An Automatic Tuning Method for Multiloop PID Controllers

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Abstract: In this paper we propose a new automatic tuning technique for multiloop PID controllers applied to MIMO processes. The process parameters are estimated by employing simple closed-loop experiments (where roughly tuned PID controllers are in place), namely, by evaluating the response of the system to a sequence of set-point step signals (one for each reference input). Once the process transfer function is obtained, a (IMC-based) PID tuning rule already proposed in the literature is then applied to suitably re-design the PID controllers. Simulation examples are given to illustrate the methodology and to show its effectiveness.

1. INTRODUCTION

It is well-known that many industrial processes are basically multiple-input-multiple-output (MIMO) systems, namely, they are affected by interactions among various control loops and plant variables. Although Multivariable Predictive Control has been successfully employed in many applications (typically in the refining processes), decentralised multiloop proportional-integral-derivative (PID) controllers remain the standard for many industries because they are often capable to provide a satisfactory performance in spite of their simple structure and intuitiveness. However, because of the above mentioned interactions, tuning the controllers can be a difficult task (Wang et al., 2008; Vazquez et al., 1999). For this reason, it is desirable to have an automatic tuning methodology in order to help the operator to properly design the control system.

Different methodologies have been proposed in the literature in this context, mainly for two-inputs-two-outputs (TITO) processes, which represent the most common case in practical applications. For example, the use of open-loop step tests (one for each input) for the model identification purpose has been proposed in (Wang et al., 2000). After having employed a suitably designed decoupler, a sequential tuning method is then employed for decentralised PID controllers. Autotuning technique based on relay-feedback experiments has also been proposed. In particular, simultaneous (Halevi et al., 1997) and sequential relay techniques (see (Shiu and Hwang, 1998; Marchetti et al., 2002) and references therein contained) have been developed. An approach based on finite frequency response data has been presented in (Gilbert et al., 2003). In this case, the closed-loop experiment to be performed consists of applying a square wave excitation input to one set-point at a time (note that this means that the feedback controllers have to be previously tuned, possibly roughly).

It can be noted that these kinds of methods require special experiments to be employed and the identification part cannot rely on routine operating data. In this paper we propose a new automatic tuning technique in which the identification experiment consists of evaluating the response of the system to a sequence of standard set-point step signals (one for each reference input). Actually, we extend to MIMO processes a methodology that has already been proven to be effective for SISO processes (Veronesi and Visioli, 2009, 2010a,b).

Once the a process model based on first-order-plus-dead-time (FOPDT) transfer functions has been estimated, a tuning rule can be adopted to determine the parameters of the PID controllers. In this paper we suggest the use of the tuning rule proposed in (Lee et al., 2004), which is based on the Internal Model Control (IMC) design (Morari and Zafiropulo, 1980) and on a MacLaurin series expansion. In particular, the tuning procedure aims at obtaining a desired closed-loop transfer function and in this context the dominant time constant is chosen by taking into account the identification technique employed.

The paper is organised as follows. In Section 2 the problem is formulated. The method for estimating the process parameters by using the closed-loop set-point step responses is explained in Section 3, while the employed tuning rules is reviewed in Section 4. Simulation examples are given in Section 5 and conclusions are drawn in Section 6.

2. PROBLEM FORMULATION

For the sake of simplicity, two-inputs-two-outputs (TITO) processes are considered from now on. We consider therefore a linear, time-invariant, overdamped, continuous-time TITO system whose matrix transfer function is:

\[ P(s) = [P_{ij}(s)] \quad i, j = 1, 2 \] (1)

where \((i, j = 1, 2)\)

\[ P_{ij}(s) = \frac{\mu_{ij}e^{-\theta_{ij}}}{q_{ij}(s)} \] (2)

and

\[ q_{ij}(s) \]

\[ \prod_{k \neq i, j} P_{ik}(s) \]
The estimation of the process parameters can be performed by evaluating the response of the system to a sequence of step signals applied to each of the set-point starting from steady-state conditions. This means that the transient response caused by a set-point step has to be terminated before applying another set-point step and should not be perturbed by external disturbances. By considering a setpoint step change in the loop 1 (i.e. to \( r_1 \)), the following relations can be easily derived:

\[
U_1(s) = C_1(s)(R_1(s) - (P_{11}(s)U_1(s) + P_{12}(s)U_2(s))),
\]

