A Simple Repetitive Learning Control for Asymptotic Tracking of Robot Manipulators with Actuator Saturation

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Abstract: In this paper, we provide a simple decentralized saturated repetitive learning controller for asymptotic tracking of robot manipulators under actuator saturation. The proposed control consists of a saturated nonlinear proportional plus derivative action and a saturated learning-based feedforward compensation term. A Lyapunov-like stability argument is employed to show semiglobal asymptotic tracking. Advantages of the proposed controller include an absence of modeling parameter in the control law formulation and an ability to ensure actuator constraints are not breached. This is accomplished by selecting control gains a priori, removing the possibility of actuator failure due to excessive torque input levels. The effectiveness of the proposed approach is illustrated via simulations.

Keywords: Actuator saturation, asymptotic stability, repetitive learning control, robot control, tracking.

1. INTRODUCTION

Given the myriad of industrial applications that require a robot to move in repetitive manner, researchers have been investigated control methods that exploit the periodic nature of the robot dynamics, and hence, many types of learning controllers have been developed to increase link position tracking performance. The advantages of learning control include the ability to compensate for disturbances without high frequency or high gains and the ability to compensate for time-varying disturbances and parametric effects (Arimoto, Kawamura, & Miyazaki, 1984; Dixon, Zergeroglu, Dawson, & Costic, 2002).

Under the assumption that the accelerations are measured and a resetting procedure is performed at the beginning of each trial, learning control laws were initially proposed in (Arimoto, Kawamura, & Miyazaki, 1984). Motivated by this seminar work, several learning controls for robot manipulators have been subsequently proposed. Horowitz (1993) gives a nice history of the development and usage of learning control for tracking of robot manipulators. Messner, Horowitz, Kao, & Boals (1991) propose adaptive learning controllers and achieve local asymptotic tracking. Kasac, Novakovic, Majetic, & Brezak (2008), Liuzzo & Tomei (2008), and Tang, Cai, & Huang (2000) exploit the idea of developing in Fourier series expansion of a periodic reference signal for perfect tracking. Kuc & Han (2000) integrate the project repetitive learning control into the PID and adaptive control for global asymptotic tracking. Dixon, Zergeroglu, Dawson, & Costic (2002) develop a hybrid control schemes that utilize learning-based feedforward terms to compensate for periodic dynamics and other Lyapunov-based approaches to compensate for nonperiodic dynamics and ensure global asymptotic tracking. Sun, Ge, & Mareels (2006) develop an adaptive repetitive learning control that removes the resetting assumption. On the other hand, another type of iterative learning controllers was developed using Lyapunov-like method to improve the link position tracking performance (Cheah, Wang, & Soh, 1996; Norrlof, 2002; Tayebi, 2004).

While these control schemes are elegant and intuitively appealing, there is an implicit assumption in the development of these schemes that the robot system actuators are able to provide any requested joint torque. This assumption can lead to difficulties in practice since the available torque amplitude is limited in actual manipulators. Moreover, it is known that control design approaches which do not incorporate input constraints directly into the design suffer from important performance limitations (Bernstein & Michel, 1995; Laib, 2000; Saberi, Lin, & Teel, 1996; Su, Müller, & Zheng, 2010).

Recognizing these difficulties, several solutions that take into account actuator constraints for tracking of robot manipulators have been proposed. For example, Loria and Nijmeijer (1998) combine the well-known PD plus (PD+) scheme with the commonly used “dirty derivative” technique to resolve the asymptotic tracking control of Euler-Lagrange systems. This result was later extended in Santibanez and Kelly (2001) and Aguinga-Ruiz, Zavala-Rio, Santibanez, & Reyes (2009) to the global asymptotic stability relay heavily on the excessively restrictive and limitative assumption that the inherent friction on each joints is larger than the upper boundedness of the tracked trajectories. Dixon, de Queiroz, Zhang, & Dawson (1999) formulate a saturated output feedback PD+ scheme and saturated adaptive control and obtain semiglobal asymptotically tracking. The main drawback of the existing saturated control strategies is that the control design requires exact knowledge of system dynamics. In general, an exact dynamic model is not
available due to system uncertainty. Therefore it is more desirable to design a completely model-free saturated control.

