Abstract: In this paper, a new distributed receding horizon formation control scheme is introduced using relative dynamical model instead of absolute dynamical model of each member robot in most existing formation control algorithms. Convergence of the proposed distributed receding horizon formation control (DRHFC) is implemented using some additional control input constraints, and the relative dynamical model is used to relieve the performance degradation due to huge computational burden and heavy measurement noises. In order to verify the feasibility and validity of the proposed algorithm, a simulation is conducted and compared to the formation control with absolute dynamical models.

Keywords: distributed receding horizon control, formation control, relative dynamics.

1. INTRODUCTION

Formation control of multiple vehicle systems has been widely researched in the past decades, and several typical formulations have been studied and present their great validity in both theory and reality, such as leader-follower method (Vidal, Shakerinia, and Sastry, 2004), behaviour based method (Balch, and Arkin, 1998), and virtual structure method (Lewis, and Tan, 1997), etc. However, formation control of multiple vehicles is far from maturity in both theories and real applications due to some unsolved but important problems, such as the constraints and optimality.

Most recently, receding horizon control has been paid more and more attentions in the field of multiple robots formation control due to its abilities of handling constraints and optimization. Unfortunately, one of the key disadvantages of receding horizon formation control (RHFC) is the huge computational burden due to the required online optimization algorithm. Distributed RHFC (DRHFC) seems a good method to solve this problem and some corresponding researching works have been published.

Some previous work on distributed receding horizon control address unconstrained coupled LTI subsystem dynamics with quadratic separable cost functions as Camponogara et al. (2002). In another work, Jia and Krogg (2002) solve a min-max problem for each subsystem, where again coupling comes in the dynamics and the neighboring subsystem states are treated as bounded contracting disturbances. Richards and How (2004) examines the multi-vehicle case of linear dynamically decoupled subsystems and coupling constraints, e.g., collision avoidance constraints. Keviczky, Borrelli, and Balas (2004) have formulated a distributed model predictive scheme where each subsystem optimizes locally for itself and every neighbour at each update. In Dunbar and Murray (2006), a particular structure in the centralized cost and by appropriate decomposition in defining the distributed integrated costs, asymptotic stability is proven under stated conditions.

Compared with a centralized version receding horizon solution, DRHFC is desirable for potential scalability and improved tractability. Thus, the convergence of DRHFC has been an important and difficult problem. In Dunbar’s work, an original idea is given to ensure the convergence of the formation algorithm using only local information and local communication.

However, there are two problems in Dunbar’s work: 1) The use of global information in terminal state constraints term make Dunbar’s algorithm requires more information than those referred to; 2) Algorithm in Dunbar (2006) requires all vehicles’ absolute states which are difficult to be obtained in most real applications and will result problems such as heavier computational burden and formation accuracy. Actually, the second problem influences the formation performance in most of DRHFC algorithms.

It should be noted that, for multi-robot formation system, relative position and orientation is more important than absolute position and orientation. Thus, in this paper, we proposed a new kind of DRHFC algorithm by introducing the concept of relative model into Dunbar’s strategy.

We begin in Section 2 by defining the formation problem with relative dynamics and the distributed optimal problem for each vehicle. In Section 3, the distributed receding horizon control algorithm is given, and its convergence results are analyzed in Section 4. Subsequently, a simulation is conducted in section 5, and the results are compared with that of Dunbar and Murray’s scheme (2006) in section 5. Finally, Section 6 concludes this paper and gives some possible future’s work.
2. PROBLEM STATEMENT

Suppose \( N (N \geq 2) \) vehicles are considered to form a formation, and dynamical model of each vehicle can be denoted as following equations,

\[
\dot{x}_i = f(x_i, u_i)
\]

where \( x_i \in \mathbb{R}^n \) \((i=1,2,\ldots,N)\) and \( u_i \in U \subset \mathbb{R}^n \) are the state vector and the control input vector, respectively; \( f(\cdot, \cdot) \) are some nonlinear smooth functions with pre-defined structure.

