = Z_{m2} + \frac{\zeta_{m2} C_{m2}}{Z_{m2} s + \zeta_{m2}} \]

Table 3. Set of parameters for Position-Position architecture with the tool.

\[ d_{fp} = s^2 Z_0 Z_{cs1} Z_{cs2} + s(Z_0 Z_{cs1} k_2 + Z_0 Z_{cs2} k_1) + \]
\[ + Z_{cs1} Z_{cs2} (k_1 + k_2)) + (Z_0 + Z_{cs1} + Z_{cs2}) k_1 k_2 \]
\[ h f_{11} = \frac{k_1 C_{m1}}{(Z_{cs1} s + k_1) (k_1 C_{s1} d_f p)} \]
\[ h f_{22} = \frac{k_2 C_{m2}}{(Z_{cs2} s + k_2) (k_2 C_{s2} d_f p)} \]
\[ h p_{11} = \frac{k_1 c_{m1}}{(Z_{cs1} s + k_1)} \]
\[ h p_{22} = \frac{k_2 c_{m2}}{(Z_{cs2} s + k_2)} \]

From the parameters listed in Table 2 and Table 3, it is possible to compare the two architectures. The position tracking, as for the bilateral systems, is the same for both architectures since it depends only on the slave devices (which are both under position control). In both architectures, the force tracking depends on the choice of the gains of the PD controllers. This is due to the presence of large values of \( k_1 \) and \( k_2 \). For the same reason both the impedance in free motion and the maximum transmittable impedance depend on the systems dynamics and on the choice of the control gains.

4. STABILITY ANALYSIS

A stability analysis of the cooperative system is now performed. In the study, the master velocities are considered as input to the system, while the tool velocity is the output. In this case, the cooperative system can be described as

\[ x_0 = G_1 x_{m1} + G_2 x_{m2} \]

where \( G_1 = \frac{s a_{mx1}}{s a_{mx2}} \) and \( G_2 = \frac{s a_{mx2}}{s a_{mx2}} \) are defined as:

\[ G_1 = \frac{(Z_{cs2} s + k_2)}{D(s)}, \quad G_2 = \frac{(Z_{cs1} s + k_1)}{D(s)} \]

and

\[ D(s) = s^2 Z_0 Z_{cs1} Z_{cs2} + (Z_0 Z_{cs1} k_2 + Z_0 Z_{cs2} k_1) + \]
\[ + Z_{cs1} Z_{cs2} (k_1 + k_2)) + (Z_0 + Z_{cs1} + Z_{cs2}) k_1 k_2 \]

Therefore, stability is achieved if the characteristic equation of \( G_1 \) and \( G_2 \) (i.e. \( D(s) = 0 \)) has no positive roots. By substituting the values of \( Z_0 \), \( Z_{cs1} \) and \( Z_{cs2} \) in the expression of \( D(s) \), one obtains:

\[ D(s) = a_6 s^6 + a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0 = 0 \]

with:

\[ a_6 = M_0 M_1 M_2 \]
\[ a_5 = M_0 M_1 (b_2 + k_2) + M_2 (b_1 + k_1) \]
\[ a_4 = M_0 M_1 M_2 (b_2 + k_2) + M_0 (b_1 + k_1) (b_2 + k_2) + \]
\[ + M_0 M_2 (M_2 k_2 + M_1 k_1) + M_1 M_2 (k_1 + k_2) \]
\[ a_3 = M_0 (b_2 + k_2) (b_1 + k_1) + k_1 (b_2 + k_2) + M_0 (b_1 + k_1) k_2 + \]
\[ + M_0 (b_2 + k_2) k_1 + (k_1 + k_2) (M_1 (b_1 + k_2) + M_2 (b_2 + k_2)) \]
\[ a_2 = M_0 (b_2 + k_2) (b_1 + k_1) + k_1 (b_2 + k_2) + k_2 (b_2 + k_2) \]
\[ + (k_1 + k_2) (M_1 (b_1 + k_2) + M_2 (b_2 + k_2)) \]
\[ a_1 = (b_1 + k_1) (b_2 + k_2) + k_1 (b_2 + k_2) \]
\[ + (b_2 + k_2) (k_1 + k_2) + (k_1 + k_2) k_1 k_2 \]
\[ a_0 = k_1 k_2 (b_1 + k_2) + (k_1 + k_2) k_1 k_2 \]
By applying the Routh-Hurwitz criterion to (26), the necessary and sufficient conditions for asymptotic stability of the teleoperator are derived as:

\[ \Delta_0 = a_0 > 0 \]
\[ \Delta_5 = a_5 > 0 \]
\[ \Delta_4 = \frac{a_4a_5 - a_0a_4}{a_2} > 0 \]
\[ \Delta_3 = a_3 - \frac{a_2(a_4a_5 - a_0a_4)}{a_2a_4 - a_0a_3} > 0 \]
\[ \Delta_2 = \frac{a_2a_4 - a_0a_3}{a_2} - \frac{a_1(a_4a_5 - a_0a_4)}{a_2a_4 - a_0a_3} - a_5^2 > 0 \]
\[ \Delta_1 = a_1 - \frac{a_2a_4 - a_0a_3}{a_2} > 0 \]