In this paper, we propose a simple model-free saturated learning-based repetitive control scheme for asymptotic tracking of robot manipulators with actuator saturation. The proposed saturated repetitive learning control is motivated by the work given in (Dixon, Zergeroglu, Dawson, & Costic, 2002). Specifically, we modify the learning estimate rule presented in (Dixon et al, 2002) to a saturated one and integrated it into a simple saturated PD control. The resulting control has a decentralized frame and is fully model-free-free and depends only on the period of the reference signals and some constant bounds extracted from the robot dynamics, and thus, it is readily implemented. The fact that the proposed controller can be a priori bounded is a significant added advantage. The practical implications are that the actuators can be appropriately sized without an ad hoc saturation module.

2. ROBOT MANIPULATOR MODEL AND PROPERTIES

The dynamics of a rigid revolute joint robot manipulator can adequately be described using Euler-Lagrange equations of motion as (Arimoto, 1996; Sciavicco & Siciliano, 2000)

\[ M(q)\ddot{q} + C(q, \dot{q})\dot{q} + D\ddot{q} + g(q) = \tau \]  

(1)

where \( q, \dot{q}, \ddot{q} \in \mathbb{R}^n \) denote the link position, velocity, and acceleration, respectively, \( M(q) \in \mathbb{R}^{n \times n} \) represents the symmetric inertia matrix, \( C(q, \dot{q}) \in \mathbb{R}^{n \times n} \) denotes the centrifugal-Coriolis matrix, \( D \in \mathbb{R}^{n \times n} \) represents the matrix composed of damping friction coefficients for each joint, \( g(q) \in \mathbb{R}^n \) is a gravity force, and \( \tau \in \mathbb{R}^n \) denotes the torque input vector. Recalling the robot manipulators are being considered, the following properties can be established (Arimoto, 1996; Sciavicco & Siciliano, 2000).

Property 1. The inertia matrix \( M(q) \) is symmetric positive definite and satisfies the following inequality:

\[ m_1\|\xi\| \leq \|M(q)\xi\| \leq m_2\|\xi\|, \quad \forall \xi \in \mathbb{R}^n \]

(2)

where \( m_1 \) and \( m_2 \) are known positive constants.

Property 2. There exist positive scalar constants \( \zeta_d \) and \( \zeta_{c1} \) such that

\[ \|D\| \leq \zeta_d, \quad \|C(q, \dot{q})\| \leq \zeta_{c1}\|\dot{q}\|, \quad \forall q, \dot{q} \in \mathbb{R}^n \]

(3)

Property 3. The centrifugal-Coriolis matrix \( C(q, \dot{q}) \) satisfies the following relationship:

\[ C(q, \dot{q})\dot{q} = C(q, \dot{q})\dot{q}, \quad \forall q, \dot{q} \in \mathbb{R}^n \]

(4)

Property 4 (Zhang, Dawson, de Queiroz, & Dixon, 2000). There exist some known positive constants \( \zeta_m, \zeta_g \) and \( \zeta_{c2} \) for all \( \xi, \vartheta \in \mathbb{R}^n \) such that

\[ \|M(\xi) - M(\vartheta)\| \leq \zeta_m\|\tanh(\xi - \vartheta)\| \]

\[ \|g(\xi) - g(\vartheta)\| \leq \zeta_g\|\tanh(\xi - \vartheta)\| \]

\[ \|C(\xi, \dot{q}) - C(\vartheta, \dot{q})\| \leq \zeta_{c2}\|\dot{q}\|\|\tanh(\xi - \vartheta)\| \]

(5)

where \( \tanh(\cdot) \in \mathbb{R}^n \) is defined as

\[ \tanh(\xi) = \left[\tanh(\xi_1), \ldots, \tanh(\xi_n)\right]^T \]

(6)

with \( \tanh(\cdot) \) being the standard hyperbolic tangent function.