Since only relative states are necessary for most of vehicles in formation, it will be convenient and beneficial to construct relative models which describe the transition of relative states. In this paper, we denote the relative dynamical model of two vehicles (vehicle \( i \) and \( j \)) as follows,

\[
\dot{x}_{ij} = f_{ij}(x_{ij}, u_{ij})
\]

where \( x_{ij} \in \mathbb{R}^n \) is the relative state vector; \( u_i \) and \( u_j \) are the control input vector of vehicle \( i \) and vehicle \( j \), respectively.

Assume there are two roles in the formation, \( Na(Na \leq N) \) leader vehicles and \( N-Na \) follower vehicles. Leader vehicles denote the systems that track their own desired trajectory and followed by one or more follower vehicles, while the so-called follower vehicles are those which follow other vehicles (For the purpose of simplification, we suppose that each follower only has one leader in this paper). In such a formulation, leader and follower’s dynamics can be respectively described by two different models (1) or (2), and the dynamical model of the formation-multiple-vehicle system can be denoted as

\[
\dot{x} = f(x, u)
\]

where \( x=(x_1, x_2, \ldots, x_N) \) contains all absolute states of Leader vehicles and the relative states of Follower vehicles; \( u = (u_1, \ldots, u_N) \) contains control inputs for all vehicles in the formation. Then, the formation control problem and the distributed formation control problem can be described as,

\textbf{Distributed formation control problem:} Design some controllers

\[
u = k(x) = \begin{cases} 
  k_i(x_i) & \text{if } i\text{th} \text{ robot is a leader} \\
  k_j(x_j) & \text{if } j\text{th} \text{ robot is not a leader}
\end{cases}
\]

to make system (3) converge to the desired state \( x^c \).

3. DISTRIBUTED RECEEDING HORIZON FORMATION CONTROL

In most DRHFC algorithm, formation is achieved by the minimization of some cost function. In this paper, we will utilize cost function with the following form,

\[
L(x, u) = \sum_{i=1}^{N} (1-\gamma) \|x_i - x_i^c\|_{Q_i} + \gamma \|x - x^c\|_{Q} + \|u\|_{R} \]

where

\[
\gamma = \begin{cases} 
  1 & i \in \{1, \ldots, Na\} \\
  0 & \text{otherwise}
\end{cases}
\]

is a positive constant for distinguishing leaders and followers; \( Q, Q_i \in \mathbb{R}^{n \times n} \) and \( R \in \mathbb{R}^{m \times m} \) are all positive definite matrices. Given \( Q=\text{diag}(Q_0, Q_0, \ldots) \) and \( R=\text{diag}(R_0, R_0, \ldots) \), the integrated cost (4) can be equivalently rewritten as

\[
L(x, u) = \|x - x^c\|_{Q} + \|x - x^c\|_{Q} + \|u\|_{R} \]

Thus, the distributed integrated cost in the optimal control problem for any vehicle \( i \in \{1, \ldots, N\} \) can be defined as

\[
L(x, u) = \sum_{i=1}^{N} L_i(x_i, x_{ij}, u_{ij})
\]

where

\[
L_i(x_i, x_{ij}, u_{ij}) = (1-\gamma) \|x_i - x_i^c\|_{Q_i} + \gamma \|x - x^c\|_{Q} + \|u\|_{R} \]

and \( x_i \) means relative state vector between vehicle \( i \) and its leader.

At each time interval, every vehicle conducts optimization only with respect to its own cost function based on its current state \( x_i \) or relative states \( x_j \). The relative states \( x_j \) can be obtained through \( u_j \) that is received from its leader through communication. (Suppose \( j \) is the unique leader of \( i \), that is \( x_j = x_{ij} \) and \( u_{ij} = u_j \)).

