An Enhanced ILC-based PI Controller for Batch Processes

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Abstract: A novel combination of PI control and iterative learning control (ILC), referred to ILC-based PI control, is proposed in this study. This algorithm belongs to set-point-related (SPR) indirect ILC. Based on a 2-dimensional (2D) Roesser's system description, a sufficient condition for asymptotical stability of multi-input multi-output linear batch processes under the ILC-based PI controller was derived in this paper. Applications on injection packing pressure control show that the proposed method can achieve the design objective well, with performance improvement along both time and batch direction, and also owns good robustness to disturbances and measurement noises.

Keywords: Iterative learning control (ILC); indirect ILC; PI control; Roesser’s system; asymptotical stability.

1. INTRODUCTION

According to various application modes, ILC methods can be divided into two types as shown in Figure 1 [Wang et al. (2009)]: 1) ILC is used to design the control signal directly, for short, termed as direct ILC; 2) ILC is used to adjust some parameters for a local controller, for short, called indirect ILC. The indirect ILC consists of two loops: a local controller in the inner loop and an ILC in the outer loop. This two-loop structure induces two key issues for indirect ILC: what algorithm is used to design the local controller, and which parameters of the local controller are adjusted by the ILC. In general, ILC could be used to adjust the set-point [Wu and Ding (2007)], control gain [Xu and Yan (2004)], weight [Jiang et al. (2007)], and other parameters [Bone (1995); Tayebi and Chien (2007)] for the local controller.

In principle, the indirect ILC has several prominent advantages over the direct one. First, the existing process structure need not change; only an outer loop module is added to update some parameters of the existing local controller. Second, in many cases, indirect ILC has better robustness than the direct form; this is because direct ILC must have a feedforward term, which is sensitive to batch-wise variations, but a feedforward term is unnecessary for the local controller of an indirect ILC. In addition, the idea of indirect ILC indicates the development trend of control engineering: stability and robustness are not the only requirements for control design, and an optimization scheme should be utilized to improve the performance.

However, the following literature overview will show the scarcity of studies on the indirect ILC, which adds additional motivation to the current work. In Wang et al. (2009), 207 articles from the Web of Science that featured “iterative learning control” in the title were reviewed, and only 16 of them belong to the indirect ILC. Among the 16 indirect-ILC-related papers, ILC was used to update the set-point for the local controller in only two studies [Wu and Ding (2007); Tan et al. (2007)]. In Wu and Ding (2007), an anticipatory-type ILC (A-ILC) was used to...
adjust the set-point for a PID controller, and the proposed scheme was implemented on an X-Y platform. In Tan et al. (2007), ILC was used to update the set-point for a PID controller, and then a standard PID with adaptive gain was used to replace the ILC-based PID. For short, an indirect ILC that updates the set-point for the local controller is termed as set-point-related (SPR) indirect ILC.

In the above-mentioned studies on SPR indirect ILC, however, the stability of the closed-loop system was not addressed. Clearly, stability analysis for SPR indirect ILC is very important for both theory development and practical implementation. Because SPR indirect ILC consists of two loops, its stability and robustness are difficult to study. This issue was first addressed in Wang and Doyle (2009b), where the local controller is limited to be P-type. P-type control. An ILC-based PI controller was proposed and its structure need not change; only an outer loop module is added to update the set-point of the PI controller.

2. PROBLEM FORMULATION

2.1 System Description

Consider the following multi-input multi-output (MIMO) batch process,

\[
\begin{aligned}
    x(t+1,k) &= Ax(t,k) + Bu(t,k) + w(t,k) \\
    y(t,k) &= Cx(t,k) \\
    x(0,k) &\equiv x_0; \quad t = 0, 1, \ldots T - 1; \quad k = 1, 2, \ldots
\end{aligned}
\]

where \( t \) denotes time; \( k \) denotes batch index; \( x(t,k) \in \mathbb{R}^n \), \( u(t,k) \in \mathbb{R}^p \), and \( y(t,k) \in \mathbb{R}^m \) represent, respectively, the states, outputs, and inputs of the process at time \( t \) of the \( k \)th batch run; \( w(t,k) \in \mathbb{R}^n \) denotes disturbances; \( A, B, \) and \( C \) are the system matrices with appropriate dimensions; \( x_0 \) is the identical initial condition for each batch.

