Nonlinear feedback control of Vehicle Speed

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Abstract: This paper deals with the development of a nonlinear static state feedback control applied to the vehicle speed. The control is obtained by suitable change of variable and after some preliminary feedback. The main objective in this study is the stabilization of vehicle velocities, i.e. longitudinal, lateral and yaw rate using Lyapunouv stability theory and LaSalle invariance principle.

Keywords: Nonlinear control, stabilization, vehicle dynamics, simulation.

NOMENCLATURE

|M| Vehicle mass.
|g| Gravitational acceleration.
|CG| Centre of gravity of the vehicle.
|a, b| Distances from CG to front and rear wheel axes.
|h| Height of CG.
|k_{ax}, k_{ay}| Aerodynamic coefficients in x and y directions.
|f| Rolling friction coefficient.
|f_r| Force distribution coefficient.
|F_l| Longitudinal (Acceleration or braking) forces.
|C_f| Cornering stiffness coefficients of front tires.
|C_r| Cornering stiffness coefficients of rear tires.
|δ| Steering front wheels angle.
|ψ| Yaw angle.
|V_ψ = ˙ψ| Yaw rate.
|V_x, V_y| Longitudinal and lateral velocity in x and y directions.
|I_z| Yaw moment inertia.

1. INTRODUCTION

Models considered to describe the vehicle dynamics are highly nonlinear and do not raise any kind of the traditional classes of the nonlinear systems studied in the literature. Indeed, the vehicle dynamics has various properties which make that their control is a difficult and interesting problem.

There exist several types of models for the longitudinal, lateral and yaw motions, see Abbassi et al. (2007) and B. D’andrea Novel et al. (2005). The majority of the movements are characterized by an absolutely nonlinear behavior which does not allow to use the traditional reliable linear methods, such pole placement. Several authors were interested in the problem of control of vehicle dynamics. D.C. Liaw (2008) has proposed a feedback linearization for the control of the lateral dynamic. However, deficiencies associated with the linear approaches appears when one moves away from the set point of linearization.

In this work, we are interested by the problem of the control of the vehicle speed by feedback. The originality here consist to consider all the inherent non linearities of the system without any kind of linearization.

Note that, the problem of the control in nonlinear systems has been a subject of research for several authors. Thus, a great number of results were proposed. These results are in the form of necessary, as in R. W. Brockett et al. (1983) and J. Tsinias (1989), or sufficient conditions. Sufficient ones, are closed to some classes of systems, like homogeneous systems or partially linear systems, see Z. Arstein (1983), or dissipative systems in V. Jurjevic et al. (1978) and R. Outbib et al. (1999) etc.

The paper is organized as follows. We present a nonlinear vehicle model suitable for the present study. Next, some variables transformations are proposed to simplify the structure of the model. Then, a feedback nonlinear control for the stabilization of the velocities is proposed. Simulation results showing various speeds of the controlled vehicle are presented.

2. VEHICLE MODEL

There exist several different types of models for the lateral and yaw motions of a vehicle, but, a model suitable for the present study is derived from Abbassi et al. (2007) and A. Alloum et al. (1995). The vehicle dynamics are developed in two coordinate directions x, y and one rotation around the z axis. Besides, the vehicle is considered as a front wheels driving and steering. The equations of motion are given by:
\[
\begin{align*}
\dot{V}_x &= \frac{1}{M} \left( F_l + c_1 + MV_y V_y + C_f \delta V_y + a C_f \delta V_x + k_{ax} V_x^2 \right) \\
\dot{V}_y &= \frac{1}{M} \left( F_l - MV_x V_y + C_f \delta + c_2 V_y \right) + a V_y \frac{V_y}{V_x} - k_{ay} V_y^2 \\
\dot{V}_\psi &= \frac{1}{I_z} \left( a F_l \delta + c_4 V_x V_y + a C_f \delta + c_5 \right)
\end{align*}
\]

where \( k_{ax} = \frac{1}{2} \rho C_a S_l \) and \( k_{ay} = \frac{1}{2} \rho C_a S_l \) are the aerodynamic drag coefficient, \( \rho \) is the mass density of air, \( C_a \) is the aerodynamic drag coefficient. \( S_l \) and \( S_t \) are respectively the front and lateral area of the vehicle.