Some notation that will be used in the following are defined as follows: \( u(\cdot:t_i) \), \( u^*(\cdot:t_i) \), \( \hat{u}(\cdot:t_i) \) are the predictive, optimal and assumed control vectors for all vehicle (definition of assumed control vectors can be referenced in (Dunbar and Murray, 2006)), respectively; for each vehicle, \( u(\cdot:t_i) \) denotes the predicted control trajectory, \( u^*(\cdot:t_i) \) and \( \hat{u}(\cdot:t_i) \) are the optimal predicted control trajectory and the assumed control trajectory, respectively; \( x(\cdot:t_i) \) is the optimal state trajectory obtained in time instant \( t_i \); \( \hat{x}(\cdot:t_i) \) and \( \hat{x}_i(\cdot:t_i) \) denote different relative state trajectories under control input profile \( u(\cdot:t_i) \), \( \hat{u}(\cdot:t_i) \), \( u^*(\cdot:t_i) \), and \( (u^*(\cdot:t_i), \hat{u}_i(\cdot:t_i)) \).

With cost function (6), the DRHFC problem with relative dynamical model can be denoted as follows,

\textbf{Problem 1:} For each vehicle \( i \in \{1, \ldots, N\} \) and at any update time \( t_i \), given initial conditions \( x(t_i) \) or \( x(t_i) \), and assumed controls \( \hat{u}_i(\cdot:t_i) \), for all \( s \in [t_i, t_i+T]\), find

\[
J_i^*(x_i(t_i), x_j(t_i), T) = \min_{u(t_i)} J_i(x_i(t_i), x_j(t_i), u_s(\cdot:t_i), T),
\]

where

\[
J_i(x_i(t_i), x_j(t_i), u_s(\cdot:t_i), T) = \int_{t_i}^{t_i+T} L_i(x_i(s), x_j(s), \hat{x}_i(s), u(s), T) ds
\]

subject to dynamics constrains.
\[ \dot{x}(t;\tau;\tau) = f_i(x(t;\tau;\tau), u_i(t;\tau;\tau)) \]
\[ \dot{x}_i(t;\tau;\tau) = f_i(x_i(t;\tau;\tau), u_i(t;\tau;\tau), u_{-i}(t;\tau;\tau)) \]

input constrains
\[ u_i(t;\tau;\tau) \in U \]
compatibility constrains
\[ \|u_i(t;\tau;\tau) - \hat{u}_i(t;\tau;\tau)\| \leq \delta^2 \kappa \]
and terminal constrains
\[ \dot{x}_i(t_i + T;\tau;\tau) \in \Omega_i(e_i) \]
\[ \dot{x}_i(t_i + T;\tau;\tau) \in \Omega_i(e_i) \]

Terminal cost function is defined as
\[ T(\dot{x}_i(t_i + T;\tau;\tau), \dot{x}_i(t_i + T;\tau;\tau)) = \gamma \|\dot{x}_i(t_i + T;\tau;\tau) - x_i\|^2 + (1 - \gamma) \|\dot{x}_i(t_i + T;\tau;\tau) - x_i\|^2 \]
where \( \kappa, \epsilon_i \in (0, \infty) \), weighting matrix \( P_i \) and \( P_d \) are all pre-designed parameters; \( \Omega_i(e_i) = \{x \|x - x_i\|_2 \leq \epsilon_i\} \) is the terminal state constraints.

The applied control for \( i \) is constrained to be at most a distance of \( \delta^2 \kappa \) from the assumed control in (10), this is very important to ensure the convergence of the whole algorithm (Dunbar and Murray, 2006).

Then we state the control algorithm following the succinct presentation in Michalska and Mayne (1993).

Algorithm 1. At time \( t_0 \) with \( x(t_0) \in X \), the distributed receding horizon controller for any vehicle \( i \in \{1,…,N\} \) is as follows:

Data: \( x(t_0) \) or \( x_i(t_0) \), \( T \in (0, \infty), \delta \in (0, T] \).

Initialization: At time \( t_0 \), solve Problem 1 for vehicle \( i \), setting \( \hat{u}_i(s; t_0) = 0 \) and \( \dot{u}_i(s; t_0) = 0 \) for all \( s \in [t_0, t_0+T] \) and removing constraint (10).

