Decentralized $H_\infty$ based control of a web transport system

N.I. Giannoccaro*, T. Nishida**, T. Sakamoto**

* Department of Innovation Engineering, University of Salento, 73100 Lecce, Italy (Tel: +39-0832297813; e-mail: ivan.giannoccaro@unisalento.it).
** Faculty of Engineering, Kyushu Institute of Technology, Tobata, Kitakyushu 804-8550, Japan (e-mail: sakamoto@cntl.kyutech.ac.jp; nishida@cntl.kyutech.ac.jp)

Abstract: Robust and good tracking control of the speed and the tension in web handling systems in spite of perturbations such as changes of web material is surely one of the important challenges in the web transport systems future development. In this paper, the authors experimentally demonstrate the applicability of a decentralized robust control to a multi-span web transport system, which is composed of eleven guide rollers, four main sections mutually interconnected with each other. The overlapping methodology has been applied for the system decomposition.

1. INTRODUCTION

The systems handling web materials are very common in the industry; the main objective of the control researches about these systems is to increase the web velocity while controlling the web tension. The industrial web handling systems are usually composed of a large number of rolls and may be considered as large-scale systems that are decomposed in many subsystems for web tension control and web speed control. All those subsystems have strong interactions with each other and the control of each subsystem is heavily influenced by the interactions with the neighbouring subsystems. In the large scale systems (Bakulé, 2008) the control decentralization is necessary because, in general, the systems to be controlled are too large and the problems to be solved are too complex. A good solution to the necessity of the decomposition was introduced (Sakamoto & Tanaka, 1998; Sakamoto, 1998) with a methodology based on a decomposition of the system into subsystems overlapped to take into account the mutual effects of the neighbouring subsystems. The simulation results (Sakamoto & Tanaka, 1998; Sakamoto, 1998) demonstrated that the controller design becomes easy and that the control performance is better than the conventional decentralized controller based on a disjoint decomposition.

The experimental performances of the decentralized controllers on an experimental web platform were recently shown in Pagilla, Siraskar, & Dwivedula, (2007), who investigated the possibility of a gain scheduling technique determined by the speed and roll radius.

Another quite important aspect in the industrial web handling systems is that there exist many sources of disturbances (roller non-circularity, web slipping, vibrations, variation of the radius and of the inertia of the winder and unwinder rolls etc.), which can be a dangerous damaging to the web. Then, various robust control strategies have been investigated (Benlatreche, Knittel & Osterbag, 2008; Knittel, Gigan & Laroche, 2002; Koc, Knittel, de Mathelin, & Abba, 2002; Laroche & Knittel, 2005; Claveau, Chevrel & Knittel, 2008). The results (Koc, Knittel, de Mathelin, & Abba, 2002) demonstrated the performance improvement of a multivariable $H_\infty$ robust centralized controller compared to PID controllers. Some simulations demonstrated that $H_\infty$ decentralized controllers, with one (Benlatreche, Knittel & Osterbag, 2008) or two degree of freedom (Claveau, Chevrel & Knittel, 2008) or with the model based feedforward (Knittel, Arbogast, Vedrines & Pagilla, 2006) could improve the system performance by reducing the coupling effects between two consecutive subsystems especially when the decomposition was carried out by introducing the overlapping control strategy (Sakamoto, 1998).

This work is aimed to apply a decentralized robust control to an experimental multi span web transport system. The experimental system has been specially designed and realized at Kyushu Institute of Technology for creating a situation of a large scale system with several sections (four main sections) mutually interconnected to one another. The overlapping decentralized control methodology (Sakamoto, 1998) facilities the controller design, and decreases the order of each controller with many advantages in terms of computational effort and of system quickness.

