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Port-Hamiltonian and power-based integral type control of a manipulator system

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Abstract: Steady-state errors usually occur in the implementation of control algorithms on a system, caused by uncertainties and/or modeling errors. Such errors are traditionally eliminated by adding a simple integrator. However, a simple integrator for nonlinear systems usually spoils the structure of the modeling framework. For passive systems it has been shown that passivity loses its global character. Here, integral type controllers are presented for standard nonlinear mechanical systems, based on the port-Hamiltonian and power-based modeling frameworks. Experimental results are given for a planar manipulator system.

Keywords: Port-Hamiltonian systems, Brayton-Moser systems, nonlinear control, integral control, mechanical manipulators.

1. INTRODUCTION

The port-Hamiltonian (PH) modeling framework of Maschke and van der Schaft (1992) has received a considerable amount of interest in the last decade because of its insightful physical structure. It is well known that a large class of (nonlinear) physical systems can be described in the PH framework. The popularity of PH systems can be largely accredited to its application for analysis and control design of physical systems, as shown in van der Schaft (2000), Fujimoto and Sugie (2001), Fujimoto et al. (2003), Ortega et al. (2002a), Ortega et al. (2002b), Duindam et al. (2009) and many others. Control is realized by energy-shaping and damping injection. The Hamiltonian (total energy of the system) is shaped into a new function with desired properties while damping injection influences the convergence to the desired equilibrium point.

Power-based modeling is another modeling framework with a specific physical structure. In this framework a system is modeled by Brayton-Moser (BM) equations. It was developed in the early sixties to describe a large class of nonlinear electrical networks, presented in Brayton and Moser (1964). The BM equations represent a gradient system with respect to an indefinite metric defined by the dynamic part of the network and a mixed-potential function which describes the static part and has the units of power. The power-based modeling concept was later extended to include a larger class of physical systems, i.e., switched-mode networks in Jeltsema and Scherpen (2009), mechanical systems in Jeltsema and Scherpen (2007), Jeltsema and Scherpen (2009) and exothermic stirred tank reactors in Favache and Dochain (2009). Some results on control of BM systems are presented in Ortega et al. (2003) and García-Canseco et al. (2010).

In this paper the above mentioned modeling frameworks are used for nonlinear mechanical systems to design controllers that include an integral action. Steady-state errors usually occur in the actual implementation of control systems, usually caused by uncertainties and/or modeling errors. A simple integrator is known to compensate for steady-state errors, but usually spoils the modeling framework. In Ortega et al. (1998) it is shown that, for Euler-Lagrange (EL) mechanical systems, only local asymptotic stability can be proven when adding an integrator the classical way. A similar problem exists for PH systems, unless the integrator is added by a transformation of coordinates as presented in Donaire and Junco (2009). Nonlinear PID controllers are presented in Ortega et al. (1998) to achieve global stability. In Kelly (1998) global asymptotic stability is achieved by a linear PD controller, combined with an integral action of a nonlinear function of position errors.

The first main contribution of this paper is to present integral control for standard mechanical systems in the PH framework. Integral control without losing the PH structure is realized by a coordinate transformation. We also show that control via the proposed coordinate transformation improves the disturbance attenuation properties. Secondly, we develop a different integral controller based on the power-based modeling framework. The two different modeling frameworks give rise to different control structures. Both controllers realize global asymptotic stability and compensate for steady-state errors. Compared to the nonlinear controllers presented in Ortega et al. (1998) and Kelly (1998) the integrator dynamics are determined by the modeling framework and depend on the dynamics of the system. The PH method has the additional advantage of preserving the PH structure for the closed-loop system. The two control methods are applied for stabilization of a planar manipulator system. Experimental results for the two methods are presented and compared with PID control based on the classical linear approach.

Sections 2 and 3 present modeling and integral control for PH systems and BM systems respectively. The different control methods are applied on a planar manipulator experimental setup with two rotational joints, i.e., 2R planar manipulator. The experimental results are presented and compared in section 4. Concluding remarks are then given in section 5.