Controller:
1. Over any interval \( [t_k, t_{k+1}] \):
   a. Apply \( u^*_i(s; t_k) \), \( s \in [t_k, t_{k+1}+T] \).
   b. Compute \( \hat{u}_i(s; t_{k+1}) = \hat{u}_i(s; t_k) \) as \( \hat{u}_i(s; t_{k+1}) = \begin{cases} u^*_i(t;\tau;\tau) & s \in [t_k, t_{k+1}+T] \\ 0 & s \in [t_k + T, t_{k+1} + T] \end{cases} \)
   c. Transmit \( \hat{u}_i(s; t_{k+1}) \) to neighbors and receive \( u_{-i}(s; t_{k+1}) \) from neighbor.
2. At any time \( t_k \):
   a. Measure current state \( x_i(t_k) \) or \( x_i(t_k) \).
   b. Solve Problem 1 for vehicle \( i \), yielding \( u^*_i(s; t_k) \), \( s \in [t_k, t_{k+1}+T] \)

With the optimal control solution to each distributed optimal control problem \( u^*_i(t) \), the closed-loop formation system can be denoted as
\[ \dot{x}(t) = f(x(t), u^*(t)) \] with the applied distributed receding horizon control law
\[ u^*(s; t) = (u^*_1(s; t), \ldots, u^*_N(s; t)) \]

For \( s \in [t_k, t_{k+1}] \) and the receding horizon control law is updated when each new initial state update \( x(t_k) \) is available. Analysis about stability of close-loop system (12) will then given in the next section.

4. STABILITY ANALYSIS

Before stating the control algorithm formally a terminal controller associated with each terminal cost and constraint set (11) is defined to be calculated off-line.

We consider the Jacobian linearization of the system (1) at the origin
\[ \dot{x}_i = A_i x_i + B_i u_i \]
and the Jacobian linearization of the system (2) at the origin with no disturbance \( u = 0 \)
\[ \dot{x}_y = A_y x_y + B_y u_y \]

If equation (13) and (14) is stabilizable, then a linear state feedback \( u = K_i(x-x_i) \) and \( u = K(x-x_0) \) can be determined such that \( A_i + B_i K_i \) is asymptotically stable. To that end, we make an assumption. Firstly, for every system \( i \in \{1,…,N\} \), let \( x^K(\cdot; t_k) \) denote the closed-loop solution to
\[ \begin{align*}
\dot{x}^K_i(\cdot; t_k) &= f_i(x^K_i(\cdot; t_k), K_i(x^K_i(\cdot; t_k) - x_i)), \\
\dot{x}^K_y(\cdot; t_k) &= f_i(x^K_y(\cdot; t_k), K_i(x^K_y(\cdot; t_k) - x_y), 0)
\end{align*} \]

Assumption 1. The following holds for every \( i \in \{1,…,N\} \). The positive constant \( \epsilon_i \) is chosen such that \( \Omega_i(\epsilon_i) \subseteq X \) and such that for all \( x_i, x_j \in \Omega_i(\epsilon_i) \), there is an asymptotically stabilizing feedback \( u_i = K_i(x_i-x_i) \) and \( u_j = K(x_j-x_0) \) that is feasible for (1) and (2). The weighting matrix \( P_i \) and \( P_d \) satisfies
\[ \begin{align*}
\left( A_i + K_i B_i \right)^T P_i + P_i \left( A_i + K_i B_i \right) &\leq -Q_i + K_i^T R K_i \\
\left( A_i + K_i B_i \right)^T P_d + P_d \left( A_i + K_i B_i \right) &\leq -Q_d + K_i^T R K_i
\end{align*} \]

Following the logic presented in Section 2 of Michalska and Mayne (1993), it is straightforward to show that such a positive \( \epsilon_i > 0 \) exist, and an immediate consequence is that \( \Omega_i(\epsilon_i) \) is a positively invariant region of attraction for system (1) and (2). By construction \( P = \text{diag}(…, P_i, P_d, …) \), from Assumption 1, we can obtain
\[ -\frac{d}{dt} \|x - x^\ast\|_2 \geq \|x - x^\ast\|_2 - \|u\|_2 \]

Next, we will shown that the sum of the distributed optimal value functions is a Lyapunov function that does decrease at each update, enabling a proof that the distributed receding horizon control laws collectively meet the control objective.