The constants \( c_i \) are defined as:
\[
\begin{align*}
c_1 &= -M f_g, \quad c_2 = -(C_f + C_r) \\
c_3 &= c_5 = -(a C_f - b C_r), \quad c_4 = -M f_h \\
c_6 &= -(a^2 C_f + b^2 C_r)
\end{align*}
\]

with \( F_l \) and \( \delta \) denote the control inputs of the system.

### 3. VARIABLES TRANSFORMATION

For technical reasons of computation as in R. Outbib et al (2000), we carry out the following transformation
\[
\hat{V}_\psi = V_y \hat{I}_z - MV_y \Leftrightarrow \hat{V}_\psi = \left( \hat{V}_y + MV_y \right) \frac{a}{\hat{I}_z}
\]

The model (1) becomes
\[
\begin{align*}
\dot{V}_x &= \frac{1}{M} \left( c_1 + \frac{Ma}{\hat{I}_z} V_y \hat{V}_\psi + \frac{a M^2}{\hat{I}_z} V_y^2 - k_{ax} V_x^2 + a C_f \hat{V}_\psi \right) + F_l + C_f \delta \\
\dot{V}_y &= \frac{1}{M} \left( -\frac{Ma}{\hat{I}_z} V_x \hat{V}_\psi - \frac{M^2 a}{\hat{I}_z} V_y V_x + c_2 V_y \right) \\
&\quad + c_3 \frac{a}{\hat{I}_z} V_y + c_4 \left( \frac{Ma}{\hat{I}_z} \right) V_y + k_{ay} V_y^2 + a C_f \hat{V}_\psi \left( \frac{c_6}{a} - c_3 \right) + a M \frac{V_y}{\hat{I}_z} \\
\dot{\hat{V}}_\psi &= \frac{1}{\hat{I}_z} \left( c_5 \frac{\hat{V}_y}{\hat{V}_x} + a M \hat{V}_\psi + \frac{a M}{\hat{I}_z} \left( \frac{c_4}{a} + M \right) V_x \right)
\end{align*}
\]

The preliminary feedback is defined by:
\[
\begin{align*}
\sigma &= F_l + C_f \delta \Theta \\
\gamma &= F_l \delta + C_f \delta
\end{align*}
\]

where \( \Theta \) indicates the following quantity
\[
\Theta = \frac{V_y}{V_x} + \frac{a^2 \hat{V}_\psi}{\hat{I}_z \V_x} + a \frac{a^2 V_y}{\hat{I}_z V_x}
\]

According to the second equation (4), we have
\[
\gamma \Theta = \left( \frac{F_l}{C_f} + 1 \right) C_f \delta \Theta
\]

By using the first equation of (4), the equation (5) becomes
\[
\gamma \Theta = \left( \frac{F_l}{C_f} + 1 \right) (\sigma - F_l)
\]

or
\[
\frac{F_l^2}{C_f} + F_l \left( 1 - \frac{\sigma}{C_f} \right) + \gamma \Theta - \sigma = 0
\]

By a similar computation, we obtain for \( \delta \)
\[
C_f \delta^2 - \delta(C_f + \sigma) + \gamma = 0
\]

Finally, the control variables \( F_l \) and \( \delta \) can be calculated according to \( \sigma \) and \( \gamma \) to leave equations (7) and (8) have real and pertinent solution to the physical field. Precisely, the equation (7) has a real solution if its discriminant (equation considered as a polynomial of degree two in \( F_l \)) is positive, then :
\[
\Delta_1 = \left( 1 - \frac{\sigma}{C_f} \right)^2 - \frac{4}{C_f} (\gamma \Theta - \sigma) \geq 0
\]  

(C1)

In other words, it is necessary for all the following study to ensure that the condition (C1) is satisfied. Then, we have to choose among the two possible solutions according to physical criteria.