3. CONTROL DEVELOPMENT

3.1 Control Objective

Let \( q_d(t) \in \mathbb{R}^n \) be any \( C^3 \) periodic reference trajectory of \( T \) for the output of system (1) such that

\[ \frac{d^3q_d(t)}{dt^3} \in \epsilon, \quad \text{for } i = 0, 1, 2, 3 \]

(7)

\[ q_d(t) = q_d(t - T), \quad \dot{q}_d(t) = \dot{q}_d(t - T), \quad \ddot{q}_d(t) = \ddot{q}_d(t - T) \]

(8)

where \( T \) is a known constant. Let the output tracking errors \( e(t), \dot{e}(t) \in \mathbb{R}^n \) be defined as follows:

\[ e = q_d - \hat{q}, \quad \dot{e} = \dot{q}_d - \dot{\hat{q}} \]

(9)

Our control objective is to design a model-free saturated learning-based controller satisfying

\[ |e_i| \leq \tau_i, \max \]

(10)

such that \( e(t) \rightarrow 0 \) as \( t \rightarrow \infty \), where \( \tau_t \) and \( \tau_{t, \max} \) denote the \( i \)-th torque input and the maximum torque of the \( i \)-th actuator, respectively.

3.2 Control Formulation

Similar to Dixon et al. (2002), the periodic reference input \( w_d(t) \in \mathbb{R}^n \) corresponding to the reference \( q_d(t) \), can be computed as

\[ w_d(t) = M(q_d)\ddot{q}_d + C(q_d, \dot{q}_d)\dot{q}_d + D\ddot{q}_d + g(q_d) \]

(11)

Remark 1 (Dixon et al., 2002). Based on (11) and the boundedness (2), (3) and (7), it is clear that \( w_d(t) \) is periodic and upper bounded by

\[ |w_{di}(t)| \leq \delta_i, \quad \text{for } i = 1, 2, \ldots, n \]

(12)

where \( \delta_i \in \mathbb{R}^1 \) is a known, positive bounding constant.

To facilitate the subsequent design and analysis, we define a filtered tracking error variable \( \eta \in \mathbb{R}^n \) as follows:

\[ \eta = \dot{e} + \alpha_i\tanh(e) \]

(13)

where \( \alpha_i \in \mathbb{R}^1 \) is a positive constant control gain.

We propose the following saturated learning-based controller to solve the above stated problem:
\[ \tau = K \tanh(\eta) + \tanh(e) + \dot{w}_d \]  \hspace{1cm} (14)

where \( K \in \mathbb{R}^{n \times n} \) being the diagonal, positive definite control gain matrix, and \( \dot{w}_d \in \mathbb{R}^n \) is generated online according to the following saturated learning-based algorithm

\[ \dot{w}_d(t) = \alpha_2 \text{sat}(\dot{w}_d(t-T)) + k_i \tanh(\eta) \]  \hspace{1cm} (15)

\( \alpha_2, k_i \in \mathbb{R}^1 \) are positive learning gains, and \( \text{sat}(\cdot) \in \mathbb{R}^n \) is a vector function whose elements are defined as follows:

\[ \text{sat}(\xi_i) = \begin{cases} \xi_i, & \text{for } |\xi_i| \leq \beta_i \\ \beta_i \text{sgn}(\xi_i), & \text{for } |\xi_i| > \beta_i \\ \end{cases} \]  \hspace{1cm} (16)

where \( \beta_i \in \mathbb{R}^1 \) denotes a positive constant gain, and \( \text{sgn}(\cdot) \) denotes the standard signum function.

**Remark 2.** The control effort given by (14)-(16) can be explicitly upper bounded in terms of \textit{a priori} known terms as

\[ |\xi_i| \leq k_i + 1 + \alpha_2 \beta_i + k_i \]  \hspace{1cm} (17)

where \( k_i \) denotes the \( i \)-th diagonal element of matrix \( K \).

Based on this fact, the actuator constraints expressed in (10) can be satisfied by selecting the control gains \textit{a priori}

\[ k_i + \alpha_2 \beta_i + k_i \leq \tau_{i, \max} - 1 \]  \hspace{1cm} (18)

where \( \tau_{i, \max} \) was defined in (10).