At any time \( t_k \), the sum of the optimal distributed value functions is denoted as
\[ J'(x(t_i), T) = \sum_{i=1}^{N} J_i'(x_i(t_i), x_{-i}(t_i), T) \]

We begin by demonstrating that initial feasibility of the implementation implies subsequent feasibility, following the standard arguments in Michalska and Mayne (1993), and Chen and Allgöwer (1998). If \( x(t_0) \in X^N \), then there exists at least one (not necessarily optimal) input \( u^0 : [t_0, t_0 + T] \rightarrow U^N \) such that the terminal constraints in Problem 1 are satisfied.

**Lemma 1.** Suppose Assumption 1 and 2 hold and \( x(t_0) \in X^N \). Then, for any update period \( \delta \in (0, T] \), Problem 1 has a feasible solution at any update time \( t_i \).

**Proof.** By assumption, Problem 1 has a feasible solution at time \( t_0 \), and feasibility for all subsequent update times is proven by induction. For leader vehicle \( i \in \{1, \ldots, N\} \), with candidate input

\[ u_i(s) = \begin{cases} u_i^*(s; t_i), & s \in [t_{x_i}, t_i + T) \\ K_i(x_i^*(t_i + T; t_i) - x_i^*), & s \in [t_i + T, t_{x_i} + T] \end{cases} \]

(18)

can steer \( x(t_{x_i}) = x(t_{x_i} + t_i) \) to terminal region \( \Omega(\varepsilon_i) \) by quasi-infinite horizon nonlinear model predictive scheme (Chen, 1998). For follower vehicle \( i \in \{N + 1, \ldots, N\} \), a candidate input be chosen with

\[ u_i(s) = \begin{cases} u_i^*(s; t_i), & s \in [t_{x_i}, t_i + T) \\ K_i(x_i^*(t_i + T; t_i) - x_i^*), & s \in [t_i + T, t_{x_i} + T] \end{cases} \]

(19)

where \( x_i^*(t_i) \) and \( x_i^*(t_i + T) \) is defined in (15). For \( s \in [t_{x_i}, t_i + T] \) input \( u_i(s) \) and assume \( x_i(t_i + T) \) can steer \( x_i(t_i + T) = x_i^*(t_i + T; t_i) \) to terminal region \( \Omega(\varepsilon_i) \). From Assumption 1, the terminal region \( \Omega(\varepsilon_i) \) is invariant for the nonlinear system model controlled with the linear state feedback gain \( K_i \). 

As a consequence, if \( x(t_0) \in X^N \), then Algorithm 1 can be initialized and applied for all time \( \varepsilon_2t_0 \). In the analysis that follows, we require that the optimal and assumed state trajectories remain bounded.

**Assumption 2.** (a) There exists a constant \( \rho_{\text{max}} \in (0, \infty) \) such that

\[ \| x_i(s; t_i) - x_i' \| \leq \rho_{\text{max}} \text{ and } \| x_{-i}(s; t_i) - x_{-i}' \| \leq \rho_{\text{max}} \text{ for all } s \in [t_{x_i}, t_i + T] \]

(b) There exists a constant \( \ell \in (0, \infty) \) , with two pair of relative sate and input, \( (x_{ij}, u_{ij}) \) and \( (x_{ij}, u_{ij}) \) subject to (2), such that

\[ \| x_{ij} - x_{ij} \| \leq \ell \| u_i - u_i \| \text{ at invariant } u_i \]

The following lemma gives a bounding result on the decrease in \( J'(\cdot, T) \) from one update to the next.