\[
U_2(s) = C_2(s)(-P_{22}(s)U_2(s) + P_{21}(s)U_1(s)),
\]

from which it can be derived that

\[
U_1(s) = \frac{-C_1(s)(1 + C_2(s)P_{22}(s))R_1(s)}{d(s)},
\]

\[
U_2(s) = \frac{C_1(s)C_2(s)P_{21}(s)R_1(s)}{d(s)}.
\]

where

\[
d(s) = 1 + C_2(s)P_{22}(s) + C_1(s)P_{11}(s) + C_1(s)C_2(s)P_{11}(s)P_{22}(s) - C_1(s)C_2(s)P_{21}(s)P_{21}(s).
\]

Further, the two control errors are:

\[
E_{1,1}(s) = R_1(s) - P_{11}(s)U_1(s) - P_{12}(s)U_2(s),
\]

\[
E_{2,1}(s) = -P_{21}(s)U_1(s) - P_{22}(s)U_2(s).
\]

It is worth noting that \( e_{1,1} \) indicates the error variable of the loop 1 after the setpoint step change in the loop 1 itself while \( e_{2,1} \) is the error variable of the loop 2 after the same setpoint step change in the loop 1. By replacing the output of the controllers in (12) with their expressions (10) and by considering (2) and (7), after some calculations the transfer functions between the setpoint of the first loop \( R_1 \) and the two control errors can be written as:

\[
\frac{E_{1,1}(s)}{R_1(s)} = \frac{sa_1(s)e^{-s\theta_{11} + \theta_{12} + \theta_{21}}}{b_1(s)s^2 + b_2(s)b_1(s)s + b_3(s)b_5(s)},
\]

\[
\frac{E_{2,1}(s)}{R_1(s)} = \frac{b_1(s)s^2 + b_2(s)b_3(s)s + b_4(s)b_5(s)}{b_1(s)s^2 + b_2(s)b_1(s)s + b_4(s)b_5(s)},
\]

where

\[
a_1(s) = (sT_{22}q_{22}(s)e^{-s\theta_{22}} + K(p_{21}\mu_{22}c_{2}(s))T_{12}q_{11}(s)q_{12}(s)q_{21}(s)),
\]

\[
a_2(s) = K(p_{21}\mu_{22}T_{12}q_{11}(s)q_{12}(s)q_{21}(s)q_{22}(s)),
\]

\[
b_1(s) = T_{12}q_{11}(s)q_{12}(s)q_{21}(s)e^{-s(\theta_{11} + \theta_{12} + \theta_{21} + \theta_{22})},
\]

\[
b_2(s) = q_{12}(s) + q_{21}(s)e^{-s(\theta_{12} + \theta_{21})},
\]

\[
b_3(s) = T_{12}K_{p_{21}\mu_{22}q_{11}(s)c_{2}(s)e^{-s\theta_{11}}}
\]

\[
+ T_{12}K_{p_{21}\mu_{21}q_{22}(s)c_{1}(s)e^{-s\theta_{22}},}
\]

\[
b_4(s) = K(p_{21}\mu_{22}c_{1}(s)c_{2}(s)),
\]

\[
b_5(s) = \mu_{11}\mu_{22}q_{12}(s)q_{21}(s)e^{-s(\theta_{12} + \theta_{21})}
\]

\[
- \mu_{12}\mu_{21}q_{11}(s)q_{22}(s)e^{-s(\theta_{12} + \theta_{22})}.
\]
By applying the final value theorem we eventually obtain the following expressions of the integrated errors:

\[ IE_{1,1} := \int e_{1,1}(t)dt = \frac{\mu_{22}T_{11}}{K_{p1}(\mu_{11}\mu_{22} - \mu_{12}\mu_{21})}A_{s1} \]
\[ IE_{2,1} := \int e_{2,1}(t)dt = -\frac{\mu_{21}T_{12}}{K_{p2}(\mu_{11}\mu_{22} - \mu_{12}\mu_{21})}A_{s1} \]

(15)

where \( A_{s1} \) denotes the amplitude of the setpoint step change applied to the first loop.

