Asymptotic Tracking Control of Variable-Speed Wind Turbines

Zongtao Lu ∗ Wei Lin § †

∗ Emerson Network Power, Columbus, Ohio 43085
§ Dept of EECS, Case Western Reserve University, Cleveland, Ohio
† Harbin Institute of Technology, Shenzhen Graduate School, China

Abstract: This paper presents a variable speed control strategy for wind turbines in order to capture maximum wind power in region 2. Wind turbines are modeled as two-mass drive-train system with generator torque control. Based on the obtained wind turbine models, variable speed control schemes are investigated for region 2. We designed nonlinear tracking controllers to achieve asymptotic tracking control for given rotor speed reference signals so as to yield maximum wind power capture for both state feedback control and output feedback control. The proposed schemes are shown to be able to achieve smooth and asymptotic tracking and illustrated by the given simulation results.

1. INTRODUCTION

Renewable energy, especially wind energy, receives increasingly attention recently because of environmental concerns and pressures. Wind turbines, the power generation plants from wind power, are large, flexible structures operating in noisy environments. How to efficiently control wind turbines is challenging but of great significance for reducing the cost of wind energy.

Most modern utility-scale turbines operate in variable speed mode with the turbine speed changing continuously in response to wind resource speed variation. A variable speed wind turbine has several levels of control systems. On the upmost level, a supervisory controller monitors the turbine and wind resource to determine when the wind speed is sufficient to start up the turbine and when, due to high winds, the turbine must be shut down for safety. In usual, the operating modes of wind turbine are divided into three regions associated with wind speed, maximum allowable rotor speed and rated power. Figure 1 shows a power curve with three regions for a wind turbine. $V_{cut-in}$ is the cut-in wind speed at which the wind turbine starts to spin and $V_{cut-off}$ is the wind speed at which the maximum allowable rotor speed is reached. A turbine that is just starting up is considered to be operating in region 1. Region 2 is an operational mode with the objective of maximizing wind energy capture. In region 2, the pitch angle is usually kept constant and wind turbine is operating at variable speed to capture the maximized wind power. In region 3, the turbine reach its maximum allowable rotor speed and rated power. The extracted torque opposes the aerodynamic torque provided by the wind, and thus indirectly regulates the turbine speed. Pitch control, depending on the pitch actuators, changes the pitch angle to the wind inflow and consequently regulates the turbine speed. In our framework in this paper, we focus on turbine control in region 2, where pitch angle is usually kept constant and only torque control plays the role.

So far, there are some reports in the literature on modeling and control of variable speed wind turbine, such as Song et al. (2000), Sivrioglu et al. (2008), Boukhezzar and Siguerdidjane (2005b), Johnson et al. (2008), Pao and Johnson (2009), Wang and Weiss (2008), Thomsen and Poulsen (2007), Beltran et al. (2008), just name a few. Adaptive control design for wind turbines is studied by Song et al. (2000), and based on the same model, a robust backstepping approach is applied for control design by Sivrioglu et al. (2008). Sliding mode and disturbance rejection controls are proposed by Beltran et al. (2008) and Thomsen and Poulsen (2007) respectively. Boukhezzar and Siguerdidjane (2005b) considers nonlinear control approach without wind speed measurement and proposes a robust nonlinear control.

Wind turbines is modeled as a one-mass drive-train system for control purpose in many papers, see Boukhezzar and Siguerdidjane (2005b), Johnson et al. (2008) and Beltran et al. (2008), because of its simple structure. But the one-mass drive-train model cannot capture the dynamics of torsional effects, which has a significant influence on the power fluctuations and the interaction of the wind turbine with the grid. Moreover, oversimplifying the models of wind turbine drive train could introduce significant error in dynamic behavior and stability (Salman and Teo (2003)). Incorporating torsional effects, a linear system based on a two-mass drive-train modeling is proposed for stability analysis by Wang and Weiss (2008). However, taking into account the aerodynamics, the whole wind turbine plant eventually ends up with a nonlinear system that is neither

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in normal form nor in lower triangular form. This brings significant challenges for control designs.

