A Design Method of Model Reference Adaptive Formation Control

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Abstract: A design method of model reference adaptive formation control of multi-agent systems composed of unknown linear processes is presented in this paper. This is the first attempt to solve the formation control problem from the viewpoint of model reference adaptive control by output feedbacks. The proposed control schemes are constructed via backstepping procedures and state variable filters, where potential functions corresponding to desirable formations are introduced into the first step. It is shown that the resulting control systems are robust to uncertain system parameters and that the desirable formations are achieved asymptotically via adaptation mechanisms.

Keywords: Adaptive Control, Co-operative Control, Adaptive Systems.

1. INTRODUCTION

Recently, formation control problems of multi-agent systems have attracted much attentions, and several formation control schemes were proposed based on various strategies (for example, leader-follower (Shao et al. [2007]), behavior-based (Balch and Arkin [1998]), virtual structure (Tan and Lewis [1996]), and potential function approaches (Pereira and Hsu [2008], Cheah et al. [2009], Yao et al. [2006], Miyasato [2010])). Among those, the potential function approaches seem to be useful tools from the viewpoint of flexibility of configurations of swarms, automatic avoidance of collisions of agents, and stability of maintaining formations. In those research works, adaptive control or sliding mode control methodologies were applied in order to deal with uncertainties of agents, and stability of control systems was assured via Lyapunov function analysis. However, all state variables in each agent are assumed to be available for measurement in those works, but the state $x_i(t)\in\mathbb{R}^n$ and system parameters in $A_i$, $b_i$, $c_i$ are unknown. Other assumptions are written below:

(1) Each process is stabilizable and observable, and the zeros of it lie in $\mathbb{C}^-$ (minimum phase).
(2) The upper bound on the dimension of each process, which is denoted as $n$, is known a priori.
(3) The relative degree of each process is $n^*$, and $n^*$ is known a priori.
(4) The sign of the high-frequency gain of each process $b_{0i} \equiv c_i^T A_i^{n^*-1} b_i$ is known. Hereafter, it is assumed that $b_{0i} > 0$ without loss of generality.

The purpose of the present paper is to provide a design method of model reference adaptive formation control of multi-agent systems composed of unknown linear processes. As far as the author knows, this is the first attempt to solve the formation control problem from the viewpoint of model reference adaptive control by output feedbacks. The proposed control scheme is constructed via backstepping procedures and state variable filters corresponding to relative degree structures by utilizing only output data of agents, where potential functions corresponding to desirable formations of the leader-follower type are introduced into the first step. It is shown that the resulting control systems are robust to uncertain system parameters and that the desirable formations are achieved asymptotically via adaptation mechanisms.
It should be noted that the constants $r_i$ and $d_{ij}$ and the reference signal $y_o$ need to be chosen appropriately for consistent configurations of the formation.

Hereafter, it is assumed that $y_o$, $\dot{y}_o$, $\cdots$, $y_o^{(n)} \in C^\infty$, and that $y_o$, $\dot{y}_o$, $\cdots$, $y_o^{(n)}$ are available for measurement.

3. SYSTEM REPRESENTATION AND CONTROL OBJECTIVE

The input-output representation of each process (1), (2) is given by utilizing state variable filters [Ioannou and Sun [1996], Krstić et al. [1995]).

\[
\frac{d}{dt} y_i + \lambda_i y_i = \theta_i^T \omega_i + b_{oi} u_{ifn^{*-1}},
\]
where

\[
\omega_i = [\nu_{if1}^T, \nu_{if2}^T, y_{ifn^{*-1}}^T]^T,
\]
\[
\frac{d}{dt} \nu_{if1} = F_i \nu_{if1} + g_i y_{ifn^{*-1}} - 1,
\]
\[
\frac{d}{dt} \nu_{if2} = F_i \nu_{if2} + g_i y_{ifn^{*-1}} - 1,
\]
\[
\left( \frac{d}{dt} + \lambda_i \right)^k y_{ifk} = y_i, \quad (k = 1, 2, \cdots),
\]
\[
\left( \frac{d}{dt} + \lambda_i \right)^k u_{ifk} = u_i, \quad (k = 1, 2, \cdots).
\]

$(F_i, g_i)$ and $\lambda_i$ are known design parameters $(F_i$ : a matrix, $g_i$ : a vector, $\lambda$ : a scalar), $\theta_i$ and $b_{oi}$ are unknown system parameters $(\theta_i$ : a vector, $b_{oi}$ : a scalar). Signals $\omega_i$, $\nu_{if1}$, $\nu_{if2}$, $y_{ifk}$ and $u_{ifk}$ are available for measurement. Especially, $(F_i, g_i)$ is a controllable pair with a Hurwitz matrix $F_i$ (the dimensions of $(F_i, g_i)$ are $F_i \in R^{n \times n}$, $g_i \in R^n$ or $F_i \in R^{(n-1) \times (n-1)}$, $g_i \in R^{n-1}$).