2. THE EXPERIMENTAL SYSTEM

The system (photo in Fig.1 and scheme in Fig.2) is composed of an unwinder section followed by a couple of tension sensors (one for each side of the web), a lead section (master speed control) followed by a couple of guide rolls, a draw roll section controlled by means of a couple of tension sensors and, finally, a winder section. Four main sections are strongly interlaced each other and 12 rollers constitute the system. The inputs to the system are the torque reference signals ($u_1$, $u_2$, $u_3$, $u_4$) of the four servomotors (Fig.2). The servomotors of the unwinder and winder section (750W, 2.39Nm maximum torque) are much more powerful than the servomotors of the lead section and draw-roll section (100W, 0.318Nm maximum torque). The outputs of the systems are the tension forces measured by tension sensors after the unwinder section and the draw-roll section (called $T_1$ and $T_3$, respectively, in Fig.2, calculated as the average value of a couple of tension...
sensors mounted at right and left side for each section), and the speed measured by encoders mounted on the servomotors for the lead section and the winder section (called \( v_l \) and \( v_r \)). The guide rollers positioned before and after the tension sensors are necessary for having a definite slope of the web. The inputs signals \( u_i \), \( i=1...4 \) are sent to the motors by using a 4 channel D/A board, the tension signals feed a 4 channel A/D board, and the 4 motor-encoder signals (including the speed signals of unwinder section and lead section) feed a digital counter. The controller’s CPU receives signals by A/D boards and counters, performs the control algorithm (C language and Linux operating system), and outputs the command signals in real time to the motor driver through D/A boards.

The preliminary tests exhibited a considerable amount of noise of the output signals even at low frequencies; in order to reduce the noise, a classical Kalman filter has been designed and realized directly on the acquisition code.

![Fig. 1. Experimental web transport system](image)

![Fig. 2. Scheme of the experimental web transport system](image)

3. MODELLING AND DECENTRALIZED ROBUST CONTROL DESIGN

The mathematical model of the web transport systems is formulated by considering three fundamental laws (see details in Sakamoto & Fujino 1995; Sakamoto 1997; Giannoccaro, Messina & Sakamoto, 2010). For increasing the dynamic accuracy of the model, the radius variations of the unwinder and winder rolls during the system running were taken into account.

The theoretical model contains many parameters, some of which are strictly related to the geometry of the system (roll radius, section length, roll inertia, elastic modulus, web thickness, etc.). The other parameters are quite difficult to estimate (such as dry friction \( C_d \), viscous friction \( K_d \)), and often they are estimated iteratively by changing their values and testing the model behaviour. The authors proposed a validation strategy (Giannoccaro, Messina & Sakamoto, 2010) based on a multivariable optimization algorithm for solving a minimum research problem expressed in (1), where \( y_{ob} \) are the outputs of the system model \( i=1...4 \) for the considered case and \( y_{exp} \) are the experimental values of the system outputs.

\[
\min \left( \frac{y_{exp} - y_{ob}}{y_{exp}} \right)
\]  

(1)

The model validation was carried out with the same procedure shown in (Giannoccaro, Sakamoto & Nishida, 2010; Giannoccaro, Messina & Sakamoto, 2010), giving step-wise constant values to the 4 motors (inputs \( u_1, u_2, u_3, u_4 \) and measuring the system outputs \( T_1, v_2, T_3, v_4 \). An input combination (shown in Table 1) was carried out for moving the web at a speed of around 1 m/s. The geometrical data directly measured on the experimental system or assumed and used in the model are summarized in Table 2 and 3; for the moment of inertia evaluation referred to the motor shaft for the four motors, a constant torque was given to each motor to measure the angular acceleration. The other parameters (for \( k_i \) section, the dry friction torque \( C_i \) and the viscous friction coefficient \( K_i \)) have been considered as unknown parameters to be estimated with the model validation procedure; so totally 8 unknown parameters have been estimated. The estimated parameters are shown in Table 4 and the comparison between the updated model and the experimental data is shown in Fig.3. It is evident that the model matches well the experimental data for both tension forces and speeds. This good matching was found also for other combination of input different from the one used for identification and shown in Fig.3.

<table>
<thead>
<tr>
<th>Table 1. Input combination for the model validation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u_1 )[Nm]</td>
</tr>
<tr>
<td>0.159</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2. Geometrical data of the system</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter definition</td>
</tr>
<tr>
<td>Radius of drive roll [m]</td>
</tr>
<tr>
<td>Web length [m]</td>
</tr>
<tr>
<td>Moment of inertia [Nm²]</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 3. Data of the web</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter definition</td>
</tr>
<tr>
<td>Cross sectional area [m²]</td>
</tr>
<tr>
<td>Viscosity modulus [Ns/m]</td>
</tr>
<tr>
<td>Elastic modulus [N/m²]</td>
</tr>
</tbody>
</table>
Table 4. Estimated parameters of the model.