In this paper, we present a study of modeling and nonlinear control for variable speed wind turbine systems. Wind turbines are modeled as two-mass drive-train system with generator torque control, which can be reduced to one-mass drive-train system by assuming that the low speed shaft is perfectly rigid. Based on the obtained wind turbine models, variable speed control schemes are investigated for region 2. We designed nonlinear tracking controllers to achieve asymptotic tracking control for given rotor speed reference signals so as to yield maximum wind power capture. Due to the difficulty of torsional angle measurement, we design observed based control with using rotor speed information only to fulfill asymptotic tracking control. Simulation results are given to illustrate our control schemes in Section 4 and the conclusion is drawn in Section 5.

From the governing equation (2) for the generated power out of the wind turbine, we can see that that if the rotor speed is kept as a constant, then any change in the wind speed will change the tip-speed ratio, leading to the change of power coefficient $C_p$ as well as the generated power out of the wind turbine. If, however, the rotor speed is adjusted according to the wind speed variation, then the tip-speed ratio can be maintained at an optimal point, which could yield maximum power output from the system (Song et al. (2000)). As a consequence, we have the following relationship between captured wind power, rotor speed and rotor torque $T_a$

$$P_m(\omega) = k_w \omega^3$$

$$T_a = \frac{P_m}{\omega} = k_w \omega^2$$

where

$$k_w = \frac{1}{2} C_p \rho \pi R_w^3 \lambda^3$$

The dynamics of wind turbine drive-train system can be modeled as a two-mass drive-train system as shown in Figure 2. Driven by the aerodynamic torque $T_a$, the rotor of the wind turbine runs at the speed $\omega$. The aeroturbine converts wind energy into mechanical energy and the gearbox serves to increase the speed ($\omega_g$) and decrease the torque from low speed shaft to the high speed shaft. The generator is to convert mechanical energy into electrical energy. $J_r$, $J_g$, $T_{ls}$, $T_{hs}$ and $T_e$ denote rotor inertia, generator inertia, low speed shaft torque, high speed shaft torque and generator torque respectively. $b_r$, $b_g$, $C_s$ and $K_s$ are the damping and torsional coefficients.

The dynamics of the rotor is characterized by the first order differential equation

$$J_r \dot{\omega} = T_a - T_{ls} - b_r \omega$$

The low speed shaft results from the torsion and friction effects due to the difference between $\omega$ and $\omega_{ls}$

$$T_{ls} = K_s (\theta - \theta_{ls}) + C_s (\omega - \omega_{ls})$$

The generator is driven by the high speed shaft torque $T_{hs}$ and braked by the generator electromagnetic torque $T_e$.

$$J_g \dot{\omega}_g = T_{hs} - T_e - b_g \omega_g$$

Through the gearbox, the low speed shaft $\omega_{ls}$ is increased by the gearbox ratio $n_g$ to obtain the generator speed $\omega_g$ while the low speed shaft $T_{ls}$ torque is augmented.

$$n_g = \frac{T_{ls}}{T_{hs}} = \frac{\omega_g}{\omega_{ls}}$$

Let $\theta_k = \theta - \theta_{ls}$, with the equations (4)-(8), we have the following ordinary differential equations that represent the dynamics of the two-mass drive-train model

$$\dot{\theta}_k = \omega - \frac{1}{n_g} \omega_g$$

$$\dot{\omega} = \frac{1}{J_r} (-K_s \theta_k + k_w \omega^2 - C_s \omega - b_r \omega + C_s \omega_g)$$

$$\dot{\omega}_g = \frac{1}{J_g} \left( \frac{K_s}{n_g} \theta_k + C_s \omega - C_s \omega_g - b_g \omega_g - T_e \right)$$
If a perfectly rigid low speed shaft is assumed, \( \omega = \omega_{ls} \), then the two-mass drive-train model reduces to a one-mass drive-train model. The governing dynamic equation is given as

\[
J_t \dot{\omega} = T_a - b_t \omega - T_g
\]  