Denote now as \( A_{s1} \) and \( A_{s2} \) the steady-state values of the output of the controllers \( C_1 \) and \( C_2 \) respectively. By considering that, at the steady state, after a step change in the setpoint \( r_1 \), it has to be:

\[ \mu_{11}A_{s1} + \mu_{12}A_{s2} = A_{s1} \]
\[ \mu_{21}A_{s1} + \mu_{22}A_{s2} = 0 \]

(16)

we can express \( A_{s1} \) and \( A_{s2} \) in terms of the process gains as follows:

\[ A_{s1} = \frac{\mu_{22}}{\mu_{11}\mu_{22} - \mu_{12}\mu_{21}}A_{s1} \]
\[ A_{s2} = -\frac{\mu_{21}}{\mu_{11}\mu_{22} - \mu_{12}\mu_{21}}A_{s1} \]

(17)

Define now the two following variables:

\[ v(t) := \mu_{11}u_1(t) + \mu_{12}u_2(t) - y_1(t), \]
\[ w(t) := \mu_{21}u_1(t) + \mu_{22}u_2(t) - y_2(t). \]

(18)

By using (2), (3), and (18) we can derive:

\[ V(s) = \frac{\mu_{11}}{q_{11}(s)}q_{11}(s) - e^{-s\theta_{11}})U_1(s) + \frac{\mu_{12}}{q_{12}(s)}q_{12}(s) - e^{-s\theta_{12}})U_2(s), \]
\[ W(s) = \frac{\mu_{21}}{q_{21}(s)}q_{21}(s) - e^{-s\theta_{21}})U_1(s) + \frac{\mu_{22}}{q_{22}(s)}q_{22}(s) - e^{-s\theta_{22}})U_2(s). \]

(19)

Hence, by applying the final value theorem, we obtain

\[ \lim_{s \to 0} \frac{V(s)}{s} = \mu_{11} \lim_{s \to 0} \left( \frac{q_{11}(s) - 1}{s} + \frac{1 + e^{-s\theta_{11}}}{s} \right) A_{s1} + \mu_{12} \lim_{s \to 0} \left( \frac{q_{12}(s) - 1}{s} + \frac{1 + e^{-s\theta_{12}}}{s} \right) A_{s2}, \]
\[ = \mu_{11}T_{11}^0 A_{s1} + \mu_{12}T_{12}^0 A_{s2} \]
\[ \lim_{s \to 0} \frac{W(s)}{s} = \mu_{21} \lim_{s \to 0} \left( \frac{q_{21}(s) - 1}{s} + \frac{1 + e^{-s\theta_{21}}}{s} \right) A_{s1} + \mu_{22} \lim_{s \to 0} \left( \frac{q_{22}(s) - 1}{s} + \frac{1 + e^{-s\theta_{22}}}{s} \right) A_{s2}, \]
\[ = \mu_{21}T_{21}^0 A_{s1} + \mu_{22}T_{22}^0 A_{s2} \]

(20)

that is, by taking into account (17), (3), and (4), we have

\[ IV_1 := \int v(t)dt = \frac{\mu_{11}\mu_{22}T_{11}^0}{\mu_{11}\mu_{22} - \mu_{12}\mu_{21}}A_{s1} - \frac{\mu_{11}\mu_{22}T_{12}^0}{\mu_{11}\mu_{22} - \mu_{12}\mu_{21}}A_{s1} \]
\[ IW_1 := \int w(t)dt = \frac{\mu_{12}\mu_{21}T_{12}^0}{\mu_{11}\mu_{22} - \mu_{12}\mu_{21}}A_{s1} - \frac{\mu_{12}\mu_{21}T_{22}^0}{\mu_{11}\mu_{22} - \mu_{12}\mu_{21}}A_{s1} \]

(21)

which can be rewritten as

\[ IV_1 = \frac{\mu_{11}\mu_{22}T_{11}^0 - \mu_{12}\mu_{21}T_{12}^0}{\mu_{11}\mu_{22} - \mu_{12}\mu_{21}}A_{s1}, \]
\[ IW_1 = \frac{\mu_{12}\mu_{21}T_{12}^0 - \mu_{11}\mu_{22}T_{22}^0}{\mu_{11}\mu_{22} - \mu_{12}\mu_{21}}A_{s1}. \]

(22)