Here we define tracking errors between the agents $(z_{i1})$ and relative distances among the agents $(\bar{y}_{ij})$ such as

\[
z_{i1} = y_i - y_o,
\]
\[
\bar{y}_{ij} = y_i - y_j, \quad (i \neq j),
\]
and introduce positive potential functions $P_{io}(z_{i1})$ to handle the tracking specifications (3) or (4), and $P_{ij}(\bar{y}_{ij})$ to handle the specifications of relative configurations (5) or (6). It is assumed that $P_{io}(z_{i1})$ and $P_{ij}(\bar{y}_{ij})$ are $n^*$-times continuously differentiable, and that $P_{ij}(\bar{y}_{ij}) = P_{ji}(\bar{y}_{ji})$ $(i \neq j)$. The desired total configuration of the multi-agent system (3) or (4) and (5) or (6) corresponds to the minimal points of $\sum_{i=1}^{N} P_{io}(z_{i1})$ and $\sum_{i=1}^{N} \sum_{j \neq i} P_{ij}(\bar{y}_{ij})$, respectively.

4. DESIGN OF MODEL REFERENCE ADAPTIVE FORMATION CONTROL

4.1 Construction of Control System

The proposed design method is based on the interactive backstepping procedure [Krstić et al. [1995]) composed of step 1 to step $n^*$ corresponding to the relative degree structures of the agents.

Step 1) For the tracking error $z_{i1}$ (13), define a state variable $z_{i2}$ such as

\[
z_{i2} = u_{ifn^{*-1}} - \alpha_{i1}, \quad (15)
\]
where $\alpha_{i1}$ is a virtual control input to be determined later (24). We take the time derivative of $z_{i1}$.

\[
z_{i1} = -\lambda_i z_{i1} - \omega_i + \theta_i^T \omega_i + b_{oi} u_{ifn^{*-1}} - 1
\]
\[
= b_{oi} z_{i2} + b_{oi} (\alpha_{i1} + \theta_i \phi_{i1}),
\]
where

\[
\phi_{i1} = -\lambda_i z_{i1} - \theta_i^T \omega_i - \omega_o,
\]
\[
\theta_i = 1/b_{oi},
\]
\[
\omega_o = y_o + \lambda_i y_o.
\]

A positive function $V_i$ is introduced such as

\[
V_i = \sum_{i=1}^{N} P_{io}(z_{i1}) + \sum_{i=1}^{N} \sum_{j \neq i} P_{ij}(\bar{y}_{ij}) + \sum_{i=1}^{N} b_{oi} (\bar{p}_i - \bar{p}_i)^2
\]
\[
+ \frac{1}{2} \sum_{i=1}^{N} (\theta_i - \hat{\theta}_i)^T \Gamma_{i1}^{-1} (\theta_i - \hat{\theta}_i),
\]
where $\Gamma_{i1} > 0$, $\Gamma_{21} = \Gamma_{21}^T > 0$, and $\hat{\theta}_i$ and $\bar{p}_i$ are current estimates of the unknown parameters $\theta_i$ and $p_i$, respectively. The tuning laws of those estimates are obtained later (26), (62). By considering (16), we take the time derivative of $V_i$ along its trajectories.

\[
\dot{V}_i = \sum_{i=1}^{N} \left[ b_{oi} z_{i2} + b_{oi} (\alpha_{i1} + \theta_i \phi_{i1}) \right] \xi_{i1}
\]
\[
+ \sum_{i=1}^{N} (\bar{p}_i - \bar{p}_i) \Gamma_{i1}^{-1} (-\dot{\hat{p}}_i)
\]
\[
+ \sum_{i=1}^{N} (\theta_i - \hat{\theta}_i)^T \Gamma_{i1}^{-1} (-\dot{\hat{\theta}}_i),
\]
\[
\xi_{i1} = \frac{\partial P_{io}}{\partial z_{i1}} + 2 \sum_{j \neq i} \frac{\partial P_{ij}}{\partial \bar{y}_{ij}}.
\]

The virtual input $\alpha_{i1}$ is determined in order to stabilize $z_{i1}$ such as

\[
\alpha_{i1} = -\bar{p}_i \phi_{i1} - K_{i1} \xi_{i1},
\]
where $K_{i1} > 0$, and
\begin{align*}
\dot{z}_{i1} &= -\lambda_i z_{i1} + \hat{\theta}_i^T \omega_{i1} - Y_{oi}, \\
\dot{p}_i &= \Gamma_1 \dot{z}_{i1}.
\end{align*}

Then, \( \dot{V}_1 \) is evaluated as follows:

\begin{align*}
\dot{V}_1 &= \sum_{i=1}^{N} b_0 z_{i2} \xi_{i1} + \sum_{i=1}^{N} (\theta_i - \hat{\theta}_i)^T \Gamma_{2i} (\tau_{\theta i} - \dot{\theta}_i) \\
&\quad - \sum_{i=1}^{N} b_0 K_{i1} \xi_{i1}^2, \\
\tau_{\theta i} &= \Gamma_{2i} \omega_{i1} \xi_{i1}.
\end{align*}