(12)

where

\[
J_t = J_r + n_t^2 J_g
\]

\[
b_t = b_r + n_t^2 b_g
\]

\[
T_g = n_g T_e
\]

With (4), the dynamic model for the one-mass drive-train can be written as

\[
\dot{\omega} = \frac{1}{J_t} (-b_t \omega + k_w \omega^2 - T_g)
\]  

(13)

3. CONTROL DESIGN

It is known that if the rotor speed is adjusted according to the wind speed variation, the tip-speed ratio can be maintained at an optimal point, which yields maximum power output from the system. As shown in Figure 3, the rotor speed of the wind turbine is controlled by the load torque. By the feedback of the current rotor speed, adjust the control input \( T_e \) such that the rotor speed, \( \omega \), can track the desired rotor speed, \( \omega^* \). Therefore, the control problem in region 2 can be stated as follows: Design a generator control torque (\( T_g \) or \( T_e \)) such that the rotor speed \( \omega \) of the wind turbine asymptotically tracks the desired speed \( \omega^* \). It is assumed that \( \omega, \dot{\omega} \) and \( \ddot{\omega} \) are bounded.

\[
\phi_1(\xi_1) = -\frac{C_s + b_r}{J_r} \xi_1 + \frac{k_w \xi_1^2}{J_r^2}
\]

(17)

\[
\phi_2(z, \xi_1, \xi_2) = \left( \frac{K_s^2}{J_r C_s} - \frac{b_g K_s}{J_g J_r} \right) z + \left( \frac{C_s^2}{J_r J_g n_g^2} - \frac{K_s}{J_r} \right) \xi_1
\]

\[
+ \left( \frac{K_s}{C_s} - \frac{C_s}{J_g n_g^2} - \frac{b_g}{J_g} \right) \xi_2
\]

\[
v = -\frac{C_s}{J_g J_r n_g} T_e
\]

(21)

Note that the output \( \xi_1 = \omega \) is kept unchanged, \( \phi_1(\xi_1) \) is nonlinear with respect to \( \xi_1 \), and \( \phi_2(z, \xi_1, \xi_2) \) is linear with respect to \( z, \xi_1 \) and \( \xi_2 \).

Observing the system (17)-(19), it is not difficult to see that 1) the system is still neither in lower triangular form nor in normal form and 2) although it is a minimum-phase system, more than one component \( (\xi_1, \xi_2) \) are involved in zero dynamics part.

All the above mentioned bring significant challenges to our control designs. However, if we only want to achieve asymptotic tracking for \( \xi_1 \) and relax the requirements for \( z, \xi_2 \) from asymptotic stability to stability, then we can utilize the partial feedback design idea inspired by Lin and Gong (2003) to fulfill our state feedback design. At this moment, the wind turbine power capture problem boils down to a tracking problem for the system (17)-(19) with respect to a reference signal \( y_r = \omega^* \) with continuous and bounded first and second derivatives. We state our first result on state feedback control design in the following theorem.

**Theorem 1.** Consider an equivalent wind turbine system (17)-(20), for a given twice differentiable reference speed signal \( y_r = \omega^* \), there exists a state feedback controller \( v = \beta(z, \xi_1, \xi_2, y_r, \dot{y}_r) \) such that the rotor speed \( \xi_1 = \omega \) can globally track the desired speed \( y_r \) asymptotically.