Notation. All vectors are column vectors, including the gradient of a scalar function.

2. INTEGRAL CONTROL FOR PORT-HAMILTONIAN SYSTEMS

2.1 Integral type control

Consider the PH system described by

$$\begin{aligned} \dot{x} &= [J(x) - R(x)]\frac{\partial H}{\partial x}(x) + g(x)u \\ y &= g(x)^\top \frac{\partial H}{\partial x}(x) \end{aligned} \tag{1}$$

with $J(x) \in \mathbb{R}^{n\times n}$ the skew-symmetric interconnection matrix, $R(x) \in \mathbb{R}^{n\times n}$ the symmetric, positive-semidefinite, damping matrix, $x \in \mathbb{R}^n$, the Hamiltonian $H(x)$, input u and output y, with $u, y \in \mathbb{R}^m$, $m \leq n$. Consider a k degrees of freedom (dof) rigid robot manipulator. The manipulator can be described as a fully actuated standard mechanical system and is described in the form of (1) by

$$\begin{aligned} \begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} &= \begin{bmatrix} 0 & I \\ -I & -D(q,p) \end{bmatrix} \begin{bmatrix} \dfrac{\partial H}{\partial q} \\ \dfrac{\partial H}{\partial p} \end{bmatrix} + \begin{bmatrix} 0 \\ (u-d) \end{bmatrix} \\ y &= \frac{\partial H}{\partial p} = \dot{q} \end{aligned} \tag{2}$$

with $q = (q_1, ..., q_k)^\top$ the vector of generalized configuration coordinates, $p = (p_1, ..., p_k)^\top$ the vector of generalized momenta, I the identity matrix, $D(q,p)$ the damping matrix and constant input disturbance d. The Hamiltonian of the system is equal to the sum of kinetic and potential energy:

$$H(q,p) = \frac{1}{2}p^\top M^{-1}(q)p + V(q) \tag{3}$$

where $M(q) = M^\top(q) > 0$ is the system mass-inertia matrix and $V(q)$ the potential energy. Assume that $D = 0$ in (2). The problem of stabilization control is to find a control input u which brings the position q of the system to the desired constant position q_d. In theory, assuming $d = 0$, it can be proven that potential energy shaping with damping injection achieves this goal. However, as mentioned in the introduction, steady-state errors usually occur in real implementation due to the disturbances. A simple integrator can be added, resulting in the well-known PID controller

$$u = \frac{\partial V}{\partial q} - K_p\bar{q} - K_d y + \nu \tag{4}$$

$$\dot{\nu} = -K_i\bar{q} \tag{5}$$

with K_p, K_i, K_d positive definite constant matrices, ν the integrator state and $\bar{q} = q - q_d$. Unfortunately only local asymptotic stability can be proven for such a controller when applied to the nonlinear mechanical system (2). To deal with this problem, we add an integrator to the system, in such a manner that the PH structure is preserved. First, and similar to feedback linearization control, Khalil (1996), Spong et al. (2006), we define

$$u = \frac{1}{2}\frac{\partial p^\top M^{-1}(q)p}{\partial q} + \frac{\partial V}{\partial q}(q) - M_d M^{-1}(q)K_p\bar{q} + v \tag{6}$$

with M_d constant and positive definite and v the new input signal. This realizes the closed-loop system (for simplicity of notation, we leave out the arguments of H_d and M)

$$\begin{aligned} \begin{bmatrix} \dot{q} \\ \dot{p} \end{bmatrix} &= \begin{bmatrix} 0 & M^{-1}M_d \\ -M_d M^{-1} & 0 \end{bmatrix} \begin{bmatrix} \dfrac{\partial H_d}{\partial q} \\ \dfrac{\partial H_d}{\partial p} \end{bmatrix} + \begin{bmatrix} 0 \\ v-d \end{bmatrix} \\ \tilde{y} &= \frac{\partial H_d}{\partial p} \end{aligned} \tag{7}$$

