Improvement in Intersample Response of
HDD Head

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Abstract: This paper proposes a new design method for controlling a head in a hard disk drive (HDD) system. In the HDD control system, the sampling interval of the head position is restricted, but the hold interval of the control input can be arbitrarily set. Hence, this study discusses a design method for a multi-rate control system, where the sampling and the hold intervals are not equal. In this study, a pole-assignment multi-rate control law is extended using newly introduced parameters such that the sample response of the plant output is maintained. Furthermore, intersample ripples in the steady state are eliminated using the new design parameters because those can be selected independently of the sample response. As a result, the intersample response can be improved independently of the sample response. The proposed method is applied to a benchmark problem of a HDD system, and its effectiveness is demonstrated.

Keywords: Multi-Rate control system, HDD head, state-space representation, intersample response, sample response, pole assignment

1. INTRODUCTION

The sampling frequency of hard disk drive (HDD) servo systems (Chen et al., 2002; Committee, 2006; Mamun et al., 2007) is limited because of the rotational speed of the disks and the number of servo sectors used. Hence, the head position can not be obtained at an arbitrary rate, but the control input can be arbitrarily updated. Fig. 1 shows a single-rate control system, where \(T_s\) and \(T_h\) denote the sampling interval and the update interval, respectively. Both the intervals are equal in this figure. To design this conventional single-rate control system, the update interval of the control input must be equal to the sampling interval of the head position, and the ability of the actuator is not utilized effectively. However, a multi-rate control system shown in Fig. 2 has higher potential than the single-rate control system since the control input can be updated between the sampling interval of the plant output (\(T_h \neq T_s\)). Hence, numerous multi-rate control methods for controlling a HDD system have been proposed (Hirata et al., 2000, 2003; Lepez et al., 2010).

In the multi-rate control system, the intersample response might oscillate even if the sample response converges to its reference input (Tangirala et al., 1999; Sato, 2008). If a control system is designed using the integral compensation, this problem can be resolved (Scattolini, 1992). In this case, the transient response is also changed, and it may be deteriorated because of the integral action. Therefore, the intersample ripples should be eliminated without changing the transient response. Sato (2008) has proposed a design method for the multi-rate control system such that a control law can be designed independently of the discrete-time closed-loop system. Using this design method, the intersample ripples can be eliminated independently of the transient response.

In this study, the design method (Sato, 2008; Yamaguchi et al., 2007) is extended into the state-space representation, and it is applied to a benchmark problem (Committee, 2006). In the conventional method (Sato et al., 2010), the state-space design method has been proposed, but in this paper, a pole-assignment control law is extended and the extended is applied to a HDD control system to improve its control performance. Simulation results show the effectiveness of the proposed method.

![Fig. 1. Single-rate control system](image-url)

2. PROBLEM STATEMENT

A benchmark problem of a HDD has been provided by IEEJ Technical Committee for Novel Nanoscale Servo Control (Committee, 2006). In this benchmark problem (ver.2), a hard disk drive is assumed to be a 3.5 inch...
Fig. 2. Multi-Rate control system
disk and 100kTPI(tracks per inch). Its block diagram is illustrated in Fig. 3.

A transfer function from $u(t)$ to $y(t)$ is $P_f(s)$ given as:

$$P_f(s) = \begin{bmatrix} K_p P_{\text{mech}}(s) \end{bmatrix} e^{-T_d s}$$

where the parameters in (1) are shown in Table 1 and Table 2. The parameters in $P_{\text{mech}}(s)$ are not fixed and those variation range are shown in Table 3. Further, the force disturbance, the flutter disturbance, the repeatable run out disturbance and the sensor noise are defined.