**Proof.** We begin our proof by introducing the change of coordinates as

\[
e_1 = \xi_1 - y_r
\]

\[
e_2 = \xi_2 - \xi_2^d
\]

\[
u = v - v^d
\]

where

\[
\xi_2^d = \dot{y}_r - \phi_1(y_r)
\]

\[
v^d = \dot{\xi_2^d} - \phi_2(z, y_r, \xi_2^d)
\]

Consequently, the system end up with the following form

\[
\dot{z} = -\frac{K_s}{C_s} z + e_1 - \frac{J_r}{C_s} e_2 + \delta_0(y_r, \dot{y}_r)
\]  

(21)
\[ \dot{e}_1 = e_2 + \hat{\phi}_1(e_1, y_r) \]  
(22)

\[ \dot{e}_1 = e_2 + \hat{\phi}_2(e_1, e_2, y_r, \xi^d) \]  
(23)

where

\[ \delta_0(y_r, \dot{y}_r) = y_r - \frac{J_r}{C_s} \xi^d \]

\[ \hat{\phi}_1(e_1, y_r) = \phi_1(e_1 + y_r) - \phi_1(y_r) \]

\[ \hat{\phi}_2(e_1, e_2, y_r, \xi^d) = \phi_2(z, e_1 + y_r, e_2 + \xi^d) - \phi_2(z, y_r, \xi^d) \]

with

\[ \hat{\phi}_1(0, y_r) = 0, \hat{\phi}_2(0, 0, y_r, \xi^d) = 0 \]

Now, the asymptotic tracking control objective is to design a state feedback controller for (21)-(23) such that \( e_1 \to 0 \) as \( t \) tends to infinity. The state feedback controller is designed step by step based on Lyapunov stability. In the first step, consider the Lyapunov function candidate

\[ V_1 = \frac{1}{2} e_1^2 \]

and taking time derivative yields

\[ \dot{V}_1 = e_1(e_2 + \dot{\phi}_1(e_1, y_r)) \]

\[ = -e_1^2 + e_1(e_2 - e_2^2) \]

with

\[ e_2^2 = -e_1 - \dot{\phi}_1(e_1, y_r) := \beta_1(e_1, y_r) \]  
(24)

Consider another Lyapunov function candidate

\[ V_2 = V_1 + \frac{1}{2}(e_2 - e_2^2)^2 \]

and taking time derivative yields

\[ \dot{V}_2 = -e_1^2 + (e_2 - e_2^2)[e_1 + u + \dot{\phi}_2(\cdot)] \]

\[ = -e_1^2 + e_1(e_2 - e_2^2) + \partial e_2 \]  
\[ \dot{\phi}_2(e_1, e_2, y_r, \xi^d) = \partial e_2 \]  
\[ \dot{\phi}_1(e_1, y_r) - \frac{\partial e_2}{\partial y_r}(\dot{y}_r) \]

If we design a state feedback controller \( u = \beta_2(e_1, e_2, y_r, \dot{y}_r) \) as

\[ u = -(e_2 - e_2^2) - e_1 - \dot{\phi}_2(\cdot) + \frac{\partial e_2}{\partial e_1}(e_2 + \dot{\phi}_1(\cdot)) + \frac{\partial e_2}{\partial y_r}(\dot{y}_r) \]

\[ := \beta_2(e_1, e_2, y_r, \dot{y}_r) \]  
(25)

then \( \dot{V}_2 \) is negative definite with respect to \( e_1, e_2 \) as

\[ \dot{V}_2 = -e_1^2 - (e_2 - e_2^2)^2 \]

which implies asymptotic stability of \( (e_1, e_2) \) part of (21)-(23).

Examine \( z \) part of the system (21) and obviously it is ISS with respect to \( e_1 \) and \( e_2 \). As \( t \) goes to infinity, \( (e_1, e_2) \to 0 \), which implies that \( z \) must be bounded since \( \delta_0(y_r, \dot{y}_r) \) is bounded. As a result, the system (21)-(23) is stable and, in particular, the components \( (e_1, e_2) \) are asymptotically stable at the origin. Therefore, global asymptotic tracking for (17)-(19) can be achieved by the (26) and hence we complete the proof.