The control objective is to determine a control law such that the outputs tracks the given target, \( Y_R(t) \), as closely as possible. The tracking error is defined as:

\[
e(t,k) = Y_R(t) - y(t,k)
\]

2.2 ILC-Based PI Control

In this section, a novel PI control is introduced as follows,

\[
u(t,k) = u_0 + K_1 e(t,k) + K_2 I e(t,k)
\]

where \( u_0 \) is the initial value of the control signal and other terms are defined as below:

\[
\begin{aligned}
    e(t,k) &\equiv y_r(t,k) - y(t,k) \\
    I e(t,k) &\equiv I e(t-1,k) + e(t,k) \\
    I e(0,k) &\equiv 0
\end{aligned}
\]

where \( y_r(t,k) \) is the set-point, which could be different with the target \( Y_R(t) \). Therefore, \( e(t,k) \) and \( I e(t,k) \) are different: \( e(t,k) \) and \( I e(t,k) \) are termed as the tracking error and offset, respectively. For convenience, the following notation was introduced:

\[
\delta \mathbf{\square}(t,k) = \mathbf{\square}(t,k) - \mathbf{\square}(t,k-1), \quad \mathbf{\square} = x, y, u, \ldots \]

\[\delta \mathbf{\square}\] is the variation of \( \mathbf{\square} \) in the batch direction.

The set-point \( y_r(t,k) \) could be different in various batches, and it is updated by using ILC, as shown below:

\[
y_r(t,k) = y_r(t,k - 1) + L_1 \delta x(t,k) + L_2 I e(t - 1,k) + L_3 e(t + 1,k - 1)
\]

where \( L_1, L_2, \) and \( L_3 \) are the learning gain matrices. The key idea behind (6) is optimizing the set-point value using the tracking error in the previous batch and the variation of the offset integral in the batch direction.

For clarity, the block diagram of ILC-based PI controller was given in Figure 2. The proposed algorithm has the following advantages. First, PI control has been widely implemented in the practice and this existing process structure need not change; only an outer loop module is added to update the set-point of the PI controller. Second, the proposed algorithm has better robustness than classical direct ILC methods; this is because direct ILC must have a feedforward term, which is sensitive to variations in batch direction, but there is no feedforward term in the PI controller.
3. STABILITY ANALYSIS

In this section, stability of the proposed scheme is analyzed in a 2-dimensional framework. Before presenting the main results, a preliminary lemma on the 2D system stability is introduced first.

3.1 Preliminary Knowledge

Consider a Roesser’s system [Kaczorek (1985)],

\[
\begin{bmatrix}
  x_h(i + 1, j) \\
  x_v(i, j + 1)
\end{bmatrix} = \begin{bmatrix}
  A_{11} & A_{12} \\
  A_{21} & A_{22}
\end{bmatrix} \begin{bmatrix}
  x_h(i, j) \\
  x_v(i, j)
\end{bmatrix}
\]

(7)

where \( x_h \in \mathbb{R}^{n_h} \) is the horizontal state vector and \( x_v \in \mathbb{R}^{n_v} \) is the vertical state vector. The boundary condition of the Roesser’s system is

\[
\begin{bmatrix}
  x_h(0, j) \\
  x_v(i, 0)
\end{bmatrix},
\]

where \( x_h(0, j) \) and \( x_v(i, 0) \) represent the horizontal and vertical boundary conditions, respectively.