\[
F_l = \frac{\sigma - C_f}{2} \pm \frac{C_f}{2} \sqrt{\Delta_2}
\]

(S1)

The equation (8), considered as a polynomial of second degree in \( \delta \), has a real solution if its discriminant is positive
\[
\Delta_2 = (C_f + \sigma)^2 - 4 \gamma C_f \Theta \geq 0
\]  

(C2)

The real solution will be given by
\[
\delta = \begin{cases} 
\left( \frac{C_f + \sigma}{2 C_f} - Sgn \left( \frac{C_f + \sigma}{2 C_f} \right) \sqrt{\Delta_2} \right) / 2C_f \Theta & \text{if } \Theta \neq 0 \\
\frac{\gamma}{C_f + \sigma} & \text{elsewise}
\end{cases}
\]

(S2)

where \( Sgn \) indicates the sign function. It is introduced to keep only the acceptable solution in steering angle \( \delta \) (when \( \Theta \neq 0 \)).

Notice that \( \delta \) is defined as continuous function. Indeed, to prove it, we can use the following approximation
\[
\sqrt{a - x} \approx \sqrt{a} - \frac{1}{2} \sqrt{a} \frac{x}{a} 
\]

for \( x \) in a neighborhood of zero and \( a \) one positive constant.
Now we try to solve the initial problem of the stabilization by a feedback control on the system (3) where the control variables are given by \((\sigma, \gamma)\) and after that, we use the equations (S1) and (S2) to deduce the values of the origin variables, \(F_1\) and \(\delta\). However, during the synthesis of the feedback control \((\sigma, \gamma)\), it is necessary to check the both conditions (C1) and (C2).

By using the expressions of \(F_1\) and \(\delta\) involving \(\sigma\) and \(\gamma\), the system (3) becomes

\[
\begin{align*}
\dot{V}_x &= \frac{1}{M} \left( c_1 + \frac{Ma}{I_z} V_y \dot{V}_p + \frac{aM^2}{I_z} V_y^2 - k_{ax} V_x^2 + \sigma \right) \\
\dot{V}_y &= \frac{1}{M} \left( -\frac{Ma}{I_z} V_x \dot{V}_p - \frac{aM^2}{I_z} V_x V_y + c_2 V_y \right) \\
\dot{V}_p &= \frac{a}{I_z} \left[ \frac{c_4}{a} + M \right] V_x \dot{V}_p + \frac{aM}{I_z} \left[ \frac{c_4}{a} + M \right] V_x V_y \\
&\quad + \left( \frac{c_6}{a} - c_2 \right) V_y + \frac{a}{I_z} \left[ \frac{c_6}{a} - c_3 \right] \frac{\dot{V}_y}{V_x} \\
&\quad + \frac{aM}{I_z} \left[ \frac{c_6}{a} - c_3 \right] \frac{V_y}{V_x} + k_{ay} V_y^2
\end{align*}
\]

(9)

Let

\[
\sigma_1 = \frac{1}{M} \left( c_1 + \frac{Ma}{I_z} V_y \dot{V}_p + \frac{aM^2}{I_z} V_y^2 - k_{ax} V_x^2 + \sigma \right)
\]

and

\[
\gamma_1 = \frac{1}{M} \left( -\frac{Ma}{I_z} V_x \dot{V}_p - \frac{aM^2}{I_z} V_x V_y + c_2 V_y \right) \\
+ \frac{a}{I_z} \left[ \frac{c_6}{a} - c_3 \right] \frac{V_y}{V_x} + k_{ay} V_y^2
\]

(10)

the system (9) becomes

\[
\begin{align*}
\dot{V}_x &= \sigma_1 \\
\dot{V}_y &= \gamma_1 \\
\dot{V}_p &= \frac{a}{I_z} \left[ \frac{c_4}{a} + M \right] V_x \dot{V}_p + \frac{aM}{I_z} \left[ \frac{c_4}{a} + M \right] V_x V_y \\
&\quad + \left( \frac{c_6}{a} - c_2 \right) V_y + \frac{a}{I_z} \left[ \frac{c_6}{a} - c_3 \right] \frac{V_y}{V_x} \\
&\quad + \frac{aM}{I_z} \left[ \frac{c_6}{a} - c_3 \right] \frac{V_y}{V_x} + k_{ay} V_y^2
\end{align*}
\]