If we design a state feedback controller for (21)-(23) such that (17)-(19) can be achieved by the (26) and hence we complete the proof.

\[ u = \beta_2(e_1, e_2, y_r, \dot{y}_r) + \dot{\xi}^d - \phi_2(z, y_r, \xi^d) \]

\[ := \beta(z, \xi_1, \xi_2, y_r, \dot{y}_r, \ddot{y}_r) \]  
(26)

Remark 1. It is known that global asymptotic tracking can be achieved for a feedback linearizable system if its global asymptotic stabilization problem is solvable, and its negative proposition usually does not hold. The zero dynamics part, which involves more than one component, makes it impossible to global asymptotic stabilization for any state feedback controller. Our partial feedback is to take advantage of the ISS property of the zero dynamics and try to achieve asymptotic tracking for \( \xi_1, \xi_2 \) only with leaving \( z \) bounded instead of asymptotic stable.

The state feedback tracking controller (26) requires the information of all states, \( \theta_1, \omega \) and \( \omega_q \). However, the torsional angle, \( \theta_1 \), is usually difficult to measure in practice. For this reason, we propose an observer based control design, using \( \omega \) and \( \omega_q \) information only, to achieve asymptotic tracking for a given reference speed signal, \( \omega^* \).

We carry on the same change of coordinates (14)-(16) for (9)-(11), which leads to the same system (17)-(20) in \((z, \xi_1, \xi_2)\) coordinates. Note that \( \xi_2 \) is not available for feedback because it involves \( \theta_2 \) and \( \xi_1 \) is the only available state for feedback design. Thus, the problem boils down to designing an output feedback tracking controller for (17)-(20) by using \( \xi_1 \) information only. We state this result in the following theorem.

Theorem 2. Consider an equivalent wind turbine system (17)-(20), for a given twice differentiable reference speed signal \( y_r = \omega^* \), there exists an output feedback controller

\[ \dot{z} = -K_s C_s z + \eta_1 - \frac{J_r}{C_s} \eta_2 + \delta_0(y_r, \dot{y}_r) \]  
(29)

\[ \dot{\eta}_1 = M \eta_2 + \hat{\phi}_1(\eta_1, y_r) \]  
(30)

such that the rotor speed \( \xi_1 = \omega \) can globally track the desired speed \( y_r \) asymptotically.

Proof. We shall prove this theorem by designing a state feedback controller and a reduced order observer. It is well-known that design of global stabilization of the system in normal form is solvable by a linear output dynamic compensator if it satisfies linear growth condition (Qian and Lin (2003)). We shall apply this idea to our proof with some revision. We only carry on the estimation the two states \( (z, \xi_1) \) satisfying linear growth condition, but not on the output \( \xi_1 \). Another new ingredient of our output feedback design is involvement of zero dynamics, which is not considered in (Qian and Lin (2003)). Our design is involved with estimation of the zero dynamics part \((z, \xi_1)\), and thanks to its minimum phase property, it can be handled by our design.

Part I - Reduced order observer design.

Consider the equivalent error dynamic system (21)-(23) in \((z, e_1, e_2)\) coordinates. Introduce

\[ \eta_1 = e_1, \eta_2 = \frac{e_2}{M} \]

where \( M > 0 \) is to be determined later. As a result, (21)-(23) can be rewritten as

\[ \dot{z} = -\frac{K_s}{C_s} z + \eta_1 - \frac{J_r}{C_s} M \eta_2 + \delta_0(y_r, \dot{y}_r) \]  
(29)

\[ \dot{\eta}_1 = M \eta_2 + \hat{\phi}_1(\eta_1, y_r) \]  
(30)
\[ \dot{\eta}_2 = \frac{u}{M} + \frac{1}{M} \dot{\phi}_2(\eta_1, \eta_2, y_r, \dot{y}_r) \]  

Let \( \chi_1 = z - L_1 \eta_1, \chi_2 = \eta_2 - L_2 \eta_1 \) where \( L_1, L_2 > 0 \) are determined later.