**Remark 3** (Dixon et al., 2002). From the definition of \( \text{sat}(\cdot) \) given in (16), it is easy to show that

\[ (\xi_{i1} - \xi_{i2})^2 \geq (\text{sat}(\xi_{i1}) - \text{sat}(\xi_{i2}))^2 \]

\[ \forall |\xi_{i1}| \leq \beta_i, \xi_{i2} \in \mathbb{R}^1, \text{ i = 1, 2, ..., n} \]  \hspace{1cm} (19)

\section{Error System Development}

After taking the time derivative of (13), multiplying both sides of the equation by \( M(q) \), and then substituting (1) for \( M(q)\dot{q} \) in the resulting expression, yields

\[ M(q)\ddot{q} = M(q)\dot{q}_d + C(q, \dot{q})\dot{q} + D\dot{q} + g(q) - \tau \]

\[ + \alpha_1 M(q) \text{Sech}^2(\dot{e})\dot{e} \]  \hspace{1cm} (20)

where \( \text{Sech}(\cdot) \in \mathbb{R}^{n \times n} \) denotes the following diagonal matrix

\[ \text{Sech}(\xi) = \text{diag}\left(\text{sech}(\xi_1), \cdots, \text{sech}(\xi_n)\right) \]  \hspace{1cm} (21)

with \( \text{sech}(\cdot) \) being the standard hyperbolic secant function and \( \text{diag}(\cdot) \) denotes a diagonal matrix.

Upon utilizing (9), (11) and (13), the closed-loop dynamics for \( \eta \) can be formulated as follows:

\[ M(q)\dot{\eta} = -C(q, \dot{q})\eta + w_d + N + \tilde{Y} - \tau \]  \hspace{1cm} (22)

where \( N(e, \eta, t), \tilde{Y}(e, \eta, t) \in \mathbb{R}^n \) are defined as follows:

\[ N = \alpha_1 M(q) \text{Sech}^2(\dot{e})\dot{e} + C(q, \dot{q})\eta + C(q, \dot{q})\dot{q} - C(q, \dot{q}_d)\dot{q}_d \]

\[ = \alpha_1 M(q) \text{Sech}^2(\dot{e})(\eta - \alpha_1 \tanh(e)) - C(q, \dot{q}_d)(\eta - \alpha_1 \tanh(e)) \]

\[ + C(q, \alpha_1 \tanh(e))(\dot{q}_d + \alpha_1 \tanh(e) - \eta) \]  \hspace{1cm} (23)

\[ \tilde{Y} = M(q)\dot{q}_d + C(q, \dot{q}_d)\dot{q}_d + D\dot{q} + g(q) - w_d \]  \hspace{1cm} (24)

where we have utilized (4) of Property 3.

**Remark 4.** By utilizing (13), Properties 1-4, (7), (9) and the properties of hyperbolic functions, we can show \( N(\cdot) \) in (23) and \( \tilde{Y}(\cdot) \) in (24) can be upper bounded as follows:

\[ \|N\| \leq \alpha_1 |\dot{\zeta}_1| \|\| \]  \hspace{1cm} (25)

\[ \|\tilde{Y}\| \leq |\dot{\zeta}_2| \|\| \]  \hspace{1cm} (26)

where \( |\dot{\zeta}_1| \) and \( |\dot{\zeta}_2| \) are positive bounding constants, and \( z \in \mathbb{R}^{2n} \) is defined as

\[ z = [\alpha_1 \tanh(e) \ | \ \eta^T] \]  \hspace{1cm} (27)

After substituting the control law (14) into (22), we obtain the following closed-loop dynamics for \( \eta \):

\[ M(q)\dot{\eta} = -C(q, \dot{q})\eta - K \tanh(\eta) + \text{sat}(\dot{w}_d(s)) + N + \tilde{Y} \]  \hspace{1cm} (28)

where \( \text{sat}(\cdot) \in \mathbb{R}^n \) denotes the learning estimation error signal defined as follows:

\[ \text{sat}(\dot{w}_d(s)) = \begin{cases} \dot{w}_d, & \text{for } |\dot{w}_d| \leq \delta \\
\delta \text{sgn}(\dot{w}_d), & \text{for } |\dot{w}_d| > \delta \end{cases} \]  \hspace{1cm} (29)

Upon substituting (15) into (29) for \( \dot{w}_d(t) \), utilizing the fact that \( w_d(t) \) is periodic, and then utilizing (12) in Remark 1, we have

\[ w_d(t) = \alpha_2 \text{sat}(w_d(t)) = \alpha_2 \text{sat}(w_d(t-T)) \]  \hspace{1cm} (30)

By employing (15) and (30), we can rewrite (29) as:

\[ \text{sat}(w_d(t-T)) - \alpha_2 \text{sat}(\dot{w}_d(t-T)) - k_1 \tanh(\eta) \]  \hspace{1cm} (31)

\subsection{Stability Analysis}

Now we are in a position to state the following result.