The stability of the system can be studied taking also into account the presence of the remote environment with impedance \( Z_r \). In this case, the denominator of \( G_1(s) \) and \( G_2(s) \) in (25) is:

\[ D(s) = s^2(Z_0 + Z_r)Z_{c_1}Z_{c_2} + s(Z_0 + Z_r)Z_{c_1}k_2 + (Z_0 + Z_r)Z_{c_2}k_1 \]
\[ + Z_{c_1}(Z_{c_2}l_1 + k_2 + Z_{c_1}l_1 + k_1) + (Z_0 + Z_r)Z_{c_1}l_1 + Z_{c_1}l_1 + k_1 \]

Again, by substituting the values of \( Z_0, Z_{c_1} \) and \( Z_{c_2} \) and \( Z_r = b_r + \frac{b_r^2}{s} \), one obtains:

\[ e_6s^6 + e_5s^5 + e_4s^4 + e_3s^3 + e_2s^2 + e_1s + e_0 = 0 \]  (27)

where:

\[ e_6 = M_2M_1M_2 \]
\[ e_5 = M_0(M_1(b_1 + k_0)) + M_2(b_1 + k_0) \]
\[ e_4 = M_0(M_2k_2 + M_1k_1 + (b_1 + k_0)(b_2 + k_2)) \]
\[ + (M_0M_1 + M_0M_2)(k_1 + k_2) + k_0M_1M_2 \]
\[ + b_1(M_2k_2 + M_1k_1 + b_2k_2) \]
\[ e_3 = M_0((k_0 + k_1)(b_1 + k_2) + (k_0 + k_2)(b_1 + k_2)) \]
\[ + M_1((k_0 + k_1)(b_1 + k_2) + b_1k_2 + k_0b_2) \]
\[ + M_2((k_0 + k_1)(b_1 + k_2) + b_1k_2 + k_0b_2) \]
\[ + b_2(b_1 + k_0)(b_2 + k_2) \]
\[ e_2 = M_0(k_0k_2(b_1 + k_0) + k_2(b_1 + k_0)) \]
\[ + M_1(k_0k_2 + k_0k_2 + k_2(b_1 + k_0) + k_0k_2) \]
\[ + M_2(k_0k_2 + k_0k_2 + k_2(b_1 + k_0) + k_0k_2) \]
\[ + b_2((k_0 + k_2)(b_1 + k_2) + (k_0 + k_2)(b_1 + k_2)) \]
\[ + k_0(b_1 + k_2)(b_2 + k_2) \]
\[ e_1 = k_0((k_0)k_2(b_1 + k_2) + (b_1 + k_1)(k_2 + k_0)) \]
\[ + b_2(k_0k_2 + k_0k_2) \]
\[ + (k_0 + k_2)(b_1 + k_0) + k_0(b_1 + k_2) \]
\[ + k_1k_2(b_1 + k_0)(b_2 + k_2) \]
\[ e_0 = k_0(k_0 + k_0k_2)(b_1 + k_2) + k_0k_1k_2 + k_2p_2k_1(1 + k_0) \]

The necessary and sufficient conditions for asymptotic stability of the teleoperator are still given by the conditions (27), with the obvious replacement of the parameters \( a_i \) with the corresponding \( e_i \), \( i = 1, \ldots, 6 \). The input/output expressions for the two control schemes are the same, since they only depend on the slave equations, which is under position control in both cases.

5. CONCLUSIONS

This paper discusses the problem of analyzing the transparency and the stability of control architectures for cooperative teleoperation systems. The robotic systems are composed of two pairs of master/slave manipulators used in cooperative tasks, where each slave device may only receive information by its corresponding master device. Performance indices have been defined in the case of no time delay, extending to this multi-master/slave case similar criteria introduced for single bilateral telemanipulators, and some comparisons have been discussed considering in particular the force-position and position-position control schemes. Future work will consider applying the concepts to other control schemes, and a possible extension to the case of time delay.

REFERENCES