Controller Design for Automatic Micro-assembly Systems Under the Influence of Surface Forces, Hysteresis and Quantizer

Ruiyue Ouyang * Bayu Jayawardhana *

* Dept. Discrete Technology and Production Automation, University of Groningen, Groningen 9747AG, The Netherlands e-mail: r.ouyang@rug.nl; bayujw@ieee.org.

Abstract In this paper, we design nonlinear controllers for micro-magnetic levitation systems. The controller takes into account the influence of the surface forces and hysteresis/quantizer. We show the exponential convergence of the error signal and its time derivative to a compact set, which is determined by the uncertainty in the hysteresis and in the quantizer.

Keywords: Micro systems; Hysteretic feedback systems; Quantized feedback systems; Nonlinear controller design;

1. INTRODUCTION

Micro-assembly processes refer to the process of putting together various components where two of its dimensions are less than 0.1 millimeter (Fatikow (2008); Popa & Stephanou (2004)). In the current state-of-art technology of automatic micro-assembly, micro-mechanical manipulator is widely used to handle the object. In practice, the application of mechanical manipulator in micro-assembly encounters several problems: the presence of surface forces during the manipulation of micro/nano parts and the difficulty in sensing whether the object is in contact with or is released by the manipulator.

The papers (Fearing (1995); Menciassi, Eisinberg, Izzo & Drio (2004)) present an overview of the significance of van der Waals and capillary forces when the manipulated objects have dimensions less than one millimeter. Due to these surface forces, the micro/nano components can stick to the handling tools and become difficult to handle.

In order to overcome these problems, non-contact manipulation system using electromagnetic forces is studied in this paper. The electromagnetic force is applied to the object by manipulating the magnetic field through the control of electrical current in electromagnets. As a consequence, the influence of the hysteresis phenomenon must be taken into account in the controller design.

In the magnetic levitation systems for large objects, it is known that the coils’ inductance is influenced by the position of the ferromagnetic object. Thus the interaction between the applied voltage and the object position can be simply described by Euler-Lagrange formalisms (Khalil (2000); Ortega, Lória, Nicklasson & Sira-Ramírez (1998)). Several controller designs based on this formulation have been proposed, for example, in Bonivento, Gentili & Marconi (2005); Gentili & Marconi (2003); Ortega, Lória, Nicklasson & Sira-Ramírez (1998).

When the aspect ratio between the object and the coil size is large, the influence of the object distance to the inductance value becomes negligible. In this case, the applied electromagnetic force is computed based on the magnetic field gradient in the neighborhood of the object. This mechanism was used in the magnetic levitation system for a small robot in Khamese, Kato, Nomura & Nakamura (2002) and corresponds to the setup considered in this paper. Due to the different formalisms, the controller design as proposed in Bonivento, Gentili & Marconi (2005); Gentili & Marconi (2003); Ortega, Lória, Nicklasson & Sira-Ramírez (1998) cannot be directly applied to such setup.

In this paper, we focus on the design of nonlinear tracking controllers for such system which also takes into account the effect of hysteresis and quantizer. By analyzing the error system, we show that the tracking error converges to a compact set, which depends on the degree of uncertainty in the hysteresis and in the quantizer.

In Section 2, we discuss the modeling of the magnetic levitation system which incorporates the model of various surface forces that are dominant in the micro world: capillary forces, electro-static forces and van der Waals forces. In Section 3, we introduce hysteresis and quantizer operators. In Section 4, we design nonlinear tracking controller in dealing with uncertainty in the hysteresis and quantizer operator which can be embedded into set-valued mapping. The resulting controller ensures the convergence of the error signal and its time derivative to a compact set.
2. SYSTEMS MODELING

For a very small ferromagnetic ball with magnetic moment \( p \) inside a magnetic field \( B \), the magnetic force \( F \) and the magnetic torque \( T \) can be calculated by using the following equations:

\[
F_{\text{magnetic}} = (p \cdot \nabla)B, \quad T = p \times B, \quad (1)
\]

where the symbol \( \cdot \) denotes the dot product and the symbol \( \times \) denotes the cross product. These equations show that manipulation of the magnetic field can be used to regulate the position and orientation of the object by controlling the force and the torque.