**Theorem 1.** Given the robot dynamics of (1) under input constraints (10), the proposed model-free saturated learning-based control torque input given in (14)-(16) ensures asymptotic tracking in the sense that

\[ \lim_{t \to \infty} e = 0 \]  \hspace{1cm} (32)

provided the control gains are chosen as follows:

\[ \beta_i \geq \delta_i \]  \hspace{1cm} (33)

\[ 0 < \alpha_2 \leq \sqrt{m_f / m_2} \]  \hspace{1cm} (34)

\[ \alpha_1 > \max\left(m_1^{-1}, 2\right) \]  \hspace{1cm} (35)

\[ (2\lambda_2(K) + k_j)\kappa = 2m_2 \max\left[ m_1^{-1}, (1 + \alpha_1)k_c + 2\alpha_1 \beta_1 \right] \]  \hspace{1cm} (36)

where \( \lambda_2(K) \) is the second largest eigenvalue of matrix \( K \), \( k_j \) is the actuator saturation constant, \( m_1 \) and \( m_2 \) are the masses of links 1 and 2, respectively, \( \alpha_1 \) and \( \alpha_2 \) are the learning gains, \( k_c \) is the actuator saturation constant, and \( k_j \) is a positive constant.
where $\lambda_m(\cdot)$ denotes the minimum eigenvalue of a matrix, $\kappa$ is a positive constant defined by the subsequent (44), and $k_c$ is a control gain that must satisfy the following sufficient condition

$$k_c > \max \left( \frac{\sqrt{(\alpha_m(m^{-1})^2)}}{2}, 1 \right)$$

(37)

**Proof.** Theorem 1 is proved following Lyapunov’s direct method and Barbalat’s lemma is invoked to guarantee semiglobal asymptotic tracking. To this end, a nonnegative Lyapunov-like function $V$ is proposed as follows

$$V = \sum_{i=1}^{n} \left( \ln(\cosh(\eta_i)) + \alpha_i k_c \ln(\cosh(e_i)) \right)$$

$$+ \frac{\alpha^2}{2k_i} \int_0^T \left[ \text{sat}(w_d(t)) - \text{sat}(\hat{w}_d(t)) \right]^T M^{-1}(q(t+T)) \left[ \text{sat}(w_d(t)) - \text{sat}(\hat{w}_d(t)) \right] dt$$

$$\times M^{-1}(q(t+T)) \text{sat}(w_d(t)) - \text{sat}(\hat{w}_d(t)) \] d\sigma$$

(38)

After taking the time derivative of (38), we have

$$\dot{V} = \Tanh(\eta) \dot{\eta} + \alpha_i k_c \Tanh(\dot{e}) \dot{e}$$

$$+ \frac{\alpha^2}{2k_i} \left[ \text{sat}(w_d(t)) - \text{sat}(\hat{w}_d(t)) \right]^T M^{-1}(q(t+T)) \left[ \text{sat}(w_d(t)) - \text{sat}(\hat{w}_d(t)) \right]$$

$$- \frac{\alpha^2}{2k_i} \left[ \text{sat}(w_d(t-T)) - \text{sat}(\hat{w}_d(t-T)) \right]^T M^{-1}(q) \left[ \text{sat}(w_d(t-T)) - \text{sat}(\hat{w}_d(t-T)) \right]$$

(39)