**Lemma 1** [Shi et al. (2005b)]. If there exist a function \( V(\cdot) \) and a scalar \( 0 < \rho < 1 \) such that

(a) \( V(X) \geq 0 \) for \( X \in \mathbb{R}^{n_h+n_v} \), and \( V(X) = 0 \) iff \( X = 0 \);

(b) \( V(X) \to \infty \) as \( \|X\| \to \infty \);

(c) for all boundary conditions,

\[
\sum_{i+j = i_0 + j_0 + q} V \left( \begin{bmatrix} x_h(i, j) \\ x_v(i, j) \end{bmatrix} \right) \leq \rho \sum_{i+j = i_0 + j_0 + q} V \left( \begin{bmatrix} x_h(i, j) \\ x_v(i, j) \end{bmatrix} \right)
\]

(8)

then the system (7) is asymptotically stable.

3.2 Stability Analysis in 2D Framework

With the ILC-based PI controller described in (3) and (6), the system (1) can be transformed to

\[
\delta x(t+1, k) = A \delta x(t, k) + B \delta u(t, k) \\
\delta x(t, k) = \bar{K}(X(t, k)) \delta x(t, k) + B \delta u(t, k)
\]

(9)

where the definition of \( \bar{K}(X(t, k)) \) is given in (13).

\[
\Pi_0 = \begin{bmatrix}
  A + BK_1L_1 - B(K_1 + K_2)C \\
  -C[A + BK_1L_1 - B(K_1 + K_2)C] \\
  BK_2 + B(K_1 + K_2)L_2 \\
  -C[BK_2 + B(K_1 + K_2)L_2] I - CB(K_1 + K_2)I
\end{bmatrix}
\]

(13)

Hence, analyzing stability of the batch process in (1) under the ILC-based PI controller in (3) and (6) is equivalent to analyzing stability of the Roesser’s system in (12). A sufficient condition for asymptotical stability of the 2D system (12) is therefore given as below.

**Theorem 1.** If there exist matrices \( Q_1 > 0 \), \( Q_2 > 0 \), \( Q_3 > 0 \), \( Y_1 \), \( Y_2 \) and \( Y_3 \) such that the following linear matrix inequality (LMI) holds

\[
\Omega < 0
\]

(14)

(\( \Omega \) was defined in (16)) and the learning gain matrices for (6) are chosen as

\[
L_1 = Y_1Q_1^{-1}, \quad L_2 = Y_2Q_2^{-1}, \quad L_3 = Y_3Q_3^{-1}
\]

(15)

then the Roesser’s system (12) is asymptotically stable.

\[
\Omega = \begin{bmatrix}
  -Q_1 & 0 & 0 & 0 & 0 \\
  0 & -Q_2 & 0 & 0 & 0 \\
  0 & 0 & -Q_3 & 0 & 0 \\
  0 & 0 & 0 & -Q_2 & 0 \\
  0 & 0 & 0 & 0 & -Q_3
\end{bmatrix}
\]

(16)

**Proof.** The proof is presented in the Appendix.

Because of the finite-time horizon, a linear batch process is always stable; however, it might be not asymptotically stable. Theorem 1 presents a sufficient condition for asymptotical stability of the closed-loop system. On the other hand, fail of condition in (14) doesn’t mean the batch process must be not asymptotically stable, because this condition is conservative.

The convergence rate of ILC in (6) is mainly determined by \( L_3 \); a larger value of \( L_3 \) induces a faster convergence rate. Therefore, one could design \( L_3 \) prior and replace \( Y_3 \) in (10) with \( L_3Q_3 \). A experiential recommendation is choosing \( L_3 = diag \{ l_1^3, l_2^3, \ldots, l_m^3 \} \), where \( 0 < l_i^3 < 2, i = 1, 2, \ldots, m \).

4. SIMULATION RESULTS

Injection molding process is a typical batch process, which consists of three main phases: filling, packing/holding, and cooling [Shi et al. (2005a)]. In the packing phase, nozzle pressure, a key process variable, should be controlled to follow a certain profile to ensure product quality.