We assume that additional controlled-coils have been designed such that the ball always remains at the center of the \( xy \)-plane. Hence, the ball movement is restricted to \( z \) direction. By manipulating the electrical current in the primary coil, the influence of surface forces will be compensated and the object position can be controlled in \( z \) direction. However, due to the property of ferromagnetic core, the hysteresis nonlinearity from the current \( I \) to the generated magnetic field \( B \) is present in the system.

![Figure 1. Three-dimensional illustration of various forces influencing the object.](image)

Based on this configuration, let us describe the dynamical equations of the object in the \( z \)-direction. Using Newton’s law, the object’s movement can be described by

\[
m\ddot{z} = F_{\text{mag}}(z,I) - F_{\text{cap}}(z) - F_g - F_{\text{es}}(z) - F_{\text{intmol}}(z), \quad (2)
\]

where \( m \) is the object’s mass, \( z(t) \) is the distance between the bottom of the sphere to the surface and \( I(t) \) is the applied current. Here, the external forces are the electromagnetic forces \( F_{\text{mag}} \), the capillary forces \( F_{\text{cap}} \), the gravity force \( F_g \), the electro-static force \( F_{\text{es}} \) and the intermolecular forces \( F_{\text{intmol}} \) which includes the van der Waals forces and the repulsive forces. We can now describe the modeling for each of these forces.

Based on the assumption that the ferromagnetic object is always at the centre of \( xy \)-plane, Figure 2 shows the relation between the primary coil and the object. We assume that the primary coil has the radius of \( R_{\text{coil}} \) and is placed at distance of \( L \) from the surface. By Biot-Savart law for magnetic field, the magnetic field \( B \) at the position \( z \) is given by

\[
B(z,I) = \frac{\Phi(I)R_{\text{coil}}^2}{2(R_{\text{coil}}^2 + (L - z)^2)^{3/2}},
\]

where \( \Phi \) is a hysteresis operator that describes the magnetization of the ferromagnetic core. Applying (1) with the magnetic field above, we obtain

\[
F_{\text{mag}}(z,I) = \left( p \cdot \frac{d}{dz} \right)B(z,I) = \frac{3p\Phi(I)R_{\text{coil}}^2(L - z)}{2(R_{\text{coil}}^2 + (L - z)^2)^{3/2}}, \quad (3)
\]

where \( p \) is the magnetic moment of the ferromagnetic object.

For the second term on the right hand side of (2), the capillary force is modeled by the generalized logistic function:

\[
F_{\text{cap}}(z) = \alpha \left[ 1 - \frac{1}{1 + \exp(-\kappa(z-d))} \right], \quad (4)
\]

where \( \alpha \) is the capillary force in the coalescence state (i.e., when the object is at the surface and there is water condensate at the interface (Israelachvili (1991))), \( \kappa \) is the approximate rate of decrease of the force with respect to \( z \) and \( d \) is the approximate distance when the capillary bridge becomes instable. This approximation is based on the experimental observation of instabilities in the microscopic capillary bridges (Maeda, Israelachvili & Kohonen (2003)).

The gravity force is simply given by

\[
F_g = mg, \quad (5)
\]

where \( g \) is the gravity acceleration constant and \( m \) is the mass of the sphere.

The electro-static force which is caused by the free charges contained in the sphere and surface, can be represented by (see also, Van Striph, Langent & Onosato (2006))

\[
F_{\text{es}}(z) = \frac{4\pi\sigma^2}{\varepsilon_0(z + 2r)^2}, \quad (6)
\]

where \( \sigma \) is the permittivity of the free space, \( \varepsilon \) is the dielectric constant of the medium and \( r \) is the radius of the sphere.