### Table 1. Parameters in $P_f$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_d$ (input delay)</td>
<td>$1.0 \times 10^{-3}$ [s]</td>
</tr>
<tr>
<td>$K_f$ (equivalent force constant)</td>
<td>1.0 [N/A]</td>
</tr>
<tr>
<td>$m$ (equivalent mass of a head actuator)</td>
<td>$1.0 \times 10^{-3}$ [kg]</td>
</tr>
<tr>
<td>$T_p$ (width of track)</td>
<td>$2.54 \times 10^{-3}$ [m]</td>
</tr>
</tbody>
</table>

### Table 2. Parameters in $P_{\text{mech}}$

<table>
<thead>
<tr>
<th>$f_i$</th>
<th>$f_i$ [Hz]</th>
<th>$\omega_i$</th>
<th>$A_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>90</td>
<td>$0.5$</td>
<td>$1.0$</td>
</tr>
<tr>
<td>2</td>
<td>3000</td>
<td>$0.01$</td>
<td>$-0.01$</td>
</tr>
<tr>
<td>3</td>
<td>4100</td>
<td>$0.03$</td>
<td>$-1.0$</td>
</tr>
<tr>
<td>4</td>
<td>5000</td>
<td>$0.01$</td>
<td>$0.3$</td>
</tr>
<tr>
<td>5</td>
<td>7000</td>
<td>$0.01$</td>
<td>$-1.0$</td>
</tr>
<tr>
<td>6</td>
<td>12300</td>
<td>$0.005$</td>
<td>$1.0$</td>
</tr>
<tr>
<td>7</td>
<td>16400</td>
<td>$0.005$</td>
<td>$-1.0$</td>
</tr>
</tbody>
</table>

### Table 3. Variation range of the parameters in $P_{\text{mech}}$

<table>
<thead>
<tr>
<th>$f_i$</th>
<th>min [%]</th>
<th>max [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_1$</td>
<td>$-2$</td>
<td>$+1$</td>
</tr>
<tr>
<td>$f_3$</td>
<td>$-5$</td>
<td>$+2.5$</td>
</tr>
<tr>
<td>$f_{4,5,6,7}$</td>
<td>$-2$</td>
<td>$+0.5$</td>
</tr>
<tr>
<td>$A_{2,4}$</td>
<td>$-200$</td>
<td>$+0$</td>
</tr>
<tr>
<td>$A_{3}$</td>
<td>$-0$</td>
<td>$+20$</td>
</tr>
<tr>
<td>$C_{5,6,7}$</td>
<td>$-50$</td>
<td>$+100$</td>
</tr>
</tbody>
</table>

In this benchmark problem, the parameter variation and the disturbances can be considered, but the objective of this study is not to compensate the influences of the parameter variation and the disturbances but to have the head control precisely. Hence, in this study, the dynamic characteristics of the controlled plant are assumed to be given as (1), Table 1 and Table 2. However, the influences of the parameter variation and the disturbances are very important and should be considered. However, the influences are not dealt with in this study, but those will be compensated in the future.

To design a discrete-time control law using the state-space representation, (1) is transformed into a discrete-time state-space equation, and a control system is designed.

$$x[k+1] = A_{\text{d}} x[k] + b_{d} u[k] \quad (2a)$$

$$y[k] = c_{d} x[k] \quad (2b)$$

where $x[k]$ is the state variable, and $y[k]$ and $u[k]$ are the plant output and the control input, respectively.

**Assumption 1.** The control input is updated at intervals of $T_h$, but the plant output is sampled at intervals of $T_s(= lT_h)$, where $l$ is an integer.

### 3. CONTROL SYSTEM DESIGN

#### 3.1 Multi-Rate control system

Using Lifting (Chen and Francis, 1995), the SISO multi-rate control system is transformed into an l-input single-output slow-rate single-rate control system given as:

$$x[k+l] = A_{\text{d}} x[k] + B_{d} u[k] \quad (3)$$

$$A = A_{d} \quad B = [ A_{d}^{l-1} b_{d} \ldots A_{d} b_{d} \ b_{d} ]$$

$$u[k] = [ u[k] u[k+1] \ldots u[k+l-1] ]^T$$

The lifted system (3) is controlled using a multi-rate control law given as:

$$u[k] = K w[k] - F \tilde{x}[k] \quad (4)$$

where $w[k]$ is a reference input, and $K$ and $F$ are the vector and the $l \times n$ matrix, respectively. $\tilde{x}[k]$ is the estimated state variable obtained by using an observer given as:

$$\tilde{x}[k+l] = A_{\text{d}} \tilde{x}[k] + B_{d} u[k] + G(y[k] - c_{d} \tilde{x}[k]) \quad (5)$$

where $G$ is the observer gain which must be designed such that $A - Gc_{d} F$ is stabilized.