**Step 2** For the state variable \( z_{i2} \), define a new state \( \bar{z}_{i3} \) by

\begin{equation}
\bar{z}_{i3} = u_{ifn^-1} - \alpha_{i2},
\end{equation}

where \( \alpha_{i2} \) is a virtual control input to be determined later (32). Take the time derivative of \( z_{i2} \)

\begin{align*}
\bar{z}_{i3} &= \bar{z}_{i3} + \alpha_{i2} + \beta_{i2} \\
&\quad - \sum_{j=1}^{N} \frac{\partial \alpha_{i1}}{\partial z_{j1}} \left( \hat{\theta}_j^T \omega_{j1} + b_{0j} u_{jfn^-1} \right) - \frac{\partial \alpha_{i1}}{\partial \hat{\theta}_i} \hat{\theta}_i,
\end{align*}

where

\begin{equation}
\beta_{i2} = -\lambda_i u_{ifn^-1} \sum_{j=1}^{N} \frac{\partial \alpha_{i1}}{\partial z_{j1}} (-\lambda_j z_{j1} - Y_{oj}),
\end{equation}

and determine the virtual input \( \alpha_{i2} \) so as to stabilize \( z_{i1} \) and \( z_{i2} \) as follows:

\begin{align*}
\alpha_{i2} &= \beta_{i2} + \sum_{j=1}^{N} \frac{\partial \alpha_{i1}}{\partial z_{j1}} \left( \hat{\theta}_j^T \omega_{j1} + b_{0j} u_{jfn^-1} \right) \\
&\quad - b_{0i} \xi_{i1} + \frac{\partial \alpha_{i1}}{\partial \hat{\theta}_i} \tau_{\theta i} - K_{i2} \bar{z}_{i2},
\end{align*}

where \( K_{i2} > 0 \), and \( b_{0i} \) is a current estimate of \( b_0 \). The tuning law of \( b_{0i} \) is obtained later (63). Furthermore, \( \tau_{\theta i} \) is a substitute of \( \xi_{i1} \) at the stage of **Step 2**. We introduce a positive function \( V_2 \)

\begin{equation}
V_2 = V_1 + \frac{1}{2} \sum_{i=1}^{N} \bar{z}_{i3}^2 + \frac{1}{2} \sum_{i=1}^{N} \frac{1}{K_{i2}} (b_{0i} - \bar{b}_{0i})^2,
\end{equation}

and take the time derivative of \( V_2 \) along its trajectories.

\begin{align*}
\dot{V}_2 &= \sum_{i=1}^{N} b_0 K_{i1} \xi_{i1}^2 - \sum_{i=1}^{N} \frac{\partial \alpha_{i1}}{\partial z_{j1}} \left( \hat{\theta}_j^T \omega_{j1} + b_{0j} u_{jfn^-1} \right) \\
&\quad + \sum_{i=1}^{N} (\theta_i - \hat{\theta}_i)^T \Gamma_{2i}^{-1} (\tau_{\theta i} - \dot{\theta}_i) \\
&\quad + \sum_{i=1}^{N} (b_{0i} - \hat{b}_{0i}) \Gamma_{3i}^{-1} (\tau_{\theta i} - \dot{\theta}_i) \\
&\quad + \sum_{i=1}^{N} \frac{\partial \alpha_{i1}}{\partial \hat{\theta}_i} (\tau_{\theta i} - \dot{\theta}_i) \bar{z}_{i2},
\end{align*}

where

\begin{align*}
\tau_{\theta i} &= \tau_{\theta i} - \Gamma_{2i} \sum_{j=1}^{N} \frac{\partial \alpha_{i1}}{\partial z_{j1}} \omega_{j1} z_{j2} + \sum_{k=1}^{N} \frac{\partial \alpha_{i1}}{\partial z_{j1}} u_{ifn^-1} z_{j2}, \\
\tau_{\theta i} &= \Gamma_{3i} \xi_{i1} z_{i2} - \Gamma_{3i} \sum_{j=1}^{N} \frac{\partial \alpha_{i1}}{\partial z_{j1}} u_{jfn^-1} z_{j2}.
\end{align*}