with new output $\tilde{y}$ and

$$H_d = \frac{1}{2}p^\top M_d^{-1}p + \frac{1}{2}(q-q_d)^\top K_p(q-q_d) \tag{8}$$

the new Hamiltonian. The goal of input (6) is to transform the original system (2) with $D = 0$ into a system with a constant mass-inertia matrix M_d in the Hamiltonian, system (7). This simplifies the coordinate transformation in order to realize integral control. Inspired by Donaire and Junco (2009) define the coordinate transformation

$$\hat{p} = p + K_p\bar{q} \tag{9}$$

The output for the transformed system now becomes

$$\hat{y} = M_d^{-1}\hat{p} \tag{10}$$

$$= M_d^{-1}\left(M(q)\dot{q} + K_p\bar{q}\right) \tag{11}$$

The integrator dynamics can then be described by

$$\dot{z} = -K_i\hat{y} \tag{12}$$

with z the integrator state and K_i a positive definite constant matrix. The control input v with integrator state z is then given by

$$v = -K_p\dot{q} - K_d\hat{y} + z \tag{13}$$

To add an integrator and maintain the PH structure the interconnection of the integrator with the system should be made through the passive output. In the new situation interconnection is made through the new passive output, shown in (12). In the original setup the passive output is velocity, so the integrator dynamics cannot depend on position error (what we actually want). With the proposed coordinate transformation we have position error in the new output, and so we have a type of integral control while preserving the PH structure. It can then be shown that the resulting closed-loop system is asymptotically stable with zero steady-state error. This is summarized in the following theorem.

***Theorem** 1.* Consider the fully actuated mechanical system (2) with $D = 0$, K_p, K_d, K_i, M_d constant positive definite matrices and constant input disturbance d. The control input

$$u = \frac{1}{2}\frac{\partial p^\top M^{-1}(q)p}{\partial q} + \frac{\partial V}{\partial q}(q) - M_d M^{-1}(q)K_p\bar{q} - K_p\dot{q} - K_d\hat{y} + z \tag{14}$$

with integrator dynamics (12) asymptotically stabilizes system (2), with zero steady-state error.

Proof. Applying (13) to (7) with integrator dynamics (12) realizes the PH closed-loop system (for simplicity of notation, we leave out the arguments of $\hat{H}$ and M)

$$\begin{bmatrix} \dot{q} \\ \dot{\hat{p}} \\ \dot{\bar{z}} \end{bmatrix} = \begin{bmatrix} -M^{-1} & M^{-1}M_d & 0 \\ -M_dM^{-1} & -K_d & K_i \\ 0 & -K_i & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial \hat{H}}{\partial q} \\ \frac{\partial \hat{H}}{\partial \hat{p}} \\ \frac{\partial \hat{H}}{\partial \bar{z}} \end{bmatrix} \quad (15)$$

$$\hat{y} = \frac{\partial \hat{H}}{\partial \hat{p}}$$

with $\bar{z} = z - d$ [1] and Hamiltonian

$$\hat{H} = \frac{1}{2}\hat{p}^\top M_d^{-1}\hat{p} + \frac{1}{2}(q-q_d)^\top K_p(q-q_d) + \frac{1}{2}\bar{z}^\top K_i^{-1}\bar{z} \quad (16)$$

Take (16) as Lyapunov function. It can be verified that

$$\dot{\hat{H}} = -\bar{q}^\top K_p M^{-1}(q) K_p \bar{q} - \hat{y}^\top K_d \hat{y} \leq 0 \quad (17)$$

and $\dot{\hat{H}}$ is equal to zero when $\bar{q} = \hat{y} = 0$. From the closed-loop dynamics (15) the largest invariant set for $\dot{\hat{H}} = 0$ equals $\bar{q} = \hat{p} = \bar{z} = 0$ and asymptotic stability can be proven by LaSalle's principle. The control input (14) is found by substituting (13) in (6). □