**Lemma 2.** Under Assumption 1-2, for a given fixed horizon time \( T>0 \), and for the positive constant \( \xi \) defined by

\[ \xi = 2TN \max(\lambda_{\text{max}}(Q)) \rho_{\text{max}} \ell \kappa \]

The function \( J'(\cdot, T) \) satisfies

\[ J'(x(t_{x_i}), T) - J'(x(t_i)) \leq - \int_{t_i}^{t_{x_i}} \sum_{j=1}^{N} L_j(x_j^*(s; t_i), x_{-j}(s; t_i), u_j^*(s; t_i))ds + \xi \delta \]

for any \( \delta \in (0, T] \) and \( x(t_i) \in X^N \). \( \lambda_{\text{max}}(P) \) denote the largest eigenvalues of \( P \).

**Proof.** The sum of the optimal distributed value functions for a given \( x(t_i) \in X^N \) is

\[ J'(x(t_i)) = \int_{t_i}^{t_{x_i}} \sum_{j=1}^{N} L_j(x_j^*(s; t_i), x_{-j}(s; t_i), u_j^*(s; t_i))ds + \xi \delta \]

A feasible control defined as (18) and (19) at update time \( t_i+1 = t_i + \delta \), with new state update \( x(t_{x_i} + t_i) \), solve Problem 1 yielding \( J'(x(t_{x_i} + t_i)) \geq J'(x(t_{x_i})) \). By the properties stated in (17), the sum of the last three terms in the equality above is non-positive and therefore the inequality holds

\[ J'(x(t_{x_i} + t_i)) - J'(x(t_i)) \leq - \int_{t_i}^{t_{x_i}} \sum_{j=1}^{N} L_j(x_j^*(s; t_i), x_{-j}(s; t_i), u_j^*(s; t_i))ds + \xi \delta \]

(20)

From Assumption 2(b), we have that

\[ \| x_j^*(s; t_i) - x_j' \|_{t_i} - \| x_j^*(s; t_i) - x_j' \|_{t_i} \leq 2\rho_{\text{max}}(\lambda_{\text{max}}(Q)) \ell \delta \kappa \]

Defining \( \xi = 2TN \max(\lambda_{\text{max}}(Q)) \rho_{\text{max}} \ell \kappa \) and joining (21), (20) becomes

\[ J'(x(t_{x_i} + t_i)) - J'(x(t_i)) \leq - \int_{t_i}^{t_{x_i}} \sum_{j=1}^{N} L_j(x_j^*(s; t_i), x_{-j}(s; t_i), u_j^*(s; t_i))ds + \xi \delta \]

(22)

This completes the proof.

Finally, we want to show that \( J'(\cdot, T) \) decreases from one update to the next along the actual closed-loop trajectories. By making the following assumption

**Assumption 3.** (a) The interval of integration \( [t_i, t_i + \delta] \) for the expressions in equation (22) is sufficiently small that first-order Taylor series approximations of the integrands is a valid approximation for any \( x(t_i) \in X^N \). (b) For every \( i \in \{1, \ldots, N\} \), there exists a Lipschitz constant \( h \in (0, \infty) \) , such that

\[ |f(x, u) - f(x', u')| \leq h \| x - x' \| + \| u - u' \| \]

for any \( x, x' \in X^N \) and \( u, u' \in U^N \)

We have the next lemmas showing that, for sufficiently small \( \delta \), the bounding expression above can be bounded by a negative-definite function of the close-loop trajectories.

**Lemma 3.** Under Assumption 1-2, for any \( x(t_i) \in X^N \), such that at least one vehicle \( i \) satisfies \( x(t_i) \neq x_i' \), or \( x_i(t_i) \neq x_i' \), and for any positive constant \( \xi \) there exists

\[ \delta(x(t_i)) = \frac{\| x(t_i) - x_i' \|^2}{\xi + \lambda(Q) \rho_{\text{max}}(\rho_{\text{max}} + u_{\text{max}})} \]

such that
Theorem 1. Under Assumption 1-3, for a given fixed horizon time $T\geq 0$ and for any state $x(t_0) \in X^*$ at initialization, if the update time satisfies $\delta \in (0, \delta(x(t_0))]$, where $\delta(x(t_0))$ is defined in (26), then $x^*$ is an asymptotically stable equilibrium point of the closed-loop system (12) with region of attraction $X^*$, an open and connected set.