**Step r** (3 \( \leq r \leq n^2 - 1 \)) Define state variables \( z_{ir} \)

\begin{align*}
\dot{z}_{ir} &= \dot{z}_{ir+1} + \alpha_{ir} + \beta_{ir} \\
&\quad - \sum_{j=1}^{N} \gamma_{ir-1} (\theta^T \omega_{j1} + b_{0j} u_{jfn^-1}) \\
&\quad - \sum_{j=1}^{N} \gamma_{ir-1} \hat{\theta}_j - \sum_{j=1}^{N} \gamma_{ir-1} \hat{b}_{0j},
\end{align*}

where

\begin{align*}
\beta_{ir} &= -\lambda_i u_{ifn^-1} \sum_{j=1}^{N} \frac{\partial \alpha_{i1}}{\partial z_{j1}} (-\lambda_j z_{j1} - Y_{oj}) \\
&\quad - \sum_{j=1}^{N} \frac{\partial \alpha_{i1}}{\partial z_{j1}} \left( \hat{\theta}_j^T \omega_{j1} + b_{0j} u_{jfn^-1} \right) \\
&\quad + \sum_{j=1}^{N} \frac{\partial \alpha_{i1}}{\partial \hat{\theta}_i} \hat{\theta}_i,
\end{align*}

and determine the virtual input \( \alpha_{ir} \) so as to stabilize \( z_{i1} \) and \( z_{ir} \) as follows:

\begin{align*}
\alpha_{ir} &= -\beta_{ir} + \sum_{j=1}^{N} \frac{\partial \alpha_{i1}}{\partial z_{j1}} \left( \hat{\theta}_j^T \omega_{j1} + b_{0j} u_{jfn^-1} \right) \\
&\quad - b_{0i} \xi_{i1} + \frac{\partial \alpha_{i1}}{\partial \hat{\theta}_i} \tau_{\theta i} - K_{i2} \bar{z}_{i2},
\end{align*}

where \( K_{i2} > 0 \), and \( b_{0i} \) is a current estimate of \( b_0 \). The tuning law of \( b_{0i} \) is obtained later (63). Furthermore, \( \tau_{\theta i} \) is a substitute of \( \xi_{i1} \) at the stage of **Step 2**. We introduce a positive function \( V_2 \)

\begin{equation}
V_2 = V_1 + \frac{1}{2} \sum_{i=1}^{N} \bar{z}_{i3}^2 + \frac{1}{2} \sum_{i=1}^{N} \frac{1}{K_{i2}} (b_{0i} - \bar{b}_{0i})^2,
\end{equation}

and take the time derivative of \( V_2 \) along its trajectories.

\begin{align*}
\dot{V}_2 &= \sum_{i=1}^{N} b_0 K_{i1} \xi_{i1}^2 - \sum_{i=1}^{N} \frac{\partial \alpha_{i1}}{\partial z_{j1}} \left( \hat{\theta}_j^T \omega_{j1} + b_{0j} u_{jfn^-1} \right) \\
&\quad + \sum_{i=1}^{N} (\theta_i - \hat{\theta}_i)^T \Gamma_{2i}^{-1} (\tau_{\theta i} - \dot{\theta}_i) \\
&\quad + \sum_{i=1}^{N} (b_{0i} - \hat{b}_{0i}) \Gamma_{3i}^{-1} (\tau_{\theta i} - \dot{\theta}_i) \\
&\quad + \sum_{i=1}^{N} \frac{\partial \alpha_{i1}}{\partial \hat{\theta}_i} (\tau_{\theta i} - \dot{\theta}_i) \bar{z}_{i2},
\end{align*}

where

\begin{align*}
\tau_{\theta i} &= \tau_{\theta i} - \Gamma_{2i} \sum_{j=1}^{N} \frac{\partial \alpha_{i1}}{\partial z_{j1}} \omega_{j1} z_{j2} + \sum_{k=1}^{N} \frac{\partial \alpha_{i1}}{\partial z_{j1}} u_{ifn^-1} z_{j2}, \\
\tau_{\theta i} &= \Gamma_{3i} \xi_{i1} z_{i2} - \Gamma_{3i} \sum_{j=1}^{N} \frac{\partial \alpha_{i1}}{\partial z_{j1}} u_{jfn^-1} z_{j2}.
\end{align*}
\[ \sum_{j=1}^{N} \gamma_{\theta ir-1 j} \tau_{\theta jr} + \sum_{j=1}^{N} \gamma_{bir-1 j} \tau_{bjr} - K_{ir} z_{ir} + \tilde{\alpha}_{ir}, \quad (45) \]

where \( K_{ir} > 0 \), and \( \tau_{\theta ir}, \tau_{bir} \) are substitutes of \( \dot{\theta}_i \) and \( \dot{b}_0 \), respectively at the stage of Step r. \( \tilde{\alpha}_{ir} \) are auxiliary signals to compensate the differences between \( \tau_{\theta ir} \) and \( \dot{\theta}_i \), and the differences between \( \tau_{bir} \) and \( \dot{b}_0 \), and are to be determined later. A positive function is introduced

\[ V_r = V_{r-1} + \frac{1}{2} \sum_{i=1}^{N} z_{ir}^2, \quad (46) \]

and take the time derivative of \( V_r \) along its trajectories.