***Remark** 1.* Additional tuning can be added to Theorem 1 by, instead of (9), defining the transformation as

$$\hat{p} = p + A\bar{q} \quad (18)$$

with A constant positive definite and instead of the input (14)

$$u = \frac{1}{2}\frac{\partial p^\top M^{-1}(q)p}{\partial q} + \frac{\partial V}{\partial q}(q) - M_d M^{-1}(q) K_p \bar{q} - A\dot{q} - K_d \hat{y} + z \quad (19)$$

and again

$$\hat{y} = M_d^{-1}\hat{p} \quad (20)$$

However, in order to still preserve the PH structure for the closed-loop system we need the extra condition that the symmetric part of $M^{-1}(q)AK_p^{-1}$ is positive definite. In our experiments we use this possibility. ◁

We emphasize the extra tuning possibilities given by this control method. Notice how we now have the matrices K_p, K_d, M_d and K_i (and eventually A) as control tuning matrices. The next subsection further analyzes the advantages of the additional tuning and the proposed coordinate transformation.

***Remark** 2.* A different alternative is to define a transformation where the new passive output does not include the position error, but the integrator state z, i.e.,

$$\hat{p} = p + Az \quad (21)$$

with A again constant and positive definite. Feedback with the new output then injects damping and adds the integrator state. Compared to (9) the control input is simpler, but with more complex integrator dynamics. In this paper we chose the transformation (9) because of the implications for disturbance attenuation, which is explained in the next subsection. ◁

[1] Since d is constant, $\dot{\bar{z}} = \dot{z}$.

2.2 Disturbance attenuation properties

Traditionally for mechanical systems damping is modeled as a force (effort) entering the $\dot{p}$ dynamics, described in terms of a Rayleigh dissipation function. This can be seen in (2) with the damping matrix D. However, by the transformation presented in this section the closed-loop system (15) now also has a damping-like term entering the $\dot{q}$ dynamics, given by the matrix $M^{-1}(q)$. In the case of Remark 1 we have the symmetric part of $M^{-1}(q)AK_p^{-1}$. In a virtual sense we have force controlled damping causing a velocity, while usually damping is velocity controlled. This damping is referred in Jeltsema and Scherpen (2009) as the mechanical co-content associated to dissipation. It is damping in terms of the dual of the Rayleigh dissipation, Rayleigh co-dissipation. It can be shown that this additional damping also improves the disturbance attenuation properties. Towards this end we analyze the L_2-gain of the closed-loop system with respect to a disturbance δ.

For a general system described by

$$\begin{aligned} \dot{x} &= f(x) + b(x)\delta \\ y &= h(x) \end{aligned} \quad (22)$$

the L_2-gain bound $\gamma > 0$ is found if for γ there exists a smooth nonnegative solution $W(x)$ to the Hamilton-Jacobi inequality (HJI), van der Schaft (2000):

$$\frac{\partial W}{\partial x}^\top f(x) + \frac{1}{2}\frac{1}{\gamma^2}\frac{\partial W}{\partial x}^\top b(x)b^\top(x)\frac{\partial W}{\partial x} + \frac{1}{2}h^\top(x)h(x) \leq 0 \quad (23)$$

Take the closed-loop system (15), without integral control. Assume furthermore that instead of only the constant disturbance d we now have the disturbances d_1, d_2, i.e., $\delta = (d_1, d_2)^\top$. Contrary to the previous subsection the disturbance does not have to be constant. The PH closed-loop system is then given by (for simplicity of notation, we leave out the arguments of $\bar{H}$ and M)

$$\begin{bmatrix} \dot{q} \\ \dot{\hat{p}} \end{bmatrix} = \begin{bmatrix} -M^{-1} & M^{-1}M_d \\ -M_dM^{-1} & -K_d \end{bmatrix} \begin{bmatrix} \frac{\partial \bar{H}}{\partial q} \\ \frac{\partial \bar{H}}{\partial \hat{p}} \end{bmatrix} + \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \quad (24)$$

$$\bar{y} = \begin{bmatrix} \frac{\partial \bar{H}}{\partial q}^\top & \frac{\partial \bar{H}}{\partial \hat{p}}^\top \end{bmatrix}^\top$$

with Hamiltonian

$$\bar{H} = \frac{1}{2}\hat{p}^\top M_d^{-1}\hat{p} + \frac{1}{2}(q-q_d)^\top K_p(q-q_d) \quad (25)$$