Using this control law (4), the closed-loop transfer function from $w[k]$ to $y[k]$ is given as:

$$y[k] = z_{i}^{-1} c_{d}(I - z_{i}^{-1} A + z_{i}^{-1} B F)^{-1} B K w[k] \quad (6)$$

In this study, $F$ and $G$ are designed such that the closed-loop poles are placed at assigned places.

#### 3.2 Improvement in steady-state response

Design polynomial vectors $Q_y[z_{i}^{-1}]$ and $Q_y[z_{i}^{-1}]$ are newly introduced, and the multi-rate control law (4) is extended (Sato et al., 2010).

$$u[k] = K w[k] - F \tilde{x}[k]$$

$$+ \left[ Q_y[z_{i}^{-1}] - Q_y[z_{i}^{-1}] \right] \left[ c_{d} \tilde{x}[k] \right] \quad (7)$$

$$Q_y[z_{i}^{-1}] = [ Q_{y1}[z_{i}^{-1}] \ Q_{y2}[z_{i}^{-1}] \cdots \ Q_{y3}[z_{i}^{-1}]]^T$$

$$Q_y[z_{i}^{-1}] = [ Q_{y1}[z_{i}^{-1}] \ Q_{y2}[z_{i}^{-1}] \cdots \ Q_{y3}[z_{i}^{-1}]]^T$$
Using the extended control law, the closed-loop transfer function from $w[k]$ to $y[k]$ is given as:

$$
y[k] = z^{-1}c_d\hat{X}[z_1^{-1}]BK\frac{w[k]}{T[z_1^{-1}] + T[z_1^{-1}]}$$

(8)

$$X[z_1^{-1}] = I - z_1^{-1}A + z_1^{-1}BF$$

$$T[z_1^{-1}] = [I - z_1^{-1}A + z_1^{-1}BF]$$

$$\hat{T}[z_1^{-1}] = -z_1^{-1}c_d\hat{X}[z_1^{-1}]B(Q_y[z_1^{-1}] - Q_y[z_1^{-1}])$$

$$\hat{T}[z_1^{-1}] = z_1^{-1}\hat{T}[z_1^{-1}]Q[z_1^{-1}]$$

$$\hat{T}[z_1^{-1}] = -c_d\hat{X}[z_1^{-1}]B$$

$$= [T_1[z_1^{-1}] - T_2[z_1^{-1}] \ldots T_l[z_1^{-1}]]$$

$$Q[z_1^{-1}] = Q_y[z_1^{-1}] - Q_y[z_1^{-1}]$$

To maintain the closed-loop characteristics of the original multi-rate control system, design polynomials $U_i[z_1^{-1}] (i = 1, \ldots, l)$ are introduced, and $Q_y[z_1^{-1}]$ and $Q_y[z_1^{-1}]$ are set as:

$$Q_y[z_1^{-1}] = U_i[z_1^{-1}]T_{i+1}[z_1^{-1}] (i \neq l)$$

(9a)

$$Q_y[z_1^{-1}] = U_i[z_1^{-1}]T_{i}[z_1^{-1}]$$

(9b)

$$Q_y[z_1^{-1}] = U_{i-1}[z_1^{-1}]T_{i-1}[z_1^{-1}] (i \neq 1)$$

(9c)

$$Q_y[z_1^{-1}] = U_1[z_1^{-1}]T_{i}[z_1^{-1}].$$

(9d)

Because the use of (9) achieves $\hat{T}[z_1^{-1}] = 0$, the sample response is independent of the selection of $U_i[z_1^{-1}]$.

Next, $U_i[z_1^{-1}]$ is designed to improve the intersample response. The use of the extended control law and the design parameters gives the closed-loop transfer function from the reference input to the control input.

$$C[z_1^{-1}] = [z_1^{-1}(-F + (Q_y[z_1^{-1}] - Q_y[z_1^{-1}])c_d]$$

$$\cdot M[z_1^{-1}] = I - z_1^{-1}(A + B(Q_y[z_1^{-1}] - Q_y[z_1^{-1}])c_d - BF)$$

(10)

