Robust Control of a Two-Axis Piezoelectric Nano-Positioning Stage

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Abstract: This paper applies robust control to a two-axis piezoelectric nano-positioning stage. As technology develops, the precision requirement for positioning platforms is becoming increasingly stringent. Since a traditional mechanical transmission structure cannot achieve high precision, a piezoelectric actuator is usually applied to drive the mechanism because of its high resolution, high accuracy, and large driving force. However, the non-linear dynamic characteristics of piezoelectric materials, such as hysteresis, might degrade system performance. Therefore, in the present study, we model a piezoelectric stage as a linear system, and regard the nonlinear factors as system uncertainties. We then apply robust control strategies to guarantee system stability and performance. Lastly, the designed controllers are implemented for experimental verification. The results demonstrate the effectiveness of these robust controllers.

Keywords: piezoelectric, PZT, identification, robust control, loop-shaping

1. INTRODUCTION

As technology progresses, precise positioning requirements are becoming increasingly stringent. Frequently, traditional mechanical transmission structures might not meet these requirements due to friction and backlash. Therefore, piezoelectric materials are normally applied to drive position platforms because of their advantages such as high resolution, high accuracy, and large driving force. In piezoelectric materials, both direct and inverse piezoelectric effects are possible. The former occurs when the material produces an electric field in response to an applied force. The latter happens when the material undergoes a deformation in response to an applied electrical field. Using the inverse effect, piezoelectric actuators can drive positioning stages with a high degree of precision. However, non-linear characteristics of piezoelectric materials, such as hysteresis, might degrade the system performance. In order to reduce the non-linear effects, the Maxwell Slip model (Choi et al., 1997, 1999, 2002) and the Bouc-Wen model (Wen, 1976, Kuo and Low, 1995) have been proposed. However, these high-order models are difficult to construct and are not suitable for control design. Therefore, in this paper, we identify a piezoelectric stage as a linear model and regard the non-linear effects as system uncertainties and disturbances. We then apply robust control strategies to design suitable robust controllers to achieve system stability and performance.

This paper is arranged as follows: Section 2 introduces the PZT stage and obtains its transfer functions by identification techniques. Section 3 introduces robust control algorithms and designs robust controllers for the PZT stage. In Section 4, the designed robust controllers are implemented for experimental verification. Lastly, we draw conclusions in Section 5.

2. SYSTEM DESCRIPTION AND IDENTIFICATION

Fig. 1 shows a two-axis PZT stage that consists of three layers: the top layer is a loading platform, the middle moves along the Y-axis, and the bottom moves along the X-axis. We installed ceramic piezoelectric actuators in the grooves of the stage, and stage movement is controlled by regulating the input voltages of the actuators. To measure stage movement, we used Mercury 5800 (MicroE Systems, 2010), which has a resolution of 1.22nm and digital outputs. The system structure is shown in Fig. 2, where Labview and PXI-8108 (National Instruments, 2010) were used for data acquisition. Because the voltage output of PXI-8180 is limited to ±10V, a voltage amplifier, E-663, was also applied to drive the actuators.
At first, we tested the open-loop characteristics of the piezoelectric actuators. Given an input voltage, we measured the output displacement and drew the phase plot in Fig. 3, where the hysteretic effects of the actuators can be observed.

Repeated the experiments six times at each axis and obtained the following models of the PZT stage:

\[
G_x^{\text{i}} = \frac{-2.389 \times 10^4 s^4 + 1.278 \times 10^4 s^3 + 0.03251 s^2 + 73.78 s + 2472}{s^5 + 2433 s^4 + 3.698 \times 10^3 s^3 + 2.35 \times 10^2 s^2 + 6.383 \times 10^0 s + 1.8}, \tag{1}
\]

\[
G_x^{\text{r}} = \frac{-1.413 \times 10^4 s^4 + 5.818 \times 10^3 s^3 + 0.03211 s^2 + 25.48 s + 134.8}{s^5 + 1487 s^4 + 2.053 \times 10^2 s^3 + 7.64 \times 10^1 s^2 + 2.257 \times 10^0 s + 1}, \tag{2}
\]