The nozzle pressure to the proportional valve has been identified as an autoregressive model:

\[
\{1 - 1.607(\pm5\%)z^{-1} + 0.6086(\pm5\%)z^{-2}\} y(t, k) = \{1.239(\pm5\%)z^{-1} - 0.9282(\pm5\%)z^{-2}\} u(t, k) + w(t, k)
\]

(17)
where $w(t,k)$ was introduced to describe the unknown disturbances. The stability margin of the nominal model is narrow, as one of the poles located near the boundary of the united disk.

Because only input and output variables are measurable, the state-space realization of (17) is formulated as below:

$$
\begin{bmatrix}
    y(t+1,k) \\
    y(t,k) \\
    y(t-1,k)
\end{bmatrix} = \begin{bmatrix}
    1.607 & -0.6086 & -0.9282 \\
    1 & 0 & 0 \\
    1 & 0 & 0
\end{bmatrix} \begin{bmatrix}
    y(t,k) \\
    y(t,k) \\
    y(t-1,k)
\end{bmatrix} + \begin{bmatrix}
    \Delta A \\
    0 \\
    0
\end{bmatrix} \begin{bmatrix}
    u(t,k) \\
    w(t,k)
\end{bmatrix}
$$

where the uncertain parameter perturbations are expressed as

$$
\Delta A = \Phi_A \Delta A = \begin{bmatrix}
    0.0804 & -0.0304 & -0.0464 \\
    0 & 0 & 0 \\
    0 & 0 & 0
\end{bmatrix} \Delta A,
$$

$$
\Delta B = \Phi_B \Delta B = \begin{bmatrix}
    0.0620 \\
    0 \\
    0
\end{bmatrix} \Delta B,
$$

$$
\Delta A = diag(\sigma_1, \sigma_2, \sigma_3), \Delta B = \sigma_4
$$

The scalars $\sigma_i$ ($0 < \sigma_i < 1$, $i = 1, 2, 3, 4$) denote unknown perturbations. Here for illustration, the target profile takes the following form:

$$
Y_D(t) = \begin{cases}
    200, & 1 \leq t < 100 \\
    200 + 5(t-100), & 100 \leq t < 120 \\
    300, & 120 \leq t < 200
\end{cases}
$$

With a sampling time of $T_s=0.005s$, a continuous-time process model may be identified from a step response test [Liu and Gao (2010)]

$$
G(s) = \frac{k_p}{\tau s + 1} = \frac{194.076}{1.199s+1}
$$

According to the internal model control (IMC) theory [Morari and Zafiou (1989)], the optimal controller in a unity feedback control structure may be expressed as a PI control,

$$
C(s) = k_c + \frac{k_i}{s}
$$

where $k_c = \frac{\tau s}{\lambda \tau s + 1}$, $k_i = \frac{1}{\lambda \tau s}$, and $\lambda$ is the closed-loop time constant that is adjustable through the controller.

The corresponding discrete-time PI control law is

$$
u(t,k) = K_1 e(t,k) + K_2 \sum_{j=0}^{t} e(j,k)
$$

where $K_1 = k_c$ and $K_2 = T_k k_i$.

One may tune $\lambda$ to obtain different settings of PI control. The smaller is $\lambda$, the faster is closed-loop output response, but weaker stability for the presence of process uncertainty, and vice versa. In this study, this parameter is chosen as $\lambda = 0.15$. Then

$$
K_1 = 0.0412, \quad K_2 = 0.00017
$$

Choosing $L_1 = 1.6$, solving LMI (14) and then using (15), the learning gain matrices were obtained:

$$
L_1 = [-48.57 \quad 26.33 \quad 40.16], \quad L_2 = -0.0182
$$

When there is no uncertainty, i.e., $\sigma_1 = 0$, the input signals and output responses in batches 1, 10, and 50 are shown
Lemma A2 (Schur Complement). (Boyd et al., 1994) Assume that $W$, $L$, and $V$ are given matrices with appropriate dimensions, the following inequality holds:

$$XY + Y^T X^T \preceq XRX^T + X^T R^{-1} Y.$$ 

Particularly let $R = \varepsilon I$, where $\varepsilon > 0$, then

$$XY + Y^T X^T \preceq \varepsilon XX^T + \varepsilon^{-1} Y^T Y.$$ 

Lemma A1. (Boyd et al., 1994) Assume $X$ and $Y$ are matrices or vectors with appropriate dimensions. For any positive definite matrix $R > 0$ with appropriate dimensions, the following inequality holds:

$$XY + Y^T X^T \preceq XRX^T + X^T R^{-1} Y.$$ 

5. CONCLUSIONS

In this study, a new ILC-based PI controller was proposed for MIMO linear batch processes. Under this algorithm, a high-acceleration and high-precision platform via A-type local controller. Proceedings of 21st Chinese Contr. Decision Conf., 1739–1744.


Appendix A. PRELIMINARY KNOWLEDGE

REFERENCES


Fig. 5. ATE comparison between ILC-based PI control and PI-type ILC in three cases: (a) with repetitive disturbances; (b) with nonrepetitive disturbances; (c) with measurement noise.

uniformly distributed within $[0,2\pi]$; with measurement noises, where $v(t,k)$ is a random variable uniformly distributed within $[-5,5]$. Obviously, the closed-loop system is a stochastic process in the second and third cases, so Monte Carlo method was used to compare these algorithms. In each case, 100 groups of Monte Carlo simulations were done. Denote $ATE^j(k)$ as the ATE value for batch $k$ in the $i$th simulation, then the mean ATE can be defined as

$${\bar{ATE}} = \frac{1}{100} \sum_{i=1}^{100} ATE^j(k)$$

The ATE and/or mean ATE values under the proposed algorithm in three cases are shown in Figure 5. Even though there exist disturbances and/or measurement noises, the ATE values can be decreased in the batch direction, which demonstrates the proposed algorithm has good robustness performance with respect to disturbances and measurement noises.

5. CONCLUSIONS

In this study, a new ILC-based PI controller was proposed for MIMO linear batch processes. Under this algorithm, a sufficient condition for asymptotical stability of the closed-loop system was introduced and proved using 2D Roesser’s model formulation and LMI. The simulation results on an injection molding process demonstrate that the proposed algorithm can achieve the control objective well and also has excellent robustness to repetitive and nonrepetitive disturbances and measurement noises. The synthesis and analysis for ILC-based PID controller will be done in the future.
dimensions, where W and V are positive definite matrices, then
\[ LT V L - W < 0 \]
iff
\[ \begin{bmatrix} -W & L \\ L & -V^{-1} \end{bmatrix} < 0 \]
or
\[ \begin{bmatrix} -V^{-1} & L \\ L^T & -W \end{bmatrix} < 0. \]

Appendix B. PROOF OF THEOREM 1

Denote \( P_1 = Q_1^{-1}, P_2 = Q_2^{-1}, P_3 = Q_3^{-1}, P_{12} = diag\{P_1, P_2, P_3\} \), and \( P = diag\{P_1, P_2, P_3\} \). Pre- and post-multiplying the left-hand side matrix in (14) by the matrix \( diag\{P_1, P_2, P_3, I, I, I\} \), respectively, and using (15), one knows that
\[ \begin{bmatrix} -P & 0 \\ \Pi_0 & -Q \end{bmatrix} < 0. \]  
(B.1)

Based on Lemma A2, one can get
\[ \Pi_0^T P \Pi_0 - P < 0. \]  
(B.2)

Obviously, there exists a \( \rho \in (0, 1) \) such that
\[ \Pi_0^T \begin{bmatrix} P_1 & 0 & 0 \\ 0 & P_2 & 0 \\ 0 & 0 & P_3 \end{bmatrix} \Pi_0 - \rho \begin{bmatrix} P_1 & 0 & 0 \\ 0 & P_2 & 0 \\ 0 & 0 & P_3 \end{bmatrix} < 0. \]  
(B.3)