The HJI (23) for this system, with $W = \bar{H}$ and $b(x)$ identity is satisfied when

$$\left(\frac{1}{2\gamma^2} + \frac{1}{2}\right) I - M^{-1}(q) \leq 0 \quad (26)$$

$$\left(\frac{1}{2\gamma^2} + \frac{1}{2}\right) I - K_d \leq 0 \quad (27)$$

In the original setup (no coordinate transformation) only the damping matrix K_d enters the system, giving disturbance attenuation only against d_2. Now, because of the additional (virtual) damping entering the $\dot{q}$ dynamics we also have disturbance attenuation against disturbances on the $\dot{q}$ dynamics, i.e., d_1. Furthermore, if we include the additional tuning matrix A and the symmetric part of

$M^{-1}(q)AK_p^{-1}$ is positive definite (see remark 1) condition (26) changes into

$$\left(\frac{1}{2\gamma^2}+\frac{1}{2}\right)I - M^{-1}(q)AK_p^{-1} \leq 0 \tag{28}$$

In (26) the disturbance attenuation against d_1 depends on the matrix $M(q)$, so we have no influence on this part. Now, we can use the matrices A and K_p to reduce γ and improve the disturbance attenuation against d_1. Improving the disturbance attenuation against d_2 was already possible by the tuning matrix K_d.

3. INTEGRAL CONTROL FOR BRAYTON-MOSER SYSTEMS

The BM (Brayton-Moser) equations, as introduced for nonlinear electrical RLC networks, take the special gradient form

$$Q(x)\dot{x} = \frac{\partial P}{\partial x}(x) + B(x)u \tag{29}$$

with $x \in \mathbb{R}^n$ the vector of system states, $Q(x) \in \mathbb{R}^{n\times n}$ a full rank matrix, $B(x)$ the input matrix of rank $m \leq n$ and $P(x)$ the mixed-potential function (which has the units of power). In control via power-shaping an input u is found which shapes the function $P(x)$ into $P_d(x) = P(x)+P_a(x)$, with desired properties for $P_d(x)$. Stability can be proven by finding an alternative pair $\tilde{Q}(x), \tilde{P}_d$, which equivalently describe the system with $Q(x), P_d(x)$ and where $\tilde{P}_d$ is a candidate Lyapunov function. The alternative pair is defined in Brayton and Moser (1964) by

$$\tilde{Q} = \left(\lambda I + \frac{\partial^2 P_d}{\partial x^2}K\right)Q(x) \tag{30}$$

$$\tilde{P}_d = \lambda P_d(x) + \frac{\partial P_d}{\partial x}K\frac{\partial P_d}{\partial x} \tag{31}$$

with arbitrary constant $\lambda \neq 0$ and constant symmetric matrix K. In Jeltsema et al. (2003) a passive output is defined for the BM system based on the matrix $\tilde{Q}(x)$.