\[
G_x^{\text{h}} = \frac{-1.399 \times 10^4 s^4 + 5.959 \times 10^3 s^3 + 0.0323 s^2 + 24.04 s + 128.1}{s^5 + 1478 s^4 + 2 \times 10^2 s^3 + 7.205 \times 10^1 s^2 + 2.242 \times 10^0 s + 180}, \tag{3}
\]

\[
G_y^{\text{i}} = \frac{-1.582 \times 10^4 s^4 + 7.753 \times 10^3 s^3 + 0.03438 s^2 + 28.48 s + 264.6}{s^5 + 1650 s^4 + 2.184 \times 10^2 s^3 + 8.627 \times 10^1 s^2 + 5.716 \times 10^0 s + 1}, \tag{4}
\]

\[
G_y^{\text{r}} = \frac{-1.438 \times 10^4 s^4 + 6.343 \times 10^3 s^3 + 0.03259 s^2 + 24.4 s + 104.6}{s^5 + 1517 s^4 + 2.027 \times 10^2 s^3 + 7.305 \times 10^1 s^2 + 1.319 \times 10^0 s + 1}, \tag{5}
\]

\[
G_y^{\text{h}} = \frac{-1.491 \times 10^4 s^4 + 6.831 \times 10^3 s^3 + 0.03312 s^2 + 25.66 s + 165.9}{s^5 + 1745 s^4 + 2.082 \times 10^2 s^3 + 7.721 \times 10^1 s^2 + 3.176 \times 10^0 s + 1}, \tag{6}
\]

Note that the orders of the transfer functions were automatically assigned by MATLAB for best fittings.

In order to achieve precise positioning, we consider a closed-loop control structure for the stage. First, we generated chirp input voltage signals to drive the actuators and then measured the output displacement signals, as shown in Fig. 4. Using subspace system identification methods (Moor and Overschee, 1991, 1994), we can obtain the system’s transfer functions by a MATLAB command n4sid. Because the cross-coupling terms are relatively small and can be neglected, the MIMO system can be simplified as two SISO sub-systems. We
3. ROBUST CONTROLLER DESIGN

In this section, we introduce robust control algorithms and design robust controllers for the PZT system. The following Small Gain Theorem is normally considered for systems with uncertainties (Zhou et al., 1996):

Suppose \( Z \in RH_\infty \) and let \( \gamma > 0 \), the system of Fig. 5 is well posed and internally stable for all \( \Delta(s) \in RH_\infty \) with: (a) \( \| \Delta \|_\infty \leq 1/\gamma \) if and only if \( \| Z \|_\infty < \gamma \); (b) \( \| \Delta \|_\infty < 1/\gamma \) if and only if \( \| Z \|_\infty \leq \gamma \), where \( \| Z \|_\infty \) is the \( H_\infty \) norm of system Z.

![Fig. 5 Small Gain Theorem](image)

Suppose a nominal plant \( G_0 \) has a normalized left coprime factorization of \( G_0(s) = \bar{M}(s)^{-1}\bar{N}(s) \), where \( \bar{M}, \bar{N} \in RH_\infty \) and \( \bar{M}\bar{M}^* + \bar{N}\bar{N}^* = I \). Assume a perturbed system \( G_\delta \) can be expressed as:

\[
G_\delta = (\bar{M} + \Delta_\delta)^{-1}(\bar{N} + \Delta_\delta),
\]

with \( \| \Delta_\delta \|_\infty < \varepsilon \), and \( \Delta_\delta, \Delta_\delta \in RH_\infty \). Consider a closed-loop system with the controller \( K \) of Fig. 6(a), which can be rearranged as in Fig. 6(b) when \( r=0 \). Using Small-Gain Theorem, the closed-loop system is internally stable for all perturbations with \( \| \Delta_\delta \|_\infty < \varepsilon \), if and only if:

\[
\begin{bmatrix} K & I \end{bmatrix} (I - G_0K)^{-1}M^{-1} \leq \varepsilon. \tag{14}
\]

Thus, we can define the stability margin (Doyle and Zhou, 1998) as:

\[
b(G_0, K) = \| \begin{bmatrix} K & I \end{bmatrix} (I - G_0K)^{-1} \|_\infty, \tag{15}
\]

such that the closed-loop system is internally stable for all perturbations \( \Delta = [\Delta_\delta \Delta_\delta] \) with \( \| \Delta \|_\infty < \varepsilon \) if and only if \( b(G, K) \geq \varepsilon \).