Therefore, for all \( \frac{\delta x(t, k)}{\delta I_c(t - 1, k)} \neq 0 \), it follows that
\[ V_{\Pi_0^T P \Pi_0 - \rho P} \left( \begin{bmatrix} \delta x(t, k) \\ \delta I_c(t - 1, k) \\ \bar{e}(t + 1, k - 1) \end{bmatrix} \right) < 0. \]  
(B.4)

Hence,
\[ V_{\Pi_0^T P \Pi_0 - \rho P} \left( \begin{bmatrix} \delta x(t, k) \\ \delta I_c(t - 1, k) \\ \bar{e}(t + 1, k - 1) \end{bmatrix} \right) - \rho V_P \left( \begin{bmatrix} \delta x(t, k) \\ \delta I_c(t, k) \\ \bar{e}(t + 1, k - 1) \end{bmatrix} \right) < 0. \]  
(B.5)

According to (12), one gets
\[ V_P \left( \begin{bmatrix} \delta x(t + 1, k) \\ \delta I_c(t, k) \\ \bar{e}(t + 1, k) \end{bmatrix} \right) - \rho V_P \left( \begin{bmatrix} \delta x(t, k) \\ \delta I_c(t - 1, k) \\ \bar{e}(t + 1, k - 1) \end{bmatrix} \right) < 0. \]  
(B.6)

That is to say
\[ V_{P_{12}} \left( \begin{bmatrix} \delta x(t + 1, k) \\ \delta I_c(t, k) \\ \bar{e}(t + 1, k) \end{bmatrix} \right) + V_{P_3} (\bar{e}(t + 1, k)) - \rho V_{P_{12}} \left( \begin{bmatrix} \delta x(t, k) \\ \delta I_c(t - 1, k) \\ \bar{e}(t + 1, k - 1) \end{bmatrix} \right) - \rho V_{P_3} (\bar{e}(t + 1, k - 1)) < 0 \]  
(B.7)

Thus, for all \( I_0 \geq 0, J_0 \geq 0, q > 0 \), the following inequalities hold.

The sum of these inequalities leads to the following inequalities
\[ \sum_{i+j=I_0+J_0+q, I_0 \leq i \leq I_0+q, J_0 \leq j \leq J_0+q} V_{P_{12}} \left( \begin{bmatrix} \delta x(i, j) \\ \delta I_c(i - 1, j) \end{bmatrix} \right) + V_{P_3} (\bar{e}(i, j)) \leq \rho \sum_{i+j=I_0+J_0+q, I_0 \leq i \leq I_0+q, J_0 \leq j \leq J_0+q} V_{P_{12}} \left( \begin{bmatrix} \delta x(i, j) \\ \delta I_c(i - 1, j) \end{bmatrix} \right) \]
\[ + \rho \sum_{i+j=I_0+J_0+q, I_0 \leq i \leq I_0+q, J_0 \leq j \leq J_0+q-1} V_{P_3} (\bar{e}(i, j)) \]
\[ \leq \rho \sum_{i+j=I_0+J_0+q, I_0 \leq i \leq I_0+q, J_0 \leq j \leq J_0+q} V_{P_{12}} \left( \begin{bmatrix} \delta x(i, j) \\ \delta I_c(i - 1, j) \end{bmatrix} \right) + V_{P_3} (\bar{e}(i, j)) \].

In other words,
\[ \sum_{i+j=I_0+J_0+q+1, I_0 \leq i \leq I_0+q, J_0 \leq j \leq J_0+q} V_P \left( \begin{bmatrix} \delta x(i, j) \\ \delta I_c(i - 1, j) \end{bmatrix} \right) \]
\[ \leq \rho \sum_{i+j=I_0+J_0+q, I_0 \leq i \leq I_0+q, J_0 \leq j \leq J_0+q} V_P \left( \begin{bmatrix} \delta x(i, j) \\ \delta I_c(i - 1, j) \end{bmatrix} \right) \].

Because \( P_1, P_2, P_3 \) are positive definite, the function \( V_P(\cdot) \) satisfies the conditions (a) and (b) in Lemma 1. Therefore, the system (12) is asymptotically stable.