We can now use (13), (28) and (31), to simplify the expression for $V$ as follows:

$$\dot{V} \leq \Tanh(\eta) M^{-1}(q) (-K \Tanh(\eta) - \Tanh(\dot{e}) + N + \tilde{Y})$$

$$- \Tanh(\eta) M^{-1}(q) C(q, \dot{q}) \eta + \alpha_i k_c \Tanh(\dot{e})(\dot{e} - a_i \Tanh(\dot{e}))$$

$$- \frac{1}{2k_i} (\dot{w}_d + k_c \Tanh(\eta))^T M^{-1}(q) (\dot{w}_d + k_c \Tanh(\eta))$$

$$+ \frac{\alpha^2}{2k_i} \left[ \text{sat}(w_d(t)) - \text{sat}(\hat{w}_d(t)) \right]^T M^{-1}(q(t+T))$$

$$\times \left[ \text{sat}(w_d(t)) - \text{sat}(\hat{w}_d(t)) \right]$$

(40)

In light of (2)-(4) and (9), we have

$$\Tanh(\eta) M^{-1}(q) C(q, \dot{q}) \eta \leq \alpha \zeta_2 \| \eta \|^2$$

(41)

where $\zeta_2$ is some positive constant.

Substituting (25), (26) and (41) into (40), it follows that

$$\dot{V} \leq -\alpha^2 k_c \| \Tanh(\dot{e}) \|^2 - \Tanh(\eta) M^{-1}(q) K \Tanh(\eta)$$

$$+ m^{-1}(\alpha_1 \zeta_1 + \zeta_2) \| \eta \|^2 + \frac{m^{-1} + \alpha_i k_c}{2} \| \Tanh(\dot{e}) \|^2 + \| \eta \|^2$$

$$+ \alpha \zeta_2 \| \eta \|^2 - \frac{k_c}{2} \Tanh(\eta) M^{-1}(q) \Tanh(\eta)$$

$$- \frac{1}{2k_i} \left[ \text{sat}(w_d(t)) - \text{sat}(\hat{w}_d(t)) \right]^T M^{-1}(q(t+T)) \left[ \text{sat}(w_d(t)) - \text{sat}(\hat{w}_d(t)) \right]$$

(42)

where we have utilized Property 1.

Similar to Dixon et al. (2002), by exploiting the properties given in (19) and (2), and condition (37), we have

$$\dot{V} \leq -k_c \left[ \alpha_i (m^{-1} + \alpha_i) \right] \| \Tanh(\dot{e}) \|^2 - m^{-1} \lambda_m(K) \| \Tanh(\eta) \|^2$$

$$+ m^{-1}(\alpha_1 \zeta_1 + \zeta_2) \| \eta \|^2 + \frac{(m^{-1} + \alpha_i k_c + 2 \alpha \zeta_2)}{2} \| \eta \|^2$$

(43)

$$+ \frac{m^{-1} + \alpha_i k_c + 2 \alpha \zeta_2}{2} \| \eta \|^2 - \frac{\alpha_1 k_c}{2} \| \eta \|^2$$

Upon utilizing (35) and (36), the following advantageous expression for the upper bound of $\dot{V}$ can be developed

$$\dot{V} \leq -k_c \left[ \alpha_i (m^{-1} + \alpha_i) \right] \| \Tanh(\dot{e}) \|^2 - \frac{k_c}{2} \| \eta \|^2$$

(44)

$$- \alpha \zeta_2 k_c \left[ \zeta_1 + \zeta_2 \right] \| \eta \|^2 - \left( \zeta_1 + \zeta_2 \right) k_c \| \eta \|^2$$

(45)

Finally, upon completing the squares on the bracketed in (46), the final upper bound on $\dot{V}$ can be expressed as

$$\dot{V} \leq -\frac{k_c}{2} \| \eta \|^2 + \frac{\alpha_1 k_c}{4} \| \eta \|^2$$

(46)

Obviously, if $k_c$ is selected to satisfy condition given by (37) in Theorem 1, we can rewrite (47) as

$$\dot{V} \leq -\gamma \| \eta \|^2$$

(48)

where $\gamma$ is some positive constant.

As observed by Dixon et al. (2002), based on (27), (38) and (48), it is clear that $e(t), \eta(t) \in \ell_2 \cap \ell_{\infty}$. Based on the fact that $\eta(t) \in \ell_\infty$, it is clear from (13), (15) and (31) that $\hat{w}_d(t), \tilde{w}_d(t), \hat{e}(t) \in \ell_\infty$, and hence, $e(t)$ is uniformly continuous. Since $e(t) \in \ell_2 \cap \ell_{\infty}$ and uniformly continuous, we can now utilize Barbalat’s lemma (Slotine & Li, 1991) to show the result given by (32) for a neighborhood of the equilibrium state. Furthermore, from (44), and condition (36) and (37), we can see that the region of attraction can be increased arbitrarily by increasing the control gains $K$ and $k_i$. Hence, we have the semiglobal asymptotic link position tracking. This completes the proof.