\[ \dot{V}_r = - \sum_{i=1}^{N} b_{0i} K_{i1} \xi_{i1}^2 - \sum_{i=1}^{N} \sum_{l=2}^{r} K_{il} z_{il}^2 + \sum_{i=1}^{N} z_{ir} \dot{z}_{ir+1} \]

\[ + \sum_{i=1}^{N} (\theta_i - \dot{\theta}_i) \Gamma_{2i} (\tau_{\theta ir} - \dot{\theta}_i) \]

\[ + \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=3}^{r} \gamma_{\theta i1-j1} (\tau_{\theta j} - \dot{\theta}_j) z_{id} \]

\[ + \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=3}^{r} \gamma_{b i1-j1} (\tau_{b j} - \dot{b}_j) z_{id} \]

\[ + \sum_{i=1}^{N} \sum_{j=1}^{N} \tilde{\alpha}_{i1} z_{id}, \quad (47) \]

where

\[ \tau_{\theta ir} = \tau_{\theta ir-1} - \Gamma_{2i} \sum_{j=1}^{N} \gamma_{jr-1 j} \omega_{i1} z_{jr}, \quad (48) \]

\[ \tau_{bir} = \tau_{bir-1} - \Gamma_{3i} \sum_{j=1}^{N} \gamma_{jr-1 j} u_{f1 n-1} z_{jr}. \quad (49) \]

**Step n**

Define a state variable \( z_{in}^* \) by

\[ z_{in}^* = u_{i1} - \alpha_{in}^* - 1, \]

and take the time derivative of it.

\[ z_{in}^* = u_i + \beta_{in}^* - \sum_{j=1}^{N} (\theta_j^* \omega_{j1} + b_{0j}) u_{f1 n-1} - \sum_{j=1}^{N} \gamma_{\theta i1-j1} \dot{\theta}_j - \sum_{j=1}^{N} \gamma_{b i1-j1} \dot{b}_j, \quad (51) \]

where

\[ \beta_{in}^* = -\lambda_i u_{f1} - \sum_{j=0}^{n-1} \frac{\partial \alpha_{in}^* - 1}{\partial y_{i j}} y_{i j+1} \]

\[ - \sum_{j=1}^{N} \frac{\partial \alpha_{in}^* - 1}{\partial z_{j1}} (-\lambda_j z_{j1} - Y_{oj}) \]

\[ - \sum_{j=1}^{N} \sum_{l=2}^{r} \frac{\partial \alpha_{in}^* - 1}{\partial z_{jl}} (u_{f1 n-1} + \beta_{jl}) \]

\[ - \sum_{j=1}^{N} \frac{\partial \alpha_{in}^* - 1}{\partial \omega_{j1 n-1}} \dot{\omega}_{j1 n-1} - \sum_{j=1}^{N} \frac{\partial \alpha_{in}^* - 1}{\partial \dot{p}_{j1}} \dot{p}_{j1}, \quad (52) \]

\[ \omega_{in}^* - 1 = [\omega_{in}^* - 2, \dot{\theta}_{i1}, \dot{b}_0, u_{f1}]^T, \quad (53) \]

\[ \gamma_{in}^* - 1 j = \frac{\partial \alpha_{in}^* - 1}{\partial z_{j1}} \]

\[ \gamma_{in}^* - 1 j = \frac{\partial \alpha_{in}^* - 1}{\partial \omega_{j1}} \frac{\partial \omega_{j1}}{\partial \theta_{i1}} \quad \gamma_{b in}^* - 1 j = \frac{\partial \alpha_{in}^* - 1}{\partial \omega_{j1}} \frac{\partial \omega_{j1}}{\partial \theta_{i1}} \quad \gamma_{b in}^* - 1 j = \frac{\partial \alpha_{in}^* - 1}{\partial \omega_{j1}} \frac{\partial \omega_{j1}}{\partial \theta_{i1}} \]

\[ \gamma_{bn}^* - 1 j = \frac{\partial \alpha_{in}^* - 1}{\partial \omega_{j1}} \frac{\partial \omega_{j1}}{\partial \theta_{i1}} \quad \gamma_{bn}^* - 1 j = \frac{\partial \alpha_{in}^* - 1}{\partial \omega_{j1}} \frac{\partial \omega_{j1}}{\partial \theta_{i1}} \]

Then, we determine an actual control input \( u_i \) so as to stabilize \( z_{11} \sim z_{n}^* \) such as

\[ u_i = -z_{in}^* - 1 - \beta_{in}^* \]

\[ + \sum_{j=1}^{N} \gamma_{in} - 1 j \theta_j^* \omega_{j1} + b_{0j} u_{f1 n-1} \]

\[ + \sum_{j=1}^{N} \gamma_{in} - 1 j \theta_j^* \omega_{j1} + b_{0j} u_{f1 n-1} \]

\[ + \sum_{j=1}^{N} \gamma_{in} - 1 j \theta_j^* \omega_{j1} + b_{0j} u_{f1 n-1} \]