For the intermolecular forces \( F_{\text{intmol}} \) in (2), we assume that it can be decomposed into two different forces: the van der Waals force and the repulsive force, which are approximated based on the Lennard-Jones potential. The latter is the weak force that maintains the distance between two different molecules. Based on this assumption, the intermolecular forces can be given by
\[ F_{\text{intmol}}(z) = F_{\text{vdw}}(z) - F_{\text{rep}}(z), \]
where \( F_{\text{vdw}} \) and \( F_{\text{rep}} \) are the functions which represent the van der Waals forces and the repulsive forces, respectively.

By using the Lennard-Jones potential function and by assuming that the object takes the form of a sphere and the surface is flat, the van der Waals force can be given by
\[ F_{\text{vdw}}(z) = \frac{A_{\text{ham}}}{6} \left( \frac{r}{z^2} + \frac{r}{(2r + z)^2} \right), \quad (7) \]
where \( A_{\text{ham}} \) is known as the Hamaker constant. Israelachvili in Israelachvili (1991) provides a nominal value of this constant for different materials of the ball and the surface.

Similarly, using the same configuration of a sphere and a flat surface and using the Lennard-Jones potential function, the repulsive force is given by (see also, Israelachvili in Israelachvili (1991))
\[ F_{\text{rep}}(z) = \frac{7A_{\text{rep}}r}{z^8}, \quad (8) \]
where \( A_{\text{rep}} \) is the repulsive force constant. This repulsive force is dominant when the distance \( z << r \).

The inclusion of repulsive force above prevents the object to coalesce with the surface. In other words, \( z \) is always greater than 0. There is no need to include a hard constraint in the dynamical equation, in contrast to the dynamical model used in Rollot, Régnier & Guinot (1999).

Substituting (3), (4), (7), (5), (6) and (8) into (2), we obtain the dynamical equation of the magnetic levitation systems in the \( z \)-direction as follows
\[ m \ddot{z} + a \left[ 1 - \frac{1}{1 + \exp(-c(z-a))} \right] + \frac{A_{\text{ham}}}{6} \left( \frac{r}{z^2} + \frac{r}{(2r + z)^2} - \frac{2r}{z(2r + z)} \right) - \frac{7A_{\text{rep}}r}{z^8} + mg + \frac{4\pi r^4 \sigma^2}{\epsilon \sigma_0(z+2r)^2} = \frac{3\rho \Phi(I) R_{\text{coil}}^2(L-z)}{2(R_{\text{coil}}^2 + (L-z)^2)^2}. \quad (9) \]

3. HYSTERESIS OPERATOR AND QUANTIZER
An operator \( \Phi : C(\mathbb{R}_+) \to C(\mathbb{R}_+) \) is said to be causal if, for all \( \tau \geq 0 \) and all \( v_1, v_2 \in C(\mathbb{R}_+) \), \( v_1 = v_2 \) on \([0, \tau]\) implies that \( \Phi(v_1) = \Phi(v_2) \) on \([0, \tau]\).

An operator \( \Phi : C(\mathbb{R}_+) \to C(\mathbb{R}_+) \) is called rate independent if, for every time transformation \( f \),
\[ (\Phi(u \circ f))(t) = (\Phi(u)(f(t))), \forall u \in C(\mathbb{R}_+), \forall t \in \mathbb{R}_+, \]
where a function \( f : \mathbb{R}_+ \to \mathbb{R}_+ \) is called a rate transformation if it is continuous, non-decreasing and surjective.

We say that \( \Phi : C(\mathbb{R}_+) \to C(\mathbb{R}_+) \) is a hysteresis operator if \( \Phi \) is causal and rate independent.