(11)

The method consists now to stabilize this system under the conditions (C1) and (C2). The procedure is displayed in two steps. The first one consists to check that for \(\sigma_1 = \gamma_1 = 0\), both conditions (C1) and (C2) are satisfied. In the second step, one synthesizes a control law with \(|\sigma_1| \leq \epsilon\) and \(|\gamma_1| \leq \epsilon\) where \(\epsilon\) is a real positive number. So, the conditions (C1) and (C2) remain satisfied for all \(t > 0\).

4. SPEED STABILIZATION

We are interested here to the stabilization of the system describing the speeds behavior of the vehicle (11) around the set point \((V_x^0, 0, 0)\) where \(V_x^0\) is the reference value for longitudinal speed. To do that, we continue our investi-
gation by transforming the third equation of the system (11). We introduce the function \(F\) defined by

\[
F(V_x) = \frac{a}{I_z} \left( \frac{c_4}{a} + M \right) V_x + \frac{a}{I_z} \left( \frac{c_6}{a} - c_3 \right) \frac{1}{V_x}
\]

(12)

A simple reasoning shows that the system (11) can be written in the following form

\[
\begin{align*}
\dot{V}_x &= \sigma_1 \\
\dot{V}_y &= \gamma_1 \\
\dot{V}_p &= G \left( V_x, V_y, \dot{V}_p \right)
\end{align*}
\]

(13)

with

\[
G \left( V_x, V_y, \dot{V}_p \right) = \left[ F(V_x) - F(V_y^0) \right] \left[ \dot{V}_p + MV_y \right] + F(V_y^0) \left[ \dot{V}_p + MV_y \right] + \left( \frac{c_6}{a} - c_2 \right) \frac{V_y}{V_x^2} + k_{ay} V_y^2
\]

The expression of the derivative for \(V_x = V_x^0\) is:

\[
\dot{V}_x = G \left( V_x^0, V_y, \dot{V}_p \right) = \lambda_1 \left( V_y^2 + \lambda_2 V_y + \lambda_3 \dot{V}_p \right)
\]

with

\[
\begin{align*}
\lambda_1 &= \frac{k_{ay}}{\lambda_2} \\
\lambda_2 &= \frac{1}{\lambda_1} \left[ \frac{aM}{I_z} \left( \frac{c_4}{a} + M \right) V_x^0 + \left( \frac{c_6}{a} - c_2 \right) \frac{1}{V_x^0} \right] \\
\lambda_3 &= \frac{1}{\lambda_1} \left[ \frac{a}{I_z} \left( \frac{c_4}{a} + M \right) V_x^0 + \frac{a}{I_z} \left( \frac{c_6}{a} - c_3 \right) \frac{1}{V_x^0} \right]
\end{align*}
\]

Let \(\Delta V_y = \lambda_2^2 - 4\lambda_3 \dot{V}_p\), then

\[
G \left( V_x^0, V_y, \dot{V}_p \right) = \lambda_1 \left( V_y + \lambda_2 + (-1)^{(i+1)} \frac{\Delta V_y^0}{2} \right)
\]

Given the following Lyapunov function \(W\) defined by:

\[
W \left( V_x, V_y, \dot{V}_p \right) = \frac{1}{2} V_y^2 + \frac{\lambda_2 - Sgn(\lambda_2) \sqrt{\Delta V_y^0}}{2} V_y + \frac{\lambda_3 - Sgn(\lambda_3) \sqrt{\Delta V_y^0}}{2}
\]

A simple computation shows that

\[
G \left( V_x^0, V_y, \dot{V}_p \right) = \lambda_1 \left( V_y + \frac{\lambda_2 + Sgn(\lambda_2) \sqrt{\Delta V_y^0}}{2} \right) \frac{\partial W}{\partial V_y}
\]

Next, the system (13) becomes

\[
\begin{align*}
\dot{V}_x &= \sigma_1 \\
\dot{V}_y &= \gamma_1 \\
\dot{V}_p &= G \left( V_x^0, V_y, \dot{V}_p \right) + G \left( V_x, V_y, \dot{V}_p \right)
\end{align*}
\]

First, we will prove that:

\[
G(V_x, V_y, \dot{V}_p) - G(V_x^0, V_y, \dot{V}_p) = G_1(V_x, V_y, \dot{V}_p) \left( V_x - V_x^0 \right)
\]

(14)

where \(G_1\) is a regular function.