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\[ + \sum_{j=1}^{N} \gamma_{in} - 1 j \theta_j^* \omega_{j1} + b_{0j} u_{f1 n-1} \]

where \( K_{in} > 0 \), and \( \gamma_{\theta in}^* \) and \( \gamma_{bin}^* \) are chosen such that those are equal to \( \dot{\theta}_i \) and \( \dot{b}_0 \), respectively. The auxiliary signals \( \alpha_{in}^* \) are to be determined so as to compensate the differences between \( \tau_{\theta ir} \) and \( \dot{\theta}_i \), and the differences between \( \tau_{bir} \) and \( \dot{b}_0 \), and are to be determined later. A positive function \( V_{n}^* \) is introduced,

\[ V_{n}^* = V_{n-1} + \frac{1}{2} \sum_{i=1}^{N} z_{in}^2, \quad (58) \]

and take the time derivative of \( V_{n}^* \) along its trajectories.

\[ \dot{V}_{n}^* = - \sum_{i=1}^{N} b_{0i} K_{i1} \xi_{i1}^2 - \sum_{i=1}^{N} \sum_{l=2}^{r} K_{il} z_{il}^2 + \sum_{i=1}^{N} (\theta_i - \dot{\theta}_i) \Gamma_{2i} (\tau_{\theta in}^* - \dot{\theta}_i) \]

\[ + \sum_{i=1}^{N} (b_{0i} - \dot{b}_0) \Gamma_{3i} (\tau_{bin}^* - \dot{b}_0) \]
\[ \begin{align*}
&\sum_{i=1}^{N} \frac{\partial \alpha_{i1}}{\partial \hat{\theta}_{i}} (\tau_{0i2} - \hat{\theta}_{i}) z_{i2} \\
&+ \sum_{i=1}^{N} \sum_{l=1}^{n^*} \sum_{j=1}^{N} \gamma_{0il-1j} (\tau_{0j2} - \hat{\theta}_{j}) z_{il} \\
&+ \sum_{i=1}^{N} \sum_{l=3}^{n^*} \gamma_{0il-1j} (\hat{\theta}_{jl} - \hat{\theta}_{0j}) z_{il} \\
&+ \sum_{i=1}^{N} \sum_{l=3}^{n^*} \tilde{\alpha}_{il} z_{il},
\end{align*} \]

where

\[ \tau_{0i2} = \tau_{0i2} - 1 - \Gamma_{2i} \sum_{j=1}^{N} \gamma_{j2n^*} \omega_i z_{j2n^*} \tag{59} \]

\[ \tau_{0i2} = \tau_{0i2} - 1 - \Gamma_{3i} \sum_{j=1}^{N} \gamma_{j2n^*} \omega_i z_{j2n^*} \tag{60} \]

**Step \( n^* + 1 \)** Determination of adaptive laws and auxiliary signals

In the final step, we determine adaptive laws of \( \hat{\theta}_{i} \) and \( \hat{b}_{0i} \), and also determine auxiliary signals \( \tilde{\alpha}_{il} \). First, \( \hat{\theta}_{i} \) and \( \hat{b}_{0i} \) are tuned in the following way.

\[ \hat{\theta}_{i} = \tau_{0i2n^*} \tag{61} \]

\[ \hat{b}_{0i} = \tau_{0i2n^*} \tag{62} \]

Then, \( \dot{V}_{n^*} \) is evaluated as follows:

\[ \dot{V}_{n^*} = - \sum_{i=1}^{N} b_{0i} K_{i1} \xi_{i1}^2 - \sum_{i=1}^{N} n^* K_{il} z_{il}^2 \\
+ \sum_{i=1}^{N} \frac{\partial \alpha_{i1}}{\partial \hat{\theta}_{i}} (\tau_{0i2} - \tau_{0i2n^*}) z_{i2} \\
+ \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{l=1}^{n^*} \gamma_{0il-1j} (\tau_{0jl} - \tau_{0jn^*}) z_{il} \\
+ \sum_{i=1}^{N} \sum_{l=3}^{n^*} \gamma_{0il-1j} (\hat{\theta}_{jl} - \tau_{0jn^*}) z_{il} \\
+ \sum_{i=1}^{N} \sum_{l=3}^{n^*} \tilde{\alpha}_{il} z_{il}. \tag{63} \]

The auxiliary signals \( \tilde{\alpha}_{il} \) are chosen in order that the terms \( \gamma_{0il-1j} (\tau_{0jl} - \tau_{0jn^*}) z_{il} \) and \( \gamma_{0il-1j} (\hat{\theta}_{jl} - \tau_{0jn^*}) z_{il} \) in (64) are cancelled out by the terms \( \tilde{\alpha}_{il} z_{il} \) such as

\[ \tilde{\alpha}_{il} = - \sum_{j=1}^{N} \frac{\partial \alpha_{ij1}}{\partial \hat{\theta}_{j}} z_{j2} \Gamma_{2j} \gamma_{i2n^*} z_{i2} \\
- \sum_{k=1}^{N} \sum_{j=1}^{N} \sum_{l=1}^{n^*} \gamma_{krl-1j} \hat{\theta}_{k} \Gamma_{2j} \gamma_{i2n^*} z_{i2} \\
- \sum_{k=1}^{N} \sum_{j=1}^{N} \sum_{l=1}^{n^*} \gamma_{krl-1j} \hat{\theta}_{k} \Gamma_{2j} \gamma_{i2n^*} z_{i2} \\
(3 \leq l \leq n^*). \tag{64} \]