In our main result, we are interested in the hysteresis operator \( \Phi : C(\mathbb{R}_+) \to C(\mathbb{R}_+) \) which satisfies the following assumption
\[ (A) \quad \text{There exist } c_1, c_2 > 0 \text{ such that for every } u \in C(\mathbb{R}_+), \]
\[ |(\Phi(u))(t) - c_1 u(t)| \leq c_2 \quad \forall t \in \mathbb{R}_+. \quad (10) \]
An equivalent representation of (A) is that for every \( u \in C(\mathbb{R}_+), \Phi(u)(t) \in [c_1 u(t) - c_2, c_1 u(t) + c_2] \) for all \( t \). In other words, \( \Phi \) can be embedded into set-valued mapping which lies in a strip.

In the following, we describe several hysteresis operators and quantizer operator for \( \Phi \) in (9) which satisfies (A).

Backlash operator. The backlash (or play) operator, which is widely used in mechanical models (for example, gear trains), can be found in a number of references, see, for example, Logemann & Ryan (2003). Let \( h \in \mathbb{R}_+ \) and introduce the function \( b_h : \mathbb{R}^2 \to \mathbb{R} \) by
\[ b_h := \max\{v_1 - h, \min\{v_1 + h, v_2\}\}, \quad (11) \]
Let \( C_{\text{eq}}(\mathbb{R}_+) \) denote the space of continuous piecewise monotone functions defined on \( \mathbb{R}_+ \). For all \( h \in \mathbb{R}_+ \) and all \( \eta \in \mathbb{R} \),
\[ (B_{h, \eta}(u))(0) := b_h(u(0), \eta) \]
\[ (B_{h, \eta}(u))(t) := b_h(u(t), (B_{h, \eta}(u))(t_1)) \]
where \( 0 = t_0 < t_1 < t_2 < \ldots \) is a partition of \( \mathbb{R}_+ \), such that \( u \) is monotone on each of the intervals \([t_{i-1}, t_i], i \in \mathbb{N}\). Here \( \eta \) plays the role of an "initial state".

Prandtl operator. Let \( g : \mathbb{R}_+ \to \mathbb{R} \) be a compactly supported and globally Lipschitz function with Lipschitz constant \( 1 \) and let \( E \) be a compact set on \( \mathbb{R}_+ \). In this paper, we are interested in a class of Prandtl operator \( \Phi \) defined by
\[ (\Phi_{\varphi}(u))(t) = \int_E (B_{h_{\varphi}(u)}(u))(d\varphi) \]
\[ \forall u \in C(\mathbb{R}_+), \forall t \in \mathbb{R}_+. \quad (13) \]
The Prandtl operator in (13) is related to the one in Logemann & Ryan (2003). The function \( g \) describes the initial condition in the Prandtl operator. For simplicity, we consider \( g = 0 \) for the rest part of this paper and we denote the corresponding Prandtl operator by \( \Phi \) instead of \( \Phi_0 \). We remark that for every \( u \in C(\mathbb{R}_+) \), the backlash operator in (11) satisfies
\[ |(B_{h, \eta}(u))(t) - u(t)| \leq h \quad \forall t \in \mathbb{R}_+. \quad (14) \]
Based on this observation, we derive the following lemma for the Prandtl operator.

Lemma 1. Let us consider the Prandtl operator \( \Phi \) in (13). Then (A) holds with \( c_1 \) and \( c_2 \) be given by \( c_1 := \int_E d\varphi \) and \( c_2 := \int_E h d\varphi \).

The proof of the lemma follows directly from (13) and (14).

Quantizer. We denote \( q_{\eta} : \mathbb{R} \to \mathbb{R} \), parameterized by \( \eta > 0 \), is a uniform quantizer given by
\[ q_{\eta}(v) = \frac{2m\eta}{\epsilon \eta} \quad v \in (2m-1)\eta, (2m+1)\eta], \quad m \in \mathbb{Z}. \quad (15) \]

Lemma 2. Let us consider the uniform quantizer operator \( \Phi : u \mapsto q_{\eta}(u) \) with \( \eta > 0 \). Then (A) holds with \( c_1 \) and \( c_2 \) be given by \( c_1 := 1 \) and \( c_2 := \eta \).