In this section we take inspiration from Brayton and Moser (1964) to, like in the previous section, find a passive output which also includes the position error. We can then again interconnect the system with an integrator in a passivity preserving way. Take the system (2) with control input

$$u = \frac{\partial V}{\partial q} - K_p\bar{q} - K_d y + v \tag{32}$$

which then gives

$$\begin{bmatrix}\dot{q}\\ \dot{p}\end{bmatrix} = \begin{bmatrix}0 & I\\ -I & -K_d\end{bmatrix}\begin{bmatrix}\frac{\partial H_n}{\partial q}\\ \frac{\partial H_n}{\partial p}\end{bmatrix}, \quad y = \frac{\partial H_n}{\partial p} \tag{33}$$

with Hamiltonian

$$H_n = \frac{1}{2}p^\top M^{-1}(q)p + \frac{1}{2}(q-q_d)^\top K_p(q-q_d) \tag{34}$$

By taking

$$Q = \begin{bmatrix}0 & I\\ -I & -K_d\end{bmatrix}^{-1}, \quad B = \begin{bmatrix}0 & I\\ -I & -K_d\end{bmatrix}^{-1}\begin{bmatrix}0\\ I\end{bmatrix} \tag{35}$$

and $P_d = H_n$ we have the BM form for (33), i.e.,

$$Q\dot{x} = \frac{\partial H_n}{\partial x} + Bv \tag{36}$$

with $x = (q,p)^\top$.

***Remark** 3.* Contrary to Brayton and Moser (1964), the system is not defined by a mixed-potential function which has units of power. Notice that (36) is still described by the energy function H_n. Nevertheless, it still has the BM form shown in (29). In Jeltsema and Scherpen (2007) a BM description is given with a mixed-potential function with units of power. However, for systems with non-constant mass-inertia matrix such a description is rather complex, which makes the task of control design very hard. ◁

Inspired by the alternative pairs of Brayton & Moser and similar to the previous section we look for an equivalent description of (36), which has a passive output that includes position error. Define a non-singular matrix

$$T = \begin{bmatrix}I & 0\\ I & K_d\end{bmatrix} \tag{37}$$

which equivalently describes (36) by

$$TQ\dot{x} = T\frac{\partial H_n}{\partial x} + TBv \tag{38}$$

Taking $\tilde{Q} = TQ$ and $f(x) = T\frac{\partial H_n}{\partial x}$ implies the new passive output

$$y' = Q^{-1}Bf(x) \tag{39}$$

With this alternative passive output we can define integrator dynamics

$$\begin{aligned}\dot{\tau} &= -K_i y'\\ &= -K_i\left(\frac{1}{2}\frac{\partial p^\top M^{-1}(q)p}{\partial q} + K_p\bar{q} + K_d M^{-1}(q)p\right)\end{aligned} \tag{40}$$

with τ the integrator state. Recall that $M^{-1}(q)p = \dot{q}$, giving the total control input

$$u = \frac{\partial V}{\partial q} - K_p\bar{q} - K_d\dot{q} + \tau \tag{41}$$

Unfortunately, $f(x)$ is not integrable. Nevertheless, it is possible to prove that under certain conditions the control input (41) with integrator dynamics (40) realizes global asymptotic stability with zero steady-state error.

***Theorem** 2.* Consider the system (2) for a constant d and assume that $D = 0$. Choose K_i and K_d such that the matrices [2]

$$C_1 = \begin{bmatrix}M^{-1}+K_i & -I\\ -I & K_i^{-1}\end{bmatrix} > 0 \tag{42}$$

$$C_2 = \begin{bmatrix}2M^{-1}K_dM^{-1} & -K_i - M^{-1}\\ -K_i - M^{-1} & 2I\end{bmatrix} > 0 \tag{43}$$

are positive definite. Then, the control input (41) with integrator dynamics (40) asymptotically stabilizes system (2), with zero steady-state error.