A similar reasoning can be applied when a step change is applied to the setpoint of the other loop (i.e. to \( r_2 \)). Hence, \( E_{1,2}(s) = -f_{12}(s)U_1(s) - P_{12}(s)U_2(s), \)

\[ E_{2,2}(s) = R_2(s) - P_{21}(s)U_1(s) - P_{22}(s)U_2(s), \]

from which,

\[ IE_{1,2} := \int e_{1,2}(t)dt = \frac{\mu_{11}T_{12}}{K_{p2}(\mu_{11}\mu_{22} - \mu_{12}\mu_{21})}A_{s2}, \]
\[ IE_{2,2} := \int e_{2,2}(t)dt = \frac{\mu_{12}T_{21}}{K_{p1}(\mu_{11}\mu_{22} - \mu_{12}\mu_{21})}A_{s2}, \]
\[ IV_2 := \int v(t)dt = \frac{\mu_{12}\mu_{21}(T_{12}^0 - T_{11}^0)}{\mu_{11}\mu_{22} - \mu_{12}\mu_{21}}A_{s2} \]
\[ IW_2 := \int w(t)dt = \frac{\mu_{11}\mu_{22}(T_{12}^0 - T_{22}^0)}{\mu_{11}\mu_{22} - \mu_{12}\mu_{21}}A_{s2} \]

(23)

(24)

(25)

where \( A_{s2} \) denotes the amplitude of the setpoint step change applied to the second loop. Thus, by applying a step change to the setpoint of the first loop and then, at the end of the transient, a step change to the setpoint of the second loop, the four process gains and the values of the four sums of the lags and dead times \( T_{ij}^0 (i, j = 1, 2) \) can be computed. In fact, by solving the system of equations (15) and (24), it is possible to obtain:

\[ \mu_{11} = \frac{T_{11}^0 K_{p1}(IE_{1,2})}{IE_{1,2}} + \mu_{12}T_{12}^0 A_{s1}, \]
\[ \mu_{12} = \frac{K_{p2}(IE_{2,2})}{IE_{2,2}} - \frac{IE_{1,2}}{K_{p1}(IE_{1,2})}A_{s1}, \]
\[ \mu_{21} = \frac{K_{p1}(IE_{1,2})}{IE_{1,2}} - \frac{IE_{1,2}}{K_{p2}(IE_{2,2})}A_{s2}, \]
\[ \mu_{22} = \frac{K_{p2}(IE_{2,2})}{IE_{2,2}} - \frac{IE_{2,2}}{K_{p1}(IE_{1,2})}A_{s2}. \]

(26)

In a similar way, by solving the system of equations (22) and (25), it is possible to obtain:

\[ T_{11}^0 = \frac{\mu_{12}IW_2 + IV_1}{\mu_{11}A_{s1}} + \frac{IV_1}{A_{s1}} \]
\[ T_{12}^0 = \frac{\mu_{12}IW_2 + IV_1}{\mu_{11}A_{s2}} + \frac{IV_1}{A_{s2}} \]
\[ T_{21}^0 = \frac{\mu_{11}IW_1 + IV_2}{\mu_{12}A_{s1}} + \frac{IV_2}{A_{s1}} \]
\[ T_{22}^0 = \frac{\mu_{11}IW_1 + IV_2}{\mu_{12}A_{s2}} + \frac{IV_2}{A_{s2}} \]

(27)
Summarising, the identification procedure initially consists of evaluating $IE_{1,1}$, $IE_{2,1}$ (see (15)), $IE_{1,2}$, $IE_{2,2}$ (see (23)), $IV_1$, $IW_1$ (see (22)), $IV_2$, $IW_2$ (see (25)), and then by determining $\mu_{ij}$ and $T_{ij}^0$ ($i,j=1,2$) by applying (26) and (27). Note that the values of the parameters of the PID controllers employed (those that need to be retuned) are obviously known.