When \( \Phi \) is the composition of the Prandtl hysteresis and quantizer, the hypothesis (A) still holds for \( \Phi \).

4. CONTROLLER DESIGN FOR (A)

Let us define
\[ N(z) = \frac{2(R_{\text{coil}}^2 + (L - z)^2)^{\frac{3}{2}}}{3pR_{\text{coil}}(L - z)}, \]
\[ M(z) = mN(z), \]
\[ G(z) = N(z) \]
\[ \alpha \]
\[ \frac{1}{1 + \exp(-\kappa(z - \lambda e))} \]
\[ 7A_{\text{mean}} \]
\[ mg + \frac{4\pi^4\sigma^2}{\epsilon_0(z + 2r)^2} \]
and (9) can be rewritten into
\[ M(z)\ddot{z} + G(z) = \Phi(I). \]  

We assume that the domain of \( z \) is \((\rho, L - \rho)\) for sufficiently small \( \rho > 0 \). It follows that there exist \( m_1, m_2 > 0 \) such that \( \frac{1}{m_1} \leq M(z) \leq m_2 \).

Note that while (16) has the same form of a fully actuated mechanical system (see, for example, Astolfi, Limebeer, Melchiorri, Tornambe & Vinter (1997); Jayawardhana & Weiss (2008); Ortega, Loria, Nicklasson & Sira-Ramírez (1998)), it does not satisfy the standard assumption on the mass, where for the one-dimensional generalized coordinate they assume that \( M(z) - 2C(z, \dot{z}) = 0 \) with \( C(z, \dot{z})\dot{z} \) be the coriolis and centrifugal forces. Hence the controller proposed in Jayawardhana & Weiss (2008); Ortega, Loria, Nicklasson & Sira-Ramírez (1998) cannot be used directly.

**Proposition 3.** Consider the system in (9) with state \( x := [z, \dot{z}]^T \), where \( x \in (\rho, L - \rho) \times \mathbb{R} \). Assume that the reference signal \( \text{ref}_t \in C^1(\mathbb{R}_+, (\rho, L - \rho)) \) and define \( x_{\text{ref}} = [\text{ref}_t, \text{ref}_t]^T \). Suppose that \( \Phi : C(\mathbb{R}_+) \to C(\mathbb{R}_+) \) satisfies (A) with \( c_1, c_2 > 0 \). Let the control input \( I \) be given by
\[ I = \frac{1}{c_1} \left( M(z)\ddot{\zeta} + G(z) - K_d\zeta - \frac{1}{2}M(z)\zeta \right), \]
where \( K_d > 0 \),
\[ \zeta = \text{ref}_t + \lambda(z_{\text{ref}} - z), \quad \lambda > 0, \]
and \( \xi = \dot{z}_{\text{ref}} - \dot{z} - \xi \).

Then for sufficiently small \( c_2 \), there exist \( \epsilon > 0 \) and \( \theta > 0 \) such that for every \( x(0) \in (\rho, L - \rho) \times \mathbb{R} \),
\[ \|x_{\text{ref}}(t) - x(t)\| \leq \|x_{\text{ref}}(0) - x(0)\|e^{-\epsilon t} + \theta|c_2|^2 \quad \forall t \geq 0. \]

In particular, if \( c_2 = 0 \) (i.e., \( \phi \) is a linear function) then \( \|x_{\text{ref}}(t) - x(t)\| \to 0 \) as \( t \to \infty \).

**Proof.** Using \( \Phi \), (16) can be expressed as
\[ M(z)\ddot{\zeta} + G(z) = c_1 I + (\Phi(I) - c_1 I). \]

By substituting (17) into (21) and by using (18)-(19), we get
\[ M(z)\ddot{\zeta} = -K_d\zeta - \frac{1}{2}M(z)\zeta + \Phi(I) - c_1 I. \]

By denoting \( e := z_{\text{ref}} - z \), it follows from (18)-(19) that \( \zeta = -\dot{e} - \lambda e \).