$\Box$

Theorem 1 indicates that the proposed saturated learning-based repetitive controller does not utilize the modelling parameter in the control law formulation, which is the simplest and would give rise to semiglobal asymptotic link position tracking for robot manipulators in the presence of actuator saturation.
Simulations on the two-DOF robot used in (Dixon et al., 2002) were conducted to illustrate the effectiveness of the proposed simple saturated repetitive learning control. The entries to model the robot manipulator are, respectively

\[
\begin{bmatrix}
p_1 + 2p_3 \cos(q_2) & p_2 + p_3 \cos(q_2) \\
p_2 + p_3 \cos(q_2) & p_3 \\
-p_3 \sin(q_2) \dot{q}_2 & -p_3 \sin(q_2) (\dot{q}_1 + \dot{q}_2) \\
p_3 \sin(q_2) \dot{q}_1 & 0
\end{bmatrix}
\]

(49)

\[
D = \text{diag}(d_1, d_2), \quad g = [0, 0]^T
\]

with \( p_1 = 3.473 \), \( p_2 = 0.193 \), \( p_3 = 0.242 \), \( d_1 = 5.3 \), and \( d_2 = 1.1 \). The parameters are given in SI units. The desired trajectories were selected as follows:

\[
q_d(t) = \begin{bmatrix}
1.57 \sin(2t)(1 - e^{-0.05t}) \\
1.2 \sin(t)(1 - e^{-0.05t})
\end{bmatrix}
\]

(50)

By inserting (49), (50), and the above parameters into (11), the upper boundedness of \( \tau_d \) is as follows:

\[
\delta = [\delta_1, \delta_2]^T = [40, 10]^T
\]

(51)

The unbounded repetitive learning control (RLC) developed by Dixon et al. (2002) is selected for comparison and is

\[
\tau = Kr + \dot{\hat{w}}_d
\]

(52)

where \( K \in \mathbb{R}^{2 \times 2} \) is a positive constant control gain matrix, \( r \in \mathbb{R}^2 \) is the following filtered tracking error

\[
r = \dot{e} + \alpha e
\]

(53)

with \( \alpha \in \mathbb{R}^1 \) being a positive constant, and \( \dot{\hat{w}}_d \in \mathbb{R}^n \) is designed as

\[
\dot{\hat{w}}_d(t) = \text{sat}(\dot{\hat{w}}_d(t - T)) + k_I r
\]

(54)

with \( k_I \in \mathbb{R}^1 \) is a positive constant learning gain.

The sampling period was \( T_s = 1 \text{ ms} \) . We imposed \( \tau_{\text{max}} = [100, 50]^T \) Nm of actuators. The control gains of the proposed saturated repetitive learning control (SRLC) were determined as \( \alpha_1 = 15 \), \( \alpha_2 = 0.5 \), \( k_I = 10 \), \( \beta = [60, 15]^T \) and \( K = \text{diag}(55, 30) \). The gains of the unbounded RLC control were chosen as \( \alpha = 15 \), \( k_I = 10 \), \( \beta = [40, 10]^T \) and \( K = \text{diag}(55, 8) \). The initial conditions for both controllers were set as \( q(0) = (-1.0, 2.0)^T \) and \( \dot{q}(0) = (1.0, -1.5)^T \). Figs. 1 and 2 illustrate the position tracking errors and the requested input torques of the proposed SRLC. The results of the unbounded RLC are shown in Figs. 3 and 4. From the comparisons, we can see that both controllers drive the robot complete the motion successfully, and after a transient due to errors in initial condition, the position tracking errors tend asymptotically to zero. In comparison with the unbounded RLC, the proposed SRLC obtains a comparable tracking result. Furthermore, requested input torques using the proposed SRLC remain uniformly within the stated torque constraints, while the unbounded RLC control initially requests input torques that exceed the actuator limits.