When \( \sum_{l=3}^{n^*} \), the term is not added to computation. Then, \( \dot{V}_{n^*} \) is rewritten into

\[ \dot{V}_{n^*} = - \sum_{i=1}^{N} b_{0i} K_{i1} \xi_{i1}^2 - \sum_{i=1}^{N} n^* K_{il} z_{il}^2 \leq 0. \tag{66} \]

**4.2 Stability Analysis**

From the evaluation of \( \dot{V}_{n^*} \), (66), it follows that \( \dot{V}_{n^*} \in \mathcal{L}^{\infty} \), \( P_{io} \), \( P_{ij} \in \mathcal{L}^{\infty} \), \( z_{i2} \sim z_{n^*} \in \mathcal{L}^{\infty} \), \( \dot{\theta}_{i} \), \( \hat{b}_{0i} \), \( \dot{\theta}_{0i2} \in \mathcal{L}^{\infty} \), and that \( \xi_{i1} \), \( z_{i2} \sim z_{n^*} \in \mathcal{L}^{2} \). Since \( P_{io} \) and \( P_{ij} \) are continuously differentiable, it is shown that \( \xi_{i1} \in \mathcal{L}^{\infty} \), \( z_{i2} \in \mathcal{L}^{2} \) is deduced from \( P_{io} \in \mathcal{L}^{\infty} \), and \( \dot{y}_{i} \in \mathcal{L}^{\infty} \) from \( y_{i} \in \mathcal{L}^{\infty} \). Then, it is assured that \( y_{jfi} (k = 1, 2, \ldots, v_{jf1} \in \mathcal{L}^{\infty} \). On the contrary, since \( u_{ijf} \) \( G_{0i}(s)y_{i} (G_{0i}(s) \equiv e_{i} (sI - \lambda_{i}) \tilde{b}_{i} (s + \lambda_{i}) \tilde{b}_{i} \in \mathcal{L}^{\infty} \), it follows that \( u_{ijf} \in \mathcal{L}^{\infty} \), and \( v_{jfi} \in \mathcal{L}^{\infty} \) is derived from \( v_{jfi} = (sI + \lambda_{i}) (sI + \lambda_{i}) \tilde{y}_{j} \in \mathcal{L}^{\infty} \). Therefore, it is shown that \( \dot{w}_{i2} \in \mathcal{L}^{\infty} \), and that \( \xi_{i1} \in \mathcal{L}^{\infty} \). Similarly, since \( u_{ijf} = \in \mathcal{L}^{\infty} \), it follows that \( u_{ijf} \in \mathcal{L}^{\infty} \), and \( \varphi_{0i2} \in \mathcal{L}^{2} \), and since \( u_{ijf} \in \mathcal{L}^{\infty} \), it is shown that \( \dot{w}_{i2} \in \mathcal{L}^{\infty} \). By repeating the same procedures, it is seen that \( \varphi_{0i2} \sim \varphi_{n^*-1} \in \mathcal{L}^{\infty} \), \( u_{ijf} \in \mathcal{L}^{\infty} \), \( u_{ijf} \in \mathcal{L}^{\infty} \). Hence, all signals in the adaptive system are shown to be uniformly bounded, and furthermore, it follows that \( \xi_{i1} \), \( z_{i2} \sim z_{n^*} \in \mathcal{L}^{2} \), and by considering \( \xi_{i1} \), \( z_{i2} \sim z_{n^*} \in \mathcal{L}^{2} \), we obtain

\[ \lim_{t \to \infty} \xi_{i1}(t) = 0 \tag{67} \]

\[ \lim_{t \to \infty} z_{i2}(t) = \cdots = \lim_{t \to \infty} z_{n^*}(t) = 0. \tag{68} \]

Next, we consider relative configuration among agents and tracking property between agents and the virtual leader. It is assumed that the control objective is defined by (6) and (4). Since \( \xi_{i1} \to 0 \), it follows that

\[ \sum_{i=1}^{N} \xi_{i1} = \sum_{i=1}^{N} \partial P_{io} / \partial z_{i1} + 2 \sum_{i=1}^{N} \partial P_{ij} / \partial y_{ij} \to 0. \tag{69} \]

and since \( \dot{P}_{ij} (\tilde{y}_{ij}) = P_{ij} (\tilde{y}_{ij}) \), \( \dot{\tilde{y}}_{ij} = -\dot{\tilde{y}}_{ij} \), it follows that

\[ \sum_{i=1}^{N} \partial P_{ij} / \partial \tilde{y}_{ij} = 0. \tag{70} \]