Indeed, a simple reasoning shows that

\[
G(V_x, V_y, \dot{V}_p) - G(V_x^0, V_y, \dot{V}_p) = \int_{V_x^0}^{V_x} \frac{\partial G}{\partial V_x}(\tau, V_y, \dot{V}_p) d\tau
\]
Let consider the change of variable defined by
\[ \tau = tV_x + (1 - t) V_0 \]
We have:
\[ d\tau = \left( V_x - V_0 \right) dt \]
Then, it comes that
\[ G \left( V_x, V_y, \tilde{V}_\psi \right) - G \left( V_0, V_y, \tilde{V}_\psi \right) \]
\[ = \left( V_x - V_0 \right) \int_0^1 \frac{dG}{dV_x} \left( tV_x + (1 - t) V_0, V_y, \tilde{V}_\psi \right) dt \]
So, we obtain the expression (14). Now, we have to compute explicitly
\[ G_1 \left( V_x, V_y, \tilde{V}_\psi \right) = \int_0^1 \frac{dG}{dV_x} \left( tV_x + (1 - t) V_0, V_y, \tilde{V}_\psi \right) dt \]
By definition, \( G \) is given by the expression
\[
G \left( V_x, V_y, \tilde{V}_\psi \right) = \left[ \frac{a}{I_z} \left( \frac{c_4}{a} + M \right) V_x + \frac{a}{I_z} \left( \frac{c_6}{a} - c_1 \right) \frac{1}{V_x} \right] \tilde{V}_\psi \\
+ \frac{aM}{I_z} \left( \frac{c_4}{a} + M \right) V_x + \left( \frac{c_6}{a} - c_2 \right) \frac{1}{V_x} V_y + k_{ay} V_y^2
\]
A simple computations give
\[
\frac{dG}{dV_x} = \eta_1 \frac{\tilde{V}_\psi}{V_x^2} + \eta_2 \frac{V_y}{V_x^2} + \eta_3 \tilde{V}_\psi + \eta_4 V_y
\]
where the constants \( \eta_i, i = 1, \ldots, 4 \) are given by
\[
\eta_1 = - \frac{a}{I_z} \left( \frac{c_4}{a} - c_3 \right) \\
\eta_2 = - \frac{c_5}{a} + c_2 + \frac{aM}{I_z} \left( \frac{c_6}{a} - c_3 \right) \\
\eta_3 = - \frac{a}{I_z} \left( \frac{c_4}{a} + M \right) \\
\eta_4 = \frac{aM}{I_z} \left( \frac{c_4}{a} + M \right)
\]
Finally, by using the expression (15), we obtain
\[ G_1 = \int_0^1 \frac{dG}{dV_x} \left( tV_x + (1 - t) V_0, V_y, \tilde{V}_\psi \right) dt \]
\[ = \eta_1 \tilde{V}_\psi + \eta_2 V_y + \eta_3 \tilde{V}_\psi + \eta_4 V_y \]
Then, the system (13) can be rewritten as:
\[
\begin{cases}
\dot{V}_x = \sigma_1 \\
\dot{V}_y = \gamma_1 \\
\dot{\tilde{V}}_\psi = G(V_0, V_y, \tilde{V}_\psi) + G_1(V_x, V_y, \tilde{V}_\psi) \left( V_x - V_0 \right)
\end{cases}
\]
Thereafter and to prove the asymptotic stability of the feedback system, we introduce the following Lyapunov function:
\[
W_1 \left( V_x, V_y, \tilde{V}_\psi \right) = \frac{1}{2} \left( V_x - V_0 \right)^2 + \ln \left( W(V_y, \tilde{V}_\psi) + 1 \right)
\]
A simple computation shows that \( W_1 \) is a positive function on \( V_0, 0, 0 \).