To eliminate the intersample ripples in the steady state, the steady-state gains of (10) must be equal (Tangirala et al., 2001; Sato, 2008). Hence, $U_i[z_1^{-1}]$ is designed so as to satisfy the following relations:

$$C_i[1] = C_{i+1}[1] \quad (i = 1, \ldots, l-1)$$

where

$$C[z_1^{-1}] = [C_1[z_1^{-1}] \ C_2[z_1^{-1}] \cdots \ C_l[z_1^{-1}]]^T$$

4. NUMERICAL EXAMPLES

To demonstrate the effectiveness of the proposed method, two control laws (4) and (7) are applied to the HDD benchmark problem, and these control results are compared, where $l = 2$. Coefficient gains $F$ in (4) and $G$ in (5) are designed such that the poles of the closed-loop system and the observer are assigned. In design of feedforward gain $K_i$, if $K_i$ is designed, $K_j$ (i $\neq$ j) must be designed such that the steady-state error is eliminated. Hence, the design of $K_j$ depends on the selection of $K_i$.

$$K = [K_1 \ K_2]^T$$

Fig. 4 and Fig. 5 show the results of a simulation employing the designed multi-rate control law. In Fig. 4, the solid line is the output behavior in continuous time and the circles denote the sampled outputs. It follows from Fig. 4 that the control system is stabilized, and the sample response converges to the reference input without the steady-state error, although the intersample response oscillates. Further, the control input also oscillates. This is because the steady-state gains ($-1.8 \times 10^{-3}$ and $1.9 \times 10^{-3}$) are not equal.

The results of the simulation show a slow response, and so a new control law is shown to increase the speed. The control result is shown in Fig. 6 and Fig. 7. It can be seen that the output response is fast but large intersample ripples emerge since the steady-state gains of the controller are $-1.47 \times 10^{-2}$ and $1.49 \times 10^{-2}$.

To eliminate the intersample ripples without changing the sample response, the multi-rate control law is extended, where $U_1[z_1^{-1}]$ and $U_2[z_1^{-1}]$ are designed such that the steady-state gains of the closed-loop system from the reference input to the control input are equal. In the case that $U_1[z_1^{-1}] = -7.3 \times 10^{-3}$ and $U_2[z_1^{-1}] = 0$, both the steady-state gains of (10) are equal and are $1.0 \times 10^{-3}$. Fig. 8 shows that the sample response is the same as Fig. 6, and the intersample response converges to its reference input without oscillation. It follows from Fig. 9 that the control input does not oscillate and converges to a constant value. The enlarged figures of both control results are shown in Fig. 10 and Fig. 11. It can be seen that the intersample response can be improved independently of the sample response using the extended control law.

5. CONCLUSION

In the control of the HDD head, the sampling interval is restricted, but the control input can be arbitrarily updated. Hence, a multi-rate control system is designed for controlling a HDD head, where the sampling interval of the plant output is an integer multiple of the hold interval of the control input. In order to improve the intersample response without changing the sample response, an existing multi-rate control system is extended. Because the extended multi-rate control law can be designed independently of the closed-loop system of the original multi-rate control system by using newly introduced design parameters, the control law is designed such that the intersample response converges to its reference input in the steady state. As a result, the intersample ripples can be eliminated, and the sample response is maintained. Simulation results have demonstrated the effectiveness of the extended control system.

In this study, the steady-state response can be improved, but a design method which improves the transient response is not shown. Hence, our future work is to design the extended control system so as to enhance the transient response. Further, the control system is designed using the ideal environment such that the dynamic characteristics are assumed to be known and time invariant, and there is no disturbance. Therefore, the proposed method should be enhanced to account for disturbance and time-varying coefficients.

ACKNOWLEDGEMENTS

The authors gratefully acknowledges the helpful comments and suggestions of the reviewers.
Fig. 4. Output result of a multi-rate control system: slow response

Fig. 5. Input result of a multi-rate control system: slow response

Fig. 6. Output result of a multi-rate control system: fast response

Fig. 7. Input result of a multi-rate control system: fast response

Fig. 8. Output result of the extended multi-rate control system

Fig. 9. Input result of the extended multi-rate control system
REFERENCES
Technical Committee for Novel Nano Scale Servo Control, The Institute of Electrical Engineers of Japan.

Fig. 10. Output results of the conventional and the extended multi-rate control system

Fig. 11. Input results of the conventional and the extended multi-rate control system