Introduce \( \hat{z} \) and \( \hat{\eta}_2 \) as the estimations of \( z \) and \( \eta_2 \) respectively as

\[ \hat{z} = \chi_1 + L_1 \eta_1, \hat{\eta}_2 = \chi_2 + L_2 \eta_1 \]

Design a reduced order observer

\[ \dot{\chi}_1 = -K_s (\ensuremath{\tilde{z}} + \eta_1) + J_r C_s \dot{\eta}_2 + \frac{\delta_0}{\varepsilon} (\cdot) \]
\[ -L_1 M \dot{\eta}_2 - L_1 \dot{\phi}_1(\eta_1, y_r) \]
(32)

\[ \dot{\chi}_2 = \frac{u}{M} + \frac{1}{M} \dot{\phi}_2(\eta_1, \eta_2, y_r, \dot{y}_r) \]
\[ -L_2 M \dot{\eta}_2 - L_2 \dot{\phi}_1(\eta_1, y_r) \]
(33)

Introduce the estimation errors \( \varepsilon_1 \) and \( \varepsilon_2 \) as

\[ \varepsilon_1 = z - \hat{z} = \chi_1 - \hat{\chi}_1, \varepsilon_2 = \eta_2 - \hat{\eta}_2 = \chi_2 - \hat{\chi}_2 \]

and their dynamics as

\[ \dot{\varepsilon}_1 = -K_s \varepsilon_1 - J_r C_s \dot{\varepsilon}_2 + \frac{\delta_0}{\varepsilon} (\cdot) \]
\[ -L_1 M \dot{\varepsilon}_2 - L_1 \dot{\phi}_1(\eta_1, y_r) \]
(34)

\[ \dot{\varepsilon}_2 = -L_2 M \dot{\varepsilon}_2 + \frac{1}{M} \left( \dot{\phi}_2(\eta_1, \eta_2, y_r, \dot{y}_r) \right) \]
\[ -\dot{\phi}_2(\eta_1, \eta_2, y_r, \dot{y}_r) \]
(35)

Recall \( \dot{\phi}_2(\cdot) \) is linear with respect to \( \eta_1, \eta_2, y_r \) and \( \dot{y}_r \). Hence, it follows that

\[ \left| \dot{\phi}_2(\eta_1, \eta_2, y_r, \dot{y}_r) - \dot{\phi}_2(\eta_1, \eta_2, y_r, \dot{y}_r) \right| \leq c |\varepsilon_2| \]
(36)

where \( c > 0 \) is a known constant.

Consider the Lyapunov function candidate

\[ V_0 = \frac{1}{2} \varepsilon_1^2 + \frac{1}{2} |\varepsilon_2|^2 \]

and taking derivative yields

\[ \dot{V}_0 = \frac{K_s}{C_s} \varepsilon_1^2 - (J_r + L_1) \varepsilon_1 \varepsilon_2 - L_2 M \dot{\varepsilon}_2^2 \]
\[ + \frac{\varepsilon_2}{M} \left( \dot{\phi}_2(\eta_1, \eta_2, y_r, \dot{y}_r) - \dot{\phi}_2(\eta_1, \eta_2, y_r, \dot{y}_r) \right) \]
\[ \leq -\frac{K_s}{C_s} \varepsilon_1^2 - (J_r + L_1) \varepsilon_1 \varepsilon_2 - L_2 M \dot{\varepsilon}_2^2 + \frac{c}{M} \varepsilon_2^2 \]

If we choose \( M = 1, L_1 = 1, L_2 = 1 + c + (J_r + C_s) \)
\[ \quad \frac{\dot{y}_r}{4K_s C_s} \]

then it follows that \( \dot{V}_0 \) is negative definite as

\[ \dot{V}_0 \leq -\frac{K_s}{C_s} (\varepsilon_1 + \frac{J_s + C_s}{2K_s} \varepsilon_2)^2 - \varepsilon_2^2 \]

which implies asymptotic stability of the estimation error dynamics.