Proof. Consider the Lyapunov candidate function

$$\mathcal{V} = \frac{1}{2}(q-q_d)^\top K_p(q-q_d) + \frac{1}{2}\begin{bmatrix}p\\ \bar{\tau}\end{bmatrix}^\top C_1\begin{bmatrix}p\\ \bar{\tau}\end{bmatrix} \tag{44}$$

with $\bar{\tau} = \tau - d$ and which is positive definite when (42) holds. The time derivative $\dot{\mathcal{V}}$ can then be given by

$$\dot{\mathcal{V}} = -p^\top M^{-1}K_dM^{-1}p + p^\top M^{-1}\bar{\tau} + p^\top K_i\bar{\tau} - \bar{\tau}^\top\bar{\tau} \tag{45}$$

[2] For simplicity of notation, we leave out the arguments of M.

and in matrix form

$$\dot{\mathcal{V}} = -\frac{1}{2} \begin{bmatrix} \dot{q} \\ \bar{\tau} \end{bmatrix}^\top C_2 \begin{bmatrix} \dot{q} \\ \bar{\tau} \end{bmatrix} \tag{46}$$

which is negative definite when (43) holds. This implies that $\dot{q} = \bar{\tau} = 0$ when $t \to \infty$. From the dynamics it can be verified that the largest invariant set for $\dot{\mathcal{V}} = 0$ is the set with $\bar{q} = \dot{q} = \bar{\tau} = 0$, with $\bar{q} = q - q_d$. LaSalle's invariance principle then proves asymptotic stability. Furthermore, since $\dot{\bar{\tau}} = \dot{\tau} = 0$, $\bar{q}$ has to converge to zero. □

Since K_d and K_i are controller gain matrices we can tune these matrices such that (42) and (43) are satisfied. The existence of a Lyapunov function confirms the validity of the method, even after having lost the BM form for the closed-loop system. Since the idea originated from the power-based modeling framework, we still call the resulting method the power-based method.

4. EXPERIMENTAL RESULTS

The control methods described in the previous sections are implemented on a planar manipulator experimental setup. First we describe the experimental setup. Then we continue with the experimental results.

4.1 Experimental setup

A picture of the experimental setup is shown in figure 1. The figure shows both a picture of the setup and a simplified representation.

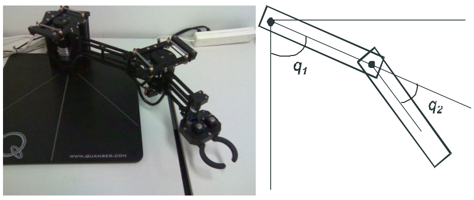

Fig. 1. 2R planar manipulator.

The manipulator has links with length l_i, angles q_i, mass m_i, the distance to the center of the mass is denoted by r_i and the moment of inertia I_i with $i = 1, 2$. The system is modeled in PH form (2) with $D = 0$. Since the system works in the horizontal plane gravity influence can be neglected, i.e., $V(q) = 0$. The mass-inertia matrix is described by

$$M(q) = \begin{bmatrix} a_1 + a_2 + 2b\cos(q_2) & a_2 + b\cos(q_2) \\ a_2 + b\cos(q_2) & a_2 \end{bmatrix} \tag{47}$$

with constants

$$a_1 = m_1 r_1^2 + m_2 l_1^2 + I_1, \qquad b = m_2 l_1 r_2$$
$$a_2 = m_2 r_2^2 + I_2$$

Remark 4. Since friction is dynamic and uncertain, using a friction model in the control design still results in a steady-state error. Since the integral action is meant to compensate for steady-state errors, we choose $D = 0$ and avoid unnecessarily complex models/controllers. ◁

The system is controlled by a Simulink model through a QuaRC interface. QuaRC is a rapid prototyping and production system for real time control from Quanser, which integrates with Simulink to allow Simulink models to be run in real-time. The sample time of control is 0.001 seconds. Digital angular position measurements of both motors are obtained by using high-resolution quadrature encoders. The values for m_i and l_i can be directly found in the reference manual of our experimental system. The values of r_i and I_i are estimated based on the specifications and geometry of the manipulator.