<table>
<thead>
<tr>
<th>Parameter definition</th>
<th>Symbol</th>
<th>$k=1$ unwinder section</th>
<th>$k=2$ lead section</th>
<th>$k=3$ draw-roll section</th>
<th>$k=4$ winder section</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscous friction coefficient [Ns]</td>
<td>$K_f$</td>
<td>0.0042</td>
<td>0.0042</td>
<td>0.0048</td>
<td>0.0048</td>
</tr>
<tr>
<td>Dry friction torque [Nm]</td>
<td>$C_s$</td>
<td>0.045</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Fig. 3. Comparison of the model with the web system

Since 1998 it was demonstrated (Sakamoto, 1998) that a decentralized controller based on overlapping decomposition permits considering some mutual interactions between subsystems. The resultant control system has better control performance compared with a decentralized controller based on disjoint decomposition, and yet it makes the controller design simpler. Following this approach, the realized web handling system has been divided in 4 overlapped subsystems. The transformation matrix (Sakamoto, 1998) $N^{-1}$ that expresses the relation (see Eq. 2) between the torque reference signals of the four servomotors $u_1, u_2, u_3, u_4$ and the overlapped subsystems controller outputs $\tilde{u}_1, \tilde{u}_2, \tilde{u}_3, \tilde{u}_4$ is given in Eq. 3. Following this approach, the realized web handling system (Fig. 4a) has been divided in 4 overlapped sections (Fig. 4b), and the new input controls (Fig. 4c) are calculated through the matrix $N^{-1}$.

\[
\begin{bmatrix}
  u_1 \\
  u_2 \\
  u_3 \\
  u_4 \\
\end{bmatrix} = N^{-1} \begin{bmatrix}
  \tilde{u}_1 \\
  \tilde{u}_2 \\
  \tilde{u}_3 \\
  \tilde{u}_4 \\
\end{bmatrix}
\]

(2)

\[
N^{-1} = \begin{bmatrix}
  \frac{T_2}{J_1} & \frac{T_2}{J_2} & 0 & 0 \\
  0 & 1 & 0 & 0 \\
  0 & 0 & \frac{T_2}{J_3} & \frac{T_2}{J_4} \\
  0 & 0 & 0 & 1 \\
\end{bmatrix}^{-1}
\]

(3)

Moreover, in order to increase the control robustness when many sources of disturbance appear, a robust $H_\infty$ control has been designed for each subsystem making use of the mixed sensitivity approach (Kwakernaak, 1993) called $S/KS/T$ scheme. In Fig. 5, $w$ is the subsystem input, $z$ the controlled signal, $W_p$, $W_w$, $W$, the frequency weighting functions, and $K(s)$ the controller to be designed.

Fig. 4. Overlapped subsystems decomposition

The closed loop transfer matrix $T_{zw}$ of this control scheme is given by (4) where $S = (I+GK)^{-1}$ is the sensitivity function and $T = (I-S)$ is the complementary sensitivity function.

\[
T_{zw} = \begin{bmatrix}
  W_pS \\
  W_wKS \\
  W_T \\
\end{bmatrix}
\]

(4)

$H_\infty$ controller $K(s)$ is calculated using the "loop-shifting formulae" (The Math Works, 2009) in order to minimize the $H_\infty$ norm of the transfer function $T_{zw}$. The controller outputs $\tilde{u}_1, \tilde{u}_2, \tilde{u}_3, \tilde{u}_4$ are then multiplied by $N^{-1}$ for calculating the servo motor torque signals $u_1, u_2, u_3, u_4$.