Part II - Output feedback controller design.

If we replace \( e_2 \) in the state feedback controller \( u = \beta_2(e_1, e_2, y_r, \dot{y}_r) \) in (25) by its estimation \( \hat{e}_2 \), which is given by

\[ \hat{e}_2 = M \hat{\eta}_2 = M \chi_2 + L_2 \eta_1 \]

then we can show that the closed-loop system (21)-(23), (32)-(33) with \( u = \beta_2(e_1, \hat{e}_2, y_r, \dot{y}_r) \) is stable, in particular, \((e_1, e_2) \rightarrow (0,0)\) as \( t \) tends to infinity.

Consider the Lyapunov function \( W_0 \)

\[ W_0 = \frac{1}{2} \hat{e}_1^2 + \frac{1}{2} (e_2 - \hat{e}_2)^2 \]

where \( e_2^2 \) is the same as in (24)

\[ e_2^2 = -e_1 - \hat{\phi}_1(\eta_1, y_r) = \beta_1(e_1, y_r) \]

Taking time derivative yields

\[ \dot{W}_0 = -e_1^2 - (e_2 - \hat{e}_2)^2 + e_2^2 \]
\[ \frac{\partial e_2^2}{\partial e_1} (e_1 + \hat{\phi}_2(\eta_1, e_2, y_r, \dot{y}_r)) + \frac{\partial e_2^2}{\partial y_r} \hat{\gamma}_r \]
\[ := \beta_2(e_1, \hat{e}_2, y_r, \dot{y}_r) \]

Design the controller \( u \) as

\[ u = -(e_2 - \hat{e}_2) - \hat{\phi}_2(\eta_1, e_2, y_r, \dot{y}_r) \]
\[ + \frac{\partial e_2^2}{\partial e_1} (e_1 + \hat{\phi}_2(\eta_1, y_r)) + \frac{\partial e_2^2}{\partial y_r} \hat{\gamma}_r \]

Consequently, with (36) in mind, \( W_0 \) yields

\[ W_0 \leq -e_1^2 - \frac{1}{2} (e_2 - \hat{e}_2)^2 + (1 + c^2) e_2^2 \]

Now, combine \( V_0 \) and \( W_0 \) together as \( W(e_1, e_2, \varepsilon_1, \varepsilon_2) \)
\[ W = (2 + c^2) V_0 + W_0 \]

and its time derivative gives

\[ \dot{W} = (2 + c^2) V_0 + W_0 \]
\[ \leq -\frac{K_s}{C_s} (2 + c^2) (e_1 + \frac{J_r + C_s}{2K_s} e_2)^2 - e_2^2 \]
\[ -e_1^2 - \frac{1}{2} (e_2 - \hat{e}_2)^2 \]

which implies asymptotic stability of \( \varepsilon \)-dynamics and \((e_1, e_2) \) subsystem at \((e, e) = (0,0)\).

Examine \( z \) part of the system (21) and obviously it is ISS with respect to \( e_1 \) and \( e_2 \). We already show that \( \varepsilon \)-dynamics and \((e_1, e_2) \) subsystem are asymptotically stable at \((e, e) = (0,0)\), hence, as \( t \) goes to infinity, \((e_1, e_2) \rightarrow 0 \), it implies that \( z \) must be bounded since \( \phi_0(y_r, \dot{y}_r) \) is bounded.