Once $\mu_{ij}$ and $T_{ij}^0$ ($i,j=1,2$) have been determined, each transfer function $P_{ij}(s)$ can be approximated as a FOPDT transfer function, namely:

$$P_{ij}(s) = \frac{\mu_{ij}e^{-s\bar{\theta}_ij}}{\tau_{ij}s + 1}. \quad (28)$$

In this context it is worth considering the so-called “half-rule” (Skogestad, 2003) which states that the largest neglected (denominator) time constant is distributed evenly to the effective dead time and the smallest retained time constant. This means that an appropriate approximation of the possibly high-order system (2) is obtained by setting

$$\tau_{ij} = \tau_{ij,1} + \frac{\tau_{ij,2}}{2}, \quad \bar{\theta}_{ij} = \theta_{ij} + \frac{\tau_{ij,2}}{2} + \sum_{k=3}^{n_{ij}} \tau_{ij,k}, \quad (29)$$

It is worth stressing that we have

$$T_{ij}^0 := \sum_{k=1}^{n_{ij}} \tau_{ij,k} + \theta_{ij} = \tau_{ij} + \bar{\theta}_{ij}, \quad (30)$$

namely, the sum of the dead time and of the time constants of the process (2) is unaltered in the reduced model. Thus, $T_{ij}^0$ is a relevant process parameter that is worth estimating for the purpose of the retuning of the PID controllers.

Finally, the apparent dead time $\theta_{ij}$ of each transfer function $P_{ij}(s)$ can be evaluated by considering respectively the time interval from the application of the step signal to the set-point $r_j$ and the time instant when the output $y_j$ attains the 2% of the steady-state value corresponding to the new set-point value $A_{ij}$. Actually, from a practical point of view, in order to cope with the measurement noise, a simple sensible solution is to define a noise band $NB$ (Åström et al., 1993) (whose amplitude should be equal to the amplitude of the measurement noise) and to rewrite the condition as $y_j > NB$.

It is clear at this point that the time constants of each transfer function can be trivially obtained as

$$\tau_{ij} = T_{ij}^0 - \bar{\theta}_{ij}, \quad i,j = 1,2. \quad (31)$$

Remark 1. The values of process parameters are determined by considering the integral of signals and therefore the method is inherently robust to the measurement noise (Veronesi and Visioli, 2009). Further, they are obtained independently on the values of the PID parameters. This is an advantage with respect to the use of other methods for the identification of the process transfer function (for example, the least squares approach), whose result depends on the control variable and process variable signals.

4. TUNING

Once the process parameters have been obtained, any of the tuning rules proposed in the literature for MIMO processes can be applied, in principle. However, since the identification technique proposed in the previous section relies on the properties of the “half-rule”, it is sensible, as in (Skogestad, 2003), to apply a tuning rule based on the IMC design. In particular, the method proposed in (Lee et al., 2004) extends the generalised IMC-PID method for SISO systems (Lee et al., 1998) to MIMO systems and consists of determining the multiloop PID controller in order to obtain a desired closed-loop response for the $i$th loop, which is specified as

$$Y_i(s) = \frac{e^{-s\theta_{ii}}}{1 + \lambda_i s} \quad i = 1,2 \quad (32)$$

The result of the design method proposed in (Lee et al., 2004), based on the expansion of the process model in a Maclaurin series, is the following set of tuning formulae ($i,j=1,2$):

$$K_{pj} = \mu_{jj}((\lambda_j + \bar{\theta}_{jj})\bar{\tau}_{jj} + \frac{\hat{\theta}_{jj}^2}{2j}),$$

$$T_{dj} = \frac{1}{2K_{pj}} \left( \frac{6\bar{\tau}_{jj}\bar{\theta}_{jj} + \hat{\theta}_{jj}^2 - 2\lambda_j\bar{\theta}_{jj} + 6\bar{\tau}_{jj}\lambda_j}{6\mu_{jj}(\lambda_j + \bar{\theta}_{jj})^3} \right).$$

$$T_{ij} = \frac{K_{pj}((\lambda_j + \bar{\theta}_{jj})\bar{\tau}_{ij} + \bar{\theta}_{ij})}{\bar{P}^{-1}(0)_{jj}} \quad (33)$$

where $\bar{P}^{-1}(0)_{jj}$ is the $i$th element of the diagonal of the inverse of the matrix $\bar{P}(s) = \begin{bmatrix} \bar{P}_{ij}(s) \end{bmatrix}$ for $s = 0$. In this way the integral action of the PID can take into account even the off-diagonal terms of $\bar{P}(s)$. This is reasonable because both the integral action and the loop interactions are more significant at low frequencies.