Since the domain of \( z, z_{\text{ref}} \) is \((\rho, L - \rho)\), the domain of \( e \) is \((-L + 2\rho, L - 2\rho)\).

Using \( e \) and \( \zeta \) as the new state variables and denoting \( w := [\dot{\zeta}] \) as the state vector, (22) can be rewritten into
\[ M_r(e, t)\dot{\zeta} = -K_d\zeta - \frac{1}{2}M_r(e, t)\zeta + (\Phi(I) - c_1 I), \]
where \( M_r(e, t) = M(z_{\text{ref}} - e) \).

Let
\[ V(e, \zeta, t) := \frac{1}{2}M_r(e, t)\zeta^2 + \frac{\lambda K_d}{2}e^2. \]

It can be checked that \( V \) satisfies
\[ \alpha_1(x) \leq V(e, \zeta, t) \leq \alpha_2(x) \]
where
\[ \alpha_1(x) := \frac{1}{2}m_1\|\zeta\|^2 + \frac{\lambda K_d}{2}\|e\|^2, \]
\[ \alpha_2(x) := \frac{1}{2}m_2\|\zeta\|^2 + \frac{\lambda K_d}{2}\|e\|^2. \]

By using \( V \), (23) and (24), it follows that
\[ \dot{V} = \langle \zeta, M_r \dot{\zeta} \rangle + \frac{1}{2} \langle \zeta, \dot{M}_r \zeta \rangle + \lambda K_d \langle e, e \rangle \]
\[ = \langle \zeta, (\Phi(I) - c_1 I) \rangle - \langle \zeta, K_d \zeta \rangle + \lambda K_d \langle e, -\zeta - \lambda e \rangle \]
\[ \leq c_2\|\zeta\| - \frac{\lambda K_d}{2} \left( \lambda^2 K_d \right) \|e\|^2, \]
\[ \leq c_2\|\zeta\| - c_1\|\zeta\|^2 - \epsilon_1\|e\|^2, \quad \epsilon_1 > 0, \]
\[ \leq \frac{c_2^2}{2}\|e\|^2 - (c_1 - \frac{1}{2}\epsilon_2)\|\zeta\|^2 - \epsilon_1\|e\|^2, \quad \epsilon_1, \epsilon_2 > 0, \]
\[ \leq \frac{|c_2|^2}{4(\epsilon_1 - \delta)} - \delta\|w\|^2, \]

where the last inequality holds if \( 0 < \delta < \epsilon_1 \), in which case, \( \epsilon_2 = \frac{4\delta(\epsilon_1 - \delta)}{(\epsilon_1 - \delta)} \). Therefore, if \( \frac{|c_2|^2}{4(\epsilon_1 - \delta)} < (L - 2\rho)^2 \) then there exists \( \epsilon_3, \epsilon_4 > 0 \) such that
\[ \dot{V} \leq -\epsilon_3\|w\|^2 \]
whenver \( \frac{|c_2|^2}{4(\epsilon_1 - \delta)} + \epsilon_4 < \|w\|^2 < (L - 2\rho)^2 \).

By applying Theorem 5.2 from Khalil (2000), we conclude that there exist \( \epsilon_5, \epsilon_6 > 0 \) such that
\[ \|w(t)\| \leq \|w(0)\|e^{-\epsilon_5 t} + \epsilon_6|c_2|^2 \quad \forall t \geq 0, \]
holds for sufficiently small \( c_2 < (L - 2\rho)\sqrt{4\delta(\epsilon_1 - \delta)} \).

The claim of the theorem follows from the definition of \( e \) and (23).

Note that the assumption (A) is equivalent to embedding \( \Phi \) into a set-valued mapping which depends on \( I \), i.e., \( \Phi(I)(t) \in [c_1I(t) - c_2, c_1I(t) + c_2] \) for all \( t \). For simplicity of notation, we do not treat the problem in the differential inclusion setting. However, the proof in the differential inclusion setting follows the same arguments.