4.2 Experimental results

The PH method (14) with (12) and the power-based method (41) with (40) are applied on the experimental setup and compared with the well-known PID controller, i.e., (4) with (5). The goal is to stabilize the manipulator joint angles to 20 degrees. Table 1 shows the control parameters used in the experiments. The control parameters

Table 1. Control parameters

Parameter	PID	Power-based	PH
K_p	$diag(30, 15)$	$diag(30, 15)$	$diag(1.5, 0.75)$
K_d	$diag(5, 1)$	$diag(5, 1)$	$diag(15, 1)$
K_i	$diag(30, 20)$	$diag(1.2, 0.4)$	$diag(40, 15)$
M_d	-	-	$diag(1, 1)$
A	-	-	$diag(3, 0.75)$

for the power-based method are chosen such that (42) and (43) are satisfied. Figure 2 shows the experimental results.

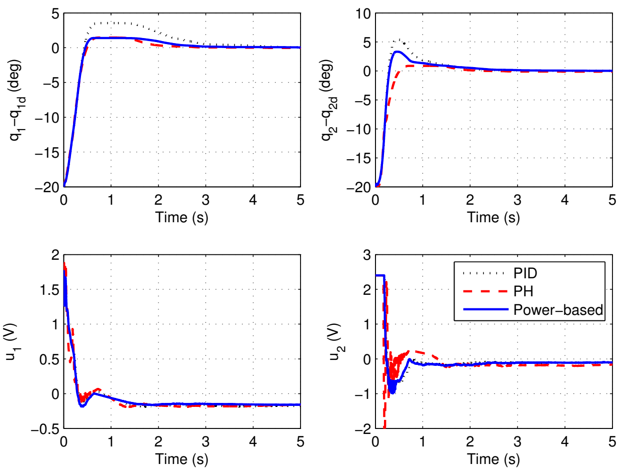

Fig. 2. Error trajectories for $q_d = (20, 20)$ degrees. Initial conditions: $[q(0)\ p(0)] = [0\ 0\ 0\ 0]$.

We try to make a fair comparison by having, approximately, similar input signals in the sense of the L_2 norm for the three methods. In practice tuning the control parameters to obtain such a situation is difficult. Notice that the PH method requires more tuning matrices compared to the other methods. Because of the coordinate transformation it is also not possible to apply the same values for K_p and K_d as with power-based and PID control. Furthermore, the second motor is relatively sensitive to noise, impeding the use of high values for the gains related to this motor. Next, we further analyze these results and compare them with the simulation results.

4.3 Performance analysis

From figure 2, it can be seen that the position errors converge to zero for all three control methods. Figure 2 also shows a faster convergence and smaller overshoots for both the PH and the power-based methods. For the second joint the PH method results in a smaller overshoot compared to the power-based method. The results also clearly show high frequency behavior in the control input for the second joint, for the PH method. This is explained by the fact that the PH control input (14) uses more velocity measurements than the control input of the other methods. The experimental setup only has position measurement encoders and velocities are obtained by using a differentiation filter. The resulting velocity signal can then be relatively noisy. Furthermore, the PH input for the second joint uses more velocity signals than the PH input for the first joint since $M(q)$ only depends on q_2. As mentioned before the second motor is small and relatively sensitive to noise.

Finally, we explain the better performance for the PH method and the power-based method compared to PID control. It can be seen in (12) and (40) that both the PH method and the power-based method include velocities in the integrator dynamics. The integrator states z and τ not only consist of the integral of the position error but also of the integral of velocity, implying some sort of proportional feedback. Proportional feedback influences the rise time and overshoot of the position. This additional proportional feedback can cause the overshoot to be smaller, without directly changing the proportional gains in matrix K_p. Lower K_p gains make the transient response slower, while higher gains are undesirable, especially for the second motor (as explained before).

5. CONCLUSION

Two nonlinear integral type control methods are presented for standard mechanical systems. The structure of the control methods is determined by either the port-hamiltonian or the power-based modeling framework. The different modeling frameworks inspire different controllers. The controllers have the advantage of providing global asymptotic stability with zero steady-state error. The methods are more physically intuitive since they depend on models with a specific physical structure. Experiments show that the methods also offer a better transient performance compared to the classical PID controller. Although popular and relatively simple, there is no good theoretical justification for PID control.

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