From equations (1–12), the transfer functions of the X-axis or Y-axis varied at each experiment. Hence, we apply the ideas of gap metric (Georgiou and Smith, 1990) to select the nominal plant for controller design. Because the coprime factorization of a transfer function is not unique, the gap between \( G_0 \) and \( G_\delta \) is defined as:

The minimum perturbation of \( \| \Delta_\delta \|_\infty \) which changes \( G_0 \) to \( G_\delta \), denoted as \( \delta(G_0, G_\delta) \).

That is, the nominal plant \( G_0 \) is chosen by:

\[
G_0 = \arg \min_{G_0} \max_{\delta(G_0, G_\delta)} \delta(G_0, G_\delta), \quad \forall G_\delta. \tag{16}
\]

Thus, we selected the nominal plants of the X-axis and Y-axis as follows:

\[
G_x = \begin{cases} \frac{-1.491 \times 10^3 s^4 + 6.831 \times 10^4 s^3 + 0.03312 s^2 + 25.66 s + 165.9}{s^4 + 1570.7 s^3 + 2.082 \times 10^7 s^2 + 7.721 \times 10^9 s + 3.176 \times 10^{11}}, & \text{for X-axis} \\
\frac{-1.316 \times 10^3 s^3 + 8.008 \times 10^7 s^2 + 0.05911 s + 1.892}{s^3 + 1.749 s^2 + 1.654 \times 10^7 s + 4.572 \times 10^9}, & \text{for Y-axis} \end{cases}
\]

These give the system gaps as \( \delta(G_x, G_x) < 0.016256 \) and \( \delta(G_y, G_y) < 0.016411 \).

In order to improve system performance, loop-shaping techniques (Glover and McFarlane, 1989, 1992) are usually applied. As shown in Fig. 7, we set weighting functions to...
shape the plant as \(G(s) = W_2(s)G(s)W_1(s)\), and then design a standard \(H_{\infty}\) robust controller \(K_{\infty}\) for the shaped plant. Finally, we implement the controller \(K = W_1K_{\infty}W_2\) for the original plant \(G(s)\).

We use root locus techniques to estimate the settling time of the system’s step responses. For example, the root loci of \(G_x\) is shown in Fig. 8(a). Therefore, we set the following weighting function

\[
W_x = \frac{3 \times 10^7(s + 600)}{s(s + 250)},
\]

with two extra poles at \(s = 0\) and \(s = -250\), and an extra zero at \(s = -600\), to adjust the root loci as Fig. 8(b). Hence, the estimated settling time is 0.001sec with a gain of \(3 \times 10^7\). Similarly, we choose the weighting function in the y-axis as

\[
W_y = \frac{10^5(s + 400)}{s(s + 300)},
\]

with two extra poles at \(s = 0\) and \(s = -300\), and an extra zero at \(s = -400\). Thus, the estimated settling time is 0.01s with a gain of \(1 \times 10^5\). The corresponding controllers are designed as :

\[
K_{\infty,x} = \frac{1.435 s^4 + 2650 s^3 + 3.609 \times 10^6 s^2 + 1.936 \times 10^8 s + 3.141 \times 10^{10}}{s^4 + 2263 s^3 + 3.174 \times 10^6 s^2 + 1.982 \times 10^8 s + 4.376 \times 10^{10}},
\]

\[
K_{\infty,y} = \frac{1.491 s^4 + 3100 s^3 + 3.296 \times 10^6 s^2 + 8.706 \times 10^7 s + 2.738 \times 10^{10}}{s^4 + 3041 s^3 + 3.928 \times 10^6 s^2 + 1.252 \times 10^8 s + 8.084 \times 10^{10}}.
\]

with the following stability margins:

\[
b_{\text{max}}(G_{e,x}, K_{\infty,x}) = 5717, \ b_{\text{max}}(G_{e,y}, K_{\infty,y}) = 0.5569.
\]

These stability margins are much greater than the system perturbations, such that robust stability can be guaranteed. These controllers will be implemented for experimental verification in section 4.

4. SIMULATIONS AND EXPERIMENTS

In this section, we implement the designed robust controllers for simulation and experimental verification. Given step inputs, the simulation results are obtained using the Simulink™ structure of Fig. 9, and the experimental results are obtained using the hardware settings of Fig. 2.