Consider the nonlinear state feedback given by
\[
\left\{ \begin{array}{c}
\sigma_1 = - \eta \frac{V_x - V_0}{1 + \left( V_x - V_0 \right)^2} - G_1 \left( V_x, V_y, \tilde{V}_\psi \right) \\
\gamma_1 = - \lambda_1 \left( V_y + \gamma_2 + Sgn(\gamma_2) \sqrt{\Delta V_0^2} \right) \frac{\partial W}{\partial V_y} - \eta \frac{\partial W}{\partial V_y} + 1 \left( \frac{\partial W}{\partial V_y} \right)^2 \end{array} \right.
\]
with:
\[
\frac{\partial W}{\partial V_y} = \frac{Sgn(\gamma_2) \gamma_2 + 4 Sgn(\gamma_2) \gamma_2 + 4 \sqrt{\Delta V_0^2}}{2 \sqrt{\Delta V_0^2}}
\]
and
\[
\frac{\partial W}{\partial V_y} = V_y + \frac{\lambda_2 - Sgn(\gamma_2) \sqrt{\Delta V_0^2}}{2}
\]
Notice that \( \eta \) is a positive parameter introduced for performance tuning and may be different in the two expressions.
The derivative of \( W_1 \) by considering the system (16) with the state feedback (17) is given by
\[
W_1 \left( V_x, V_y, \tilde{V}_\psi \right) = \frac{\partial W_1}{\partial V_y} \dot{V}_x + \frac{\partial W_1}{\partial V_y} \dot{V}_y + \frac{\partial W_1}{\partial \tilde{V}_\psi} \dot{\tilde{V}}_\psi
\]
\[ = - \eta \left( V_x - V_0 \right)^2 \frac{1}{1 + \left( V_x - V_0 \right)^2} \left( \frac{\partial W}{\partial V_y} \right)^2 \left( \frac{\partial W}{\partial V_y} \right)^2 \leq 0 \]
Then, the system is stable.
To prove that the system is attractive, we use the invariance principle of LaSalle J. P. LaSalle et al (1961).
Consider a set \( \Omega \) defined by
\[ \Omega = \{ W_1(V_x, V_y, \tilde{V}_\psi) = 0 \} \]
or
\[ \Omega = \left\{ (V_x, V_y, \tilde{V}_\psi) : V_x = V_0 \text{ and } V_y + \frac{\lambda_2 - Sgn(\gamma_2) \sqrt{\Delta V_0^2}}{2} = 0 \right\} \]
On the set \( \Omega \), we have
\[
\frac{d}{dt} \left( V_y + \frac{\lambda_2 - Sgn(\gamma_2) \sqrt{\Delta V_0^2}}{2} \right) \frac{\partial W}{\partial V_y} = - \lambda_1 \left( V_y + \frac{\lambda_2 - Sgn(\gamma_2) \sqrt{\Delta V_0^2}}{2} \right) \frac{\partial W}{\partial V_y}
\]
The greatest invariant set checks the following equations

\[
\begin{cases}
V_y + \frac{\lambda_2 - Sgn(\lambda_2) \sqrt{\Delta V^0_y}}{2} = 0 \\
\partial W \left( V_y + \frac{\lambda_2 + Sgn(\lambda_2) \sqrt{\Delta V^0_y}}{2} \right) = 0
\end{cases}
\]

or

\[
\begin{cases}
V_y + \frac{\lambda_2 - Sgn(\lambda_2) \sqrt{\Delta V^0_y}}{2} = 0 \\
2 Sgn(\lambda_2) \lambda_3 \left[ V_y + 2 \left( \lambda_2 - Sgn(\lambda_2) \sqrt{\Delta V^0_y} \right) \right] \sqrt{\lambda_3^2 - 4 \lambda_3 V_\psi} = 0
\end{cases}
\]

where

\[ R^* = \left( V_y + \frac{\lambda_2 + Sgn(\lambda_2) \sqrt{\Delta V^0_y}}{2} \right) \]