**Remark 4.1.** We remark here that if there exists external disturbance in the system, i.e., an external disturbance signal is added to (16), then using the same controller as in Proposition 3, we can have a practical local input-to-state stability of the closed-loop system for sufficiently small disturbance. See Khalil (2000) for the concept of input-to-state stability and Jiang, Teel & Praly (1994) for the concept of practical input-to-state stability.
In the simulation, the levitated object is a sphere of iron oxide ($\text{Fe}_3\text{O}_4$) with radius of $r = 100\mu m$, mass of $m = 0.263\mu g$ and the magnetic moment of $p = 1.75\mu J/T$. By assuming that the surface is made of silicon, the Hamaker constant is assumed to be $A_{\text{ham}} = 11.68 \times 10^{-20}\text{J}$ (Israelachvili (1991)). The repulsive constant is approximated by computing the molecular radii. This approximation gives us $A_{\text{rep}} = 9.2029 \times 10^{-79}\text{Jm}^6$. Consider the medium between the sphere and the surface is air, then $\epsilon = 1$. The charge density of the iron oxide ball is approximate to be $\sigma = 0.2\mu \text{Cm}^{-2}$.

The capillary forces is approximated according to the experimental observations in Israelachvili (1991); Maeda, Israelachvili & Kohonen (2003). For the value of $\alpha$, we take the maximum capillary force for a given sphere with radius $r$. Based on the calculation, $\alpha = 9\mu\text{N}$. The distance $d$ in (4) which describes the point of capillary bridge instability is approximated according to the measurement in Maeda, Israelachvili & Kohonen (2003). We assume $d = 30\text{nm}$ and $\kappa = 10^9\text{m}^{-1}$.

For the coil, we assume that $L = 10\text{mm}$ and $R_{\text{coil}} = 2\text{mm}$. The permeability is given by $\mu = 47 \times 10^{-6} \text{N/A}^2$.

The control law is as given in Section 4. With this controller, we evaluate the performance of the controller for regulating the object position. The reference signal $x_{\text{ref}}$ is a twice-differentiable signal, which is shown in Figure 3. It is a smoothed version of a piecewise-continuous signal with two desired constant positions: $7.5\mu m$ and $600\mu m$.

The simulation result shows the ability of the nonlinear controller to regulate the ball position close to the desired position in the presence of input nonlinearity. Figure 4(a) presents the error plot when the input nonlinearity is a backlash operator with the width of 0.6. Figure 4(b) presents the error plot when the input nonlinearity is a backlash operator with the width of 1.2. We remark that the width of the hysteresis operator is chosen based on the ferromagnetic property of the material for the core of the coils, which is assumed to be mu-metal.

As introduced in Section 2, the Prandtl operator is built based on the backlash operator (integration of the backlash operator). Here we also give the simulation results for the Prandtl operator for two different $E$, where $E = [0, 0.6]$ and $E = [0, 1.2]$. The results are shown in Figure 5.

6. DISCUSSIONS

In this paper, we focus on the controller design for the micro-assembly system which takes into account various nonlinearity that comes from surface forces, as well as, the hysteresis. The simulation results suggest that the nonlinear controller can steer the ball close to the desired trajectory and is robust to the nonlinearity due to ferromagnetic core.

REFERENCES


Figure 3. The reference trajectory

Figure 4. The plot of tracking errors for two different width of backlash operator. The control parameters are $Kd = 0.9 \times 10^4$, $\lambda = 200$ (a). Backlash operator $(B_{h,\eta}(u))(t)$, where $h = 0.6$ and $\eta = 0$. (b). Backlash operator $(B_{h,\eta}(u))(t)$, where $h = 1.2$ and $\eta = 0$.
Figure 5. The plot of tracking errors for two different width of Prandtl operator. The control parameters are $Kd = 10^6$, $\lambda = 180$ (a). Prandtl operator with $h = 0.6$ and $\rho = 0$. (b) Prandtl operator with $h = 1.2$ and $\rho = 0$.