The simulation and experimental results are compared in Fig. 10, and illustrated in Table 1. First, the experimental results match with the simulations. That is, the identified transfer functions have caught basic dynamics of the PZT system. Second, the experimental responses have oscillations of about 300Hz, which is resulted from the system’s noises. In the
future, we will adjust the weighting functions to suppress these oscillations. Third, the experimental responses tend to have a shorter settling time, which might be caused by extra damping effects in the physical systems.

Table 2 Statistic data of the ramp responses.

<table>
<thead>
<tr>
<th>Axis</th>
<th>error</th>
<th>100</th>
<th>200</th>
<th>500</th>
<th>1000</th>
<th>2000</th>
<th>5000</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
<td>RMSE</td>
<td>0.0072</td>
<td>0.0101</td>
<td>0.0175</td>
<td>0.0308</td>
<td>0.0586</td>
<td>0.1193</td>
</tr>
<tr>
<td></td>
<td>MAE</td>
<td>6.21</td>
<td>5.595</td>
<td>6.645</td>
<td>9.03</td>
<td>11.79</td>
<td>22.71</td>
</tr>
<tr>
<td>Y</td>
<td>RMSE</td>
<td>0.0066</td>
<td>0.0092</td>
<td>0.0157</td>
<td>0.0267</td>
<td>0.0499</td>
<td>0.1017</td>
</tr>
<tr>
<td></td>
<td>MAE</td>
<td>5.335</td>
<td>5.8</td>
<td>6.625</td>
<td>7.025</td>
<td>11.08</td>
<td>21.49</td>
</tr>
</tbody>
</table>

In addition, to test the effect of loads on the platform during operation, we place 340g and 1000g masses on the stage and again test the system performance. The results are shown in Fig. 11, which indicates that the performance seems to be irrelevant to the carrying loads.

Finally, we test the system’s ability to track three trajectories: a square, a circle, and a star, as shown in Fig. 12 (a), (c), and (e). The corresponding time trajectories of the x-axis and y-axis are shown in Fig. 12 (b), (d), and (f), which test the ability of the stage to track straight lines, sinusoidal signals, and simultaneous ramps, respectively. This analysis is illustrated in Table 3, which shows that the errors are smallest for the square, and largest for the circle. It is because the average moving speeds are 100nm/s for the square, 1257nm/s for the circle, and 200nm/s for the star. That is, the RMSEs and MAEs become bigger when the moving faster.

Table 3 Tracking errors.

<table>
<thead>
<tr>
<th></th>
<th>X-axis</th>
<th>Y-axis</th>
<th>X-axis</th>
<th>Y-axis</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMSE</td>
<td>RMSE</td>
<td>MAE</td>
<td>MAE</td>
</tr>
<tr>
<td>(nm)</td>
<td>(nm)</td>
<td>(nm)</td>
<td>(nm)</td>
<td></td>
</tr>
<tr>
<td>square</td>
<td>0.0111</td>
<td>0.01</td>
<td>3.545</td>
<td>3.355</td>
</tr>
<tr>
<td>circle</td>
<td>0.0315</td>
<td>0.0265</td>
<td>7.056</td>
<td>6.039</td>
</tr>
<tr>
<td>star</td>
<td>0.0118</td>
<td>0.0093</td>
<td>4.71</td>
<td>4.364</td>
</tr>
</tbody>
</table>

The responses to ramp inputs are also tested, and the root mean square error (RMSE) and the maximum absolute error (MAE) are illustrated in Table 2. The system errors increase with more rapid movement and the y-axis tends to have smaller error than the x-axis.
5. CONCLUDING REMARKS

This paper has demonstrated $H_{\infty}$ robust control for a piezoelectric nano-positioning stage. We first identified the stage as linear transfer functions, and regarded the system nonlinearities as model perturbations and disturbances. Then we designed robust controllers for the PZT system. The simulation and experimental results showed the effectiveness of the designed robust controllers in improving stability and performance of the piezoelectric stage. For future work, we can modify the weighting functions to suppress the system’s noises, as illustrated in (Wang et al., 2011). Furthermore, reduced-order robust control and robust PID control techniques can also be applied to simplify the controllers for system miniaturization, as shown in (Wang et al., 2009, 2010).

REFERENCES