About the selection of the weighting functions for the 4 subsystems, as described in (Beaven, Wrigth & Seaward, 1996) there are no objective set of criteria, although simple guidelines exist. The weighting functions for the four subsystems were selected considering the guidelines given in (Beaven, Wrigth & Seaward, 1996) and verifying the performance of the controlled system on the validated model. In detail, considering the symmetry of the subsystems, the same weighting functions shown in Figure 5 were selected for the subsystems 1 and 3 and for the sections 2 and 4. For the sections 1 and 3 the general structure of $W_p$ defined by (5) (Koc, Knittel, de Mathelin, & Abba, 2002).
where $M$ is the maximum peak amplitude of $S$, $||S||_\infty \leq M$, $\omega_B$ is the required bandwidth frequency and $\epsilon_0$ is the steady-state error. The choice for the selection of $W_p$ was to guarantee a steady state error equal to 0, fixing $\epsilon_0 = 0$. In addition the main control interest was related to low frequencies, so the bandwidth $\omega_B$ was chosen equal to 5 and the maximum peak amplitude $M = 1$. The denominator order necessary became equal to 2 (double poles in the origin) for permitting the solution of the Riccati equation for defining the $H_\infty$ controller (Safonov, Limebeer & Chiang, 1989). The polynomials $W_u$ and $W_f$ were defined as first order polynomials with one zero and one pole placed at high frequencies. In the actual case the subsystems 2 and 4 have a transfer function with one pole on the imaginary axis; in this case a loop transformation technique is available (Qi & Tsuji, 1996) for designing the controllers. The weighting functions used in the experimental tests are shown in Table 5.

The system decomposition considerably decreases the resulting controller order to obtain only 6th order for each subsystem; this aspect guarantees a very small computational effort of the real time controller and an agile control action.

Table 5. Weighting functions for the 4 subsystems

<table>
<thead>
<tr>
<th>Weighting function</th>
<th>Subsystem 1</th>
<th>Subsystem 2</th>
<th>Subsystem 3</th>
<th>Subsystem 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_p$</td>
<td>$\frac{(s+5)/s^2}{s^2}$</td>
<td>$\frac{(s+5)/s^2}{s^2}$</td>
<td>$\frac{(s+5)/s^2}{s^2}$</td>
<td>$\frac{(s+5)/s^2}{s^2}$</td>
</tr>
<tr>
<td>$W_u$</td>
<td>$\frac{(s+100)/s}{s+1000}$</td>
<td>$\frac{(s+100)/s}{s+1000}$</td>
<td>$\frac{(s+100)/s}{s+1000}$</td>
<td>$\frac{(s+100)/s}{s+1000}$</td>
</tr>
<tr>
<td>$W_f$</td>
<td>$\frac{(s+1100)/s}{s+1200}$</td>
<td>$\frac{(s+600)/s}{s+1200}$</td>
<td>$\frac{(s+1100)/s}{s+1200}$</td>
<td>$\frac{(s+600)/s}{s+1200}$</td>
</tr>
</tbody>
</table>

The decentralized robust controller design was realized with the first objective of reducing the start-up of the system and of having an adequate short rise time even in the starting condition with a light overshoot. In Fig.6 a simulation of the system behaviour is shown, where we used the aforementioned validated model with the decentralized robust controller. The experimental tests showed that the tension $T_3$ can be controlled with the actual system, only in a small range of setpoint variability. This can be justified considering the difference of the maximum torque between the draw role section (0.318 Nm) and the winder section (2.39 Nm) that are overlapped in the subsystem 3. The simulations (see Fig.7) confirm that $T_3$ setpoint may not be chosen in an independent way with respect to the winder velocity setpoint for having good control performances. This is an actual mechanical limit of the realised web handling system that can control the tension $T_3$ only in a small range and for small web speed acceleration; for this reason it will not be shown in the experimental results. However, this omission does not subtract any relevance to all the results that demonstrate the possibility of controlling the tension $T_1$ in the first section of the system together with the web speed.

Finally, each designed controller is discretized by the bilinear Tustin approximation at a 10 ms sampling time and used for the experimental tests. The discrete transfer functions of the designed controllers for the experimental tests are shown in Table 6.

Table 6. Controller discrete transfer functions for the 4 subsystems

<table>
<thead>
<tr>
<th>Discrete transfer functions (sampling time=0.01 s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subsystem 1</td>
</tr>
<tr>
<td>$0.0658 z^3 + 0.09478 z^2 - 0.08917 z - 0.1758 z^2 - 0.005051 z^3 + 0.08128 z + 0.02865$ \n</td>
</tr>
<tr>
<td>$0.153 z^3 - 0.08296 z^2 - 0.3451 z^3 + 0.1382 z^2 + 0.2594 z^3 - 0.05159 z^2 + 0.06735$ \n</td>
</tr>
<tr>
<td>$0.05587 z^3 + 0.07978 z^2 - 0.07602 z^2 - 0.148 z^3 - 0.003678 z^3 + 0.06454 z + 0.02402$ \n</td>
</tr>
</tbody>
</table>
| $0.05355 z - 0.06538$ \n| $z^2 - 2.897 z^2 + 2.416 z^2 + 0.2475 z^3 - 1.27 z^2 + 0.6223 z - 0.1162$ \n
4. EXPERIMENTAL RESULTS