As a result, the closed-loop system (21)-(23) and (34)-(35) with \( u = \beta_2(e_1, \hat{e}_2, y_r, \dot{y}_r) \) is stable and, in particular, the components \((e_1, e_2) \) are asymptotically stable at the origin. Therefore, asymptotic tracking for (17)-(19) can be achieved by (38)-(39) Hence we complete our proof.

\[
\begin{align*}
\dot{v} &= u + v^d \\
&= \beta_2(e_1, \hat{e}_2, y_r, \dot{y}_r) + \hat{e}_2 - \hat{\phi}_2(\hat{z}, y_r, e_2^2)
\end{align*}
\]
\[ \beta(\xi_1, \varsigma, y_r, \dot{y}_r, \dot{y}_r) \quad (38) \]
\[ \varsigma = \varsigma(\xi_1, \varsigma), \varsigma = [\hat{z} \, \hat{\xi}_1]^T \quad (39) \]

Remark 2. The computation of \( v = \beta(\xi_1, \varsigma, y_r, \dot{y}_r, \dot{y}_r) \) in (38) involves \( \phi_2(\dot{z}, y_r, \xi_1^d) \), which is guaranteed to asymptotically converge to \( \phi_2(z, y_r, \xi_1^d) \). This makes the dynamic output feedback controller (38)-(39) achieve asymptotic tracking for \( \xi_1 \) as the dynamic output controller (34)-(35), (37) guarantee asymptotic stability for \( (e, \epsilon) \) only and global stability for \( z \), which is enough to achieve global asymptotic tracking for \( \xi_1 \).

In this section, we show that we can design a nonlinear state feedback controller and an observer based output feedback controller, based on the two-mass drive-train model, to achieve global asymptotic tracking to any given rotor speed reference signal as long as the signal is two differentiable. The proposed control scheme is applicable in region 2 to realize the maximum wind power capture.

4. SIMULATION STUDY

The simulation study is to illustrate our proposed control schemes. The wind turbine under study is with 1.7 MW rated power and its diameter of the blades is 35 meters (Garcia-Sanz and Elso (2009)). The physical parameters are given in Table 1. The rotor speed is to be adjusted to follow the desired trajectory.

\[ \omega^* = 1.1 + 0.5 \times (1 + \sin(\frac{\pi}{10} \cdot t)) \]

The simulation results with the state and output feedback controllers are shown in Figure 4 and 5 respectively. It is clear that the proposed control schemes are able to achieve smooth and precise asymptotic speed tracking.

Table 1. Wind turbine’s parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J_r )</td>
<td>( 4 \times 10^6 , \text{kg} \cdot \text{m}^2 )</td>
</tr>
<tr>
<td>( b_r )</td>
<td>( 980 , \text{N} \cdot \text{m/sec} )</td>
</tr>
<tr>
<td>( n_g )</td>
<td>38.06</td>
</tr>
<tr>
<td>( K_s )</td>
<td>( 10 \times 6 , \text{N} \cdot \text{m} )</td>
</tr>
<tr>
<td>( J_g )</td>
<td>( 20 , \text{kg} \cdot \text{m}^2 )</td>
</tr>
<tr>
<td>( b_g )</td>
<td>( 0.2 , \text{N} \cdot \text{m/sec} )</td>
</tr>
<tr>
<td>( k_w )</td>
<td>( 1.14 \times 10^7 , \text{kg} \cdot \text{m}^2 )</td>
</tr>
<tr>
<td>( C_s )</td>
<td>( 500 , \text{N} \cdot \text{m/sec} )</td>
</tr>
</tbody>
</table>

Variable speed control for wind turbine is necessary to increase wind power generation efficiency. We modeled the wind turbine systems as a two-mass drive-train model and it can be reduced to a one-mass drive-train model if we assume the low speed shaft is perfectly rigid. We design both state feedback and output feedback tracking controllers to achieve global asymptotic tracking for the rotor speed. Although the nonlinear model is neither in normal form nor in lower triangular form, we can still fulfill our design to achieve asymptotic rotor speed tracking by utilizing partial feedback design.

REFERENCES