Proof. Combing Lemma 2 and Lemma 3, we have a recursively inequality that

$$J'(x(\tau)) - J'(x(t_0)) \leq -\int_{t_0}^{\tau} \|x(t) - x^*\|^2 ds$$

for any $\tau \in [t_0, t_0 + \delta)$. Then following precisely the steps in the proof of Theorem 1 in Chen and AllgöWer (1998), we observe that $\|x-x^*\| \rightarrow 0$ as $t \rightarrow \infty$. As a consequence, the compatibility constraint gets tighter, and the communication between neighbors must happen with increasing bandwidth, as the agents collectively approach the control objective. Still, the conditions above for convergence are only sufficient. \qed

5. SIMULATION RESULTS

Simulation of formation for three vehicles with the following double integrator is presented in this paper to verify the proposed method

$$\ddot{q} = u$$

Assume vehicle-1 is a leader robot; vehicle-2 and vehicle-3 are followers of vehicle-1 and Vehicle-2, respectively. So vehicle-2 and vehicle-3 is denoted as following relative dynamical model

$$\ddot{q}_i = u_i - u_j$$

where $q_i$ denotes the relative position. The formation structure is shown as a classical leader-follower scheme. So we take relative position $q_{21}$, $q_{31}$ and $q_{32}$ to analyses the precision of the formation.

We will conduct two simulations and compare the results to that in Dunbar and Murray’s work (2006). For simplification, denote Case A as the results of Dunbar’s scheme and Case B as the results of the new proposed algorithm. Control parameters in both simulations are equal with sample time $\delta=0.2s$, horizon time $T=2s$, weighting matrix $Q=I_{2x2}$, $Q_0=I_{2x2}$, $R_1=1$, $P_{2}=I_{2x2}$, $P_{3}=I_{2x2}$.

Simulation 1. (Changing formation) Three vehicles are initially at positions: $q_{20}=10.0m$, $q_{30}=2.0m$ and $q_{30}=6.0m$. In 0–3 seconds, every two vehicle keep 4.0m distance each other and the whole formation move at a constant velocity 1.0m/s. Therefore, the formation is kept at $q_{21}=8.0m$, $q_{31}=4.0m$ and $q_{32}=4.0m$ during 0–3s periods. At 3 seconds, vehicles are forced to change the formation with 2.0m distance each other and move at a constant velocity 3.0m/s. That is, the formation is kept at $q_{21}=4.0m$, $q_{31}=2.0m$ and $q_{32}=-2.0m$ after 3 seconds.

Simulation result for 10 seconds is proposed in Fig.1. The red, blue and green line denotes positions of Vehicle 1, 2 and 3 respectively. The solid line and the dotted line respectively mean positions by Case B and Case A. The transient response of the formation, reported by $q_{21}$, $q_{31}$ and $q_{32}$, is presented in Fig.2. From the simulation results, we can conclude that method in this text (Case B) have almost the same response.
capability as method in Dunbar’s (Case A) in changing formation with suitable receding time $T$.

While solving the optimal problem, e.g. Problem 1 in section III, Case A has to deal with two global individual states ($x_i$ and $x_j$) and two dynamic models (1). On the contrary, Case B solves only one relative state $x_{ij}$ and a single dynamics model (2). Thus, the computation burden will be largely reduced in every receding horizon time $t_k$ where cost time is compared in Fig.3. As in Fig.3, Case B saves much time than Case A does, and the summation time is also reduced that Case A is 32.87s contrast to 11.14s of Case B. In addition, results are simulated in conditions that: CPU for AMD Athlon 4000+ 2.10GHz, RAM for 1.00GB, Os for Windows XP and Software for Matlab Optimization Toolbox.

**CONCLUSIONS**

In this paper, a new decentralized receding horizon formation control on relative dynamic model was proposed. The new designed algorithm has the following there advantages: 1) some vehicles do not need global information while using only locally sensed information. 2) asymptotic stability is proven in the need of relative dynamic model. 3) Computation burden and noise is reduced by substituting one relative state for two global individual states.

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