Experiments were carried out on the system that is depicted in Fig.1 for testing the decentralized controllers performances and robustness. At this aim, the designed controller was used for moving the web with the speed reference values in the range 0.7-1.1 m/s starting from a standstill. Different values of the tension $T_1$ were set and tracking tests were carried out for following with the step variations of the tension $T_1$ (while maintaining a constant web speed) and of the web speed (while maintaining a constant tension $T_1$). Moreover, all these tests were carried out considering different types of web.
having different thickness. Three different thickness sizes were considered (30, 40 and 25 μm), in order to check the robustness of the controller to changes of the web characteristics. The experimental results are shown in Figs. 8-13 considering the tension and velocity measurements (variables $T_1$, $v_2$, $v_4$). Three different velocity step inputs were considered in Fig.8, which has the web thickness of 30μm: the reference speeds are 0.7 m/s (Fig.8a), 0.8 m/s (Fig. 8b) and 0.9 m/s (Fig. 8c) with a corresponding step input of tension $T_1$ equal to 15 N. The dynamic performances are very good for all the variables. The results show good step-input characteristics: in all the tests, the 10 % settling time is smaller than 2 s with a contained overshoot, and the accuracy is good with small steady state errors. The same web was tested for tracking tension changes (Fig. 9a) and velocity changes (Figs 9b and 9c). The tension tracking is very good for the requested variations of tension, while the web speed is controlled to keep the same velocity. When the web speed variations are requested, only small oscillations of the tension around the set point value are generated. As a result, Figs. 8 and 9 show that the decentralized controllers work satisfactorily with good tracking performances for the control of the web speed and tension. In order to test the robustness of the designed controllers, similar experimental tests were carried out with webs of different thickness: in Figs. 10 (step input) and 11 (double step input) the results with a web thickness of 40 μm are shown. Figs 12 (step input) and 13 (double step input) show the results with a web thickness of 25 μm.

The following considerations may be given from the experimental results:

- very good dynamic performances and tracking properties, with respect to the tension $T_1$ and the web velocity;
- the tracking of web reference speed variations (Figs 9b, 9c, 11b,11c, 13b,13c) is quick, and in all the cases provokes a slight increase of the fluctuations of $T_1$;
- the tracking of reference tension $T_1$ variation (Figs 9a,11a,13a) doesn’t provoke an appreciable increase of the fluctuations of the web speed;
- the changing of the web thickness (Figs 10-13) causes an increase of fluctuations of the controlled variables when compared with the case of the web that was used for the model validation and the controller design.
Fig. 13. Measurements of the system with decentralized H∞ controllers with a web of thickness 25 μm.

6. CONCLUSIONS

This paper shows the simulation and experimental results of a web handling system, which can be regarded as a large-scale system controlled by decentralized robust controllers. Previous studies (Sakamoto & Tanaka, 1998; Sakamoto, 1998; Giannoccaro, Messina & Sakamoto, 2010) were used to realize an accurate dynamic model of the multi-span web transport system and for designing the decentralized control. It is important to underline that the design and the realization of such decentralized control are necessarily linked to the availability of a validated model of the system: the model is necessary for tuning the subsystem controllers and for estimating the overlapping parameters of the overlapping decomposition. But the advantage of our approach, as demonstrated in this paper, lies in the absolute robustness of the designed controller that maintains its characteristics (quick dynamics and high capacity to track) for different cases of set point profiles and using different web (geometrical characteristics) also if a certain increasing of oscillation has been observed especially for the tension. This aspect is interesting for a potential industrial applicability.

Another interesting aspect is the possibility of decentralizing the controller for each subsystem that makes it of very low order (6th in this case) with easiness of realization, very low computational cost and absolute real time applicability (as demonstrated by the results of this paper).

The applicability of the decentralized H∞ controllers and the good experimental results shown in this paper may constitute the first experimental validation of the inherent studies and researches.

Acknowledgement

This work is supported by the Japan Society for the Promotion of Science that permitted to Dr Nicola Ivan Giannoccaro of spending a period of time for research at the laboratory of Kyushu Institute of Technology.

REFERENCES