Then, we obtain

\[ \sum_{i=1}^{N} \partial P_{io} / \partial z_{i1} \to 0. \tag{71} \]

Here we consider the case where all agents do not satisfy the formation constraint related to \( P_{io}(z_{i1}) \), and several agents are outside the desired region defined by (4). It should be noted that \( \partial P_{io} / \partial z_{i1} = 0 \) for the agents inside the desired region. If those agents outside the desired region are on the one side of the region, then the corresponding \( \partial P_{io} / \partial z_{i1} \) have the same sign along one axis, and this shows that (71) means the relation \( \partial P_{io} / \partial z_{i1} = 0 \) (1 \( \leq i \leq N \)). Therefore, it follows that \( \sum_{j \neq i} \partial P_{ij} / \partial y_{ij} \to 0 \) (1 \( \leq i \leq N \)). Next, we consider the case where several agents are on the opposite sides outside the desired region. If we choose a
sufficient large region related to $P_{io}(z_{i1})$, then it follows that
\[ \sum_{j \neq i} \frac{\partial P_{io}}{\partial z_{ij}} \to 0 \]
for the agents outside the region. Hence, $\frac{\partial P_{io}}{\partial z_{i1}} \to 0$ holds for the corresponding $\frac{\partial P_{io}}{\partial z_{i1}}$. In the end, by choosing appropriate formation constraints, such as an appropriate desired region related to $P_{io}(z_{i1})$ and appropriate relative distances related to $P_{ij}(\tilde{y}_{ij})$, the next equation holds for all agents
\[ \frac{\partial P_{io}}{\partial z_{i1}} \to 0, \quad \sum_{j \neq i} \frac{\partial P_{ij}}{\partial \tilde{y}_{ij}} \to 0. \]
and the desired formation of the leader-follower type is achieved asymptotically (Cheah et al. [2009]). Similar arguments are also applied to the case of the control objective (5), (3), and it is shown that the desired formations ($\frac{\partial P_{io}}{\partial z_{i1}} = 0$, $\sum_{j \neq i} \frac{\partial P_{io}}{\partial \tilde{y}_{ij}} = 0$) are attained by choosing tracking property $P_{io}(z_{i1})$ and relative configurations $P_{ij}(\tilde{y}_{ij})$ appropriately.

Then, we obtain the main theorem of the manuscript.

**Theorem 1.** The model reference adaptive formation control system is uniformly bounded, and the state variables $\xi_{i1}(t)$, $z_{i2}(t) \sim z_{in^*}(t)$ converge to zero asymptotically.

\[ \lim_{t \to \infty} \xi_{i1}(t) = 0, \]
\[ \lim_{t \to \infty} z_{i2}(t) = \cdots = \lim_{t \to \infty} z_{in^*}(t) = 0. \]

Furthermore, by choosing appropriate formation constraints, such as an appropriate desired region related to $P_{io}(z_{i1})$ and appropriate relative distances related to $P_{ij}(\tilde{y}_{ij})$, the desired formation of the leader-follower type is achieved asymptotically such that
\[ \lim_{t \to \infty} \frac{\partial P_{io}}{\partial z_{i1}} = 0, \]
\[ \lim_{t \to \infty} \sum_{j \neq i} \frac{\partial P_{ij}}{\partial \tilde{y}_{ij}} = 0. \]

**Remark 2.** In the manuscript, it is assumed that the upper bound on the dimension of each process is $n$ for simplicity of notation, but the assumption can be easily extended to the case where each process has a different upper bound $n_i$. On the contrary, in order to extend the present methodology to the case where each process has a different relative degree $n_i^*$, the total control structure should be modified substantially. That extension is left in our future study.

5. CONCLUDING REMARKS

A design methodology of model reference adaptive formation control of multi-agent systems composed of linear processes has been provided in the present paper. This is the first attempt to solve the formation control problem from the view point of model reference adaptive control by output feedbacks. The proposed control scheme is constructed via backstepping procedures and state variable filters corresponding to the relative degree structures by utilizing only input/output data of agents, where potential functions corresponding to desirable formations of the leader-follower type are introduced into the first step. It is shown that the resulting control systems are robust to uncertain system parameters and that the desirable formations are achieved asymptotically via adaptation mechanisms.

Although only the most basic adaptive control problem is discussed in the present work, and each agent in this manuscript is confined to a single-input single-output linear finite dimensional system, it is thought that the proposed design scheme can be extended to the case where agents are nonlinear systems, a certain class of infinite dimensional systems, multivariable systems, or a class of systems with nonlinear parameteric structures. Detailed discussions on such cases are left in our future study.

Furthermore, the paper focuses on the formation control problem in which tracking properties and relative configurations are specified explicitly, and the input/output data and current estimates of system parameters of other agents are needed to construct total adaptive control systems. Decentralized version of the present control scheme for limited class of control problems (such as consensus problems) together with a consideration of restricted information network structures is also a future topic.

REFERENCES