Finally, and according to (18), we have

\[
\begin{cases}
V_y + \frac{\lambda_2 - Sgn(\lambda_2) \sqrt{\Delta V^0_y}}{2} = 0 \\
V_y + \frac{\lambda_2 + Sgn(\lambda_2) \sqrt{\Delta V^0_y}}{2} = 0
\end{cases}
\]

or

\[
\begin{cases}
V_y + \frac{\lambda_2 - Sgn(\lambda_2) \sqrt{\Delta V^0_y}}{2} = 0 \\
V_y + 2 \left( \lambda_2 - Sgn(\lambda_2) \sqrt{\Delta V^0_y} \right) = 0
\end{cases}
\]

The system (19) does not have a solution in physical field. The above computation is done under the assumption such that \( \Delta V^0_y > 0 \), but it imposes a condition on \( V_\psi \).

The system (16) has \( V_y = \tilde{V}_y = 0 \) as a unique solution. Thus, the greatest invariant set contained in \( \Omega \) is limited in a neighborhood of \((V^0_y, 0, 0)\). Finally, the system is asymptotically stable. This finish the proof of the following theorem, which is the main contribution of this paper.

**Theorem 1.** The system (16) controlled by the state feedback (17) is asymptotically stable on \((V^0_y, 0, 0)\).

5. SIMULATION RESULTS

To evaluate the performance of the proposed nonlinear control systems a various simulations are proposed by using the vehicle data given in Appendix A. The main objective of the feedback control here is the stabilization of the different velocities of the vehicle.

Fig. 1, Fig. 2 and Fig. 3 show respectively the evolution of the longitudinal velocity \( V_x \), the acceleration force \( F_l \) and the steering angle \( \delta \) in the case of acceleration. We can see when \( \eta \) increases, the vehicle reaches the stabilization velocity more quickly. Obviously the acceleration force also increases as shown in Fig. 2. However, the parameter \( \eta \) was introduced in the nonlinear control (??) and its determination is done in regard of one supplement criteria as a given rise time or avoiding overshoot.

Fig.4 show the lateral velocity in the case of acceleration, i.e., evolution of the longitudinal velocity from 7 m/s to 20 m/s.

Fig.5 shows the longitudinal velocity in two deceleration cases from 30 m/s to 20 m/s and from 30 m/s to 25 m/s with \( \eta = 0.5 \), the parameter \( \eta \) can be selected for soft acceleration/braking, i.e the driver does not accelerate so fast or braking so hard.
6. CONCLUSION

We presented here a nonlinear approach method to design a nonlinear state feedback control. A stabilization of the different velocities of vehicle dynamics is chosen as an application. The stabilization is achieved following two steps. The first one consists to change variables in order to simplify the system structure. The second step consists to design a nonlinear feedback based on the combined Lyapunov-LaSalle approach applied to the simplified system structure.

Notice that, the change on variables introduce two polynomial relations of second degree between the new and the old control variables. So, the final solution, in original control variables, must consider the operating ranges where the defined equation admits real solutions and respect the real handling of the vehicle. We showed that for constant longitudinal and lateral speeds and given an operating range, each of the two polynomial relations between the controls has a positive discriminant and consequently a real solutions. The simulation results show the performance of the suggested result. This result let us to consider in for coming work the problem of trajectory tracking.

Appendix A. VEHICLE DATA

\[ M = 1480 \text{ kg}, \quad I_z = 1950 \text{ N.m/s}^2, \quad C_f = 95000 \text{ N/rad}, \]
\[ C_r = 50000 \text{ N/rad}, \quad h = 0.42 \text{ m}, \quad a = 1.421 \text{ m}, \quad b = 1.029 \text{ m}, \quad k_{ax} = 0.41 \text{ kg.m/s}^2, \quad k_{ay} = 0.54 \text{ kg.m/s}^2. \]