The choice of the desired closed-loop time constant $\lambda_j$ can be done as in (Skogestad, 2003), by taking into account the trade-off between aggressiveness and robustness, as

$$\lambda_j = \bar{\theta}_{jj}. \quad (34)$$

In this way the tuning formulae can be rewritten simply as:

$$K_{pj} = \frac{1}{8\mu_{jj}} \left( 4\bar{\tau}_{jj} + \bar{\theta}_{jj} \right),$$

$$T_{dj} = \frac{1}{12} \left( 12\bar{\tau}_{jj} - \bar{\theta}_{jj} \right) \bar{\theta}_{jj},$$

$$T_{ij} = \frac{4\bar{\tau}_{jj} + \bar{\theta}_{ij}}{4} 2(\bar{\mu}_{11}\bar{\mu}_{22} - \bar{\mu}_{12}\bar{\mu}_{21}). \quad (35)$$

Remark 2. As it is well-known in the IMC design, parameter $\lambda_j$ handles effectively the trade-off between aggressiveness and robustness (and control effort) and represents therefore a very desirable feature from the user viewpoint. Actually, the rule (34) has been selected as in (Skogestad, 2003) but the user can modify simply the value of $\lambda_j$ (that is, use (33)) in order to meet specific requirements (for example, the value of $\lambda_j$ can be increased in order to reduce the overshoot in the set-point step response).

5. SIMULATION RESULTS

5.1 Example 1

As a first example, the well-known distillation column model reported in (Wood and Berry, 1973) is considered:
As an initial controller parameters, those proposed in (Luyben, 1986) are selected: $K_{p1} = 0.375$, $T_{d1} = 8.29$, $T_{d2} = -0.075$, $T_{12} = 23.6$, $T_{d2} = 0$. With these parameters, after a setpoint step change in the first loop, the value of the integrated absolute errors defined as

$$IAE_i = \int_0^\infty |e_i(t)|dt \quad i = 1, 2$$

are $IAE_1 = 4.56$ and $IAE_2 = 16.76$, while, after a setpoint step change in the second loop, it results $IAE_1 = 3.37$ and $IAE_2 = 32.46$. By applying the identification procedure presented in Section 3 (namely, by evaluating the two step responses), the following process parameters are determined:

$$T_{11}^0 = 17.72, \quad T_{12}^0 = 24.16, \quad T_{21}^0 = 17.93, \quad T_{22}^0 = 17.83$$

$$\theta_{11} = 1.08, \quad \theta_{12} = 3.32, \quad \theta_{21} = 7.1, \quad \theta_{22} = 3.2$$

$$\mu_{11} = 12.80, \quad \mu_{12} = -18.92, \quad \mu_{21} = 6.61, \quad \mu_{22} = -19.46$$

and therefore (see (31)):

$$\tilde{\tau}_{11} = 16.64, \quad \tilde{\tau}_{22} = 14.63$$

Hence, by applying the tuning formulae (35), the following PID parameters can be used for retuning the two controllers:

$$K_{p1} = 0.61, \quad T_{11} = 8.42, \quad T_{d1} = 0.26$$

$$K_{p2} = -0.12, \quad T_{12} = 7.68, \quad T_{d2} = 0.73$$

With these values of the parameters, the performance of the controllers have been improved to $IAE_1 = 3.80$ and $IAE_2 = 5.58$ when the set-point unit step change is applied to the set-point of the first loop and to $IAE_1 = 2.12$ and $IAE_2 = 7.32$ when the unit step signal is applied to the set-point of the second loop. The obtained step responses are shown in Figures 2 and 3 where the effectiveness of the automatic tuning methodology can be evaluated.

Fig. 2. Example 1: process variable $y_1$ with the initial controller tuning (dashed line) and with the tuning obtained by applying the new method (solid line).

Fig. 3. Example 1: process variable $y_2$ with the initial controller tuning (dashed line) and with the tuning obtained by applying the new method (solid line).

5.2 Example 2

In order to verify the robustness of the methodology to modelling uncertainties, as a second example, the following high-order process model is considered:

$$P(s) = \begin{bmatrix}
12.8e^{-s} - 18.9e^{-3s} \\
1 + 16.7s - 1 + 21s \\
6.6e^{-7s} - 19.4e^{-3s} \\
1 + 10.9s - 1 + 14.4s
\end{bmatrix}$$

As an initial controller parameters, the values $K_{p1} = 0.5$, $T_{11} = 20$, $T_{d1} = 1$, $K_{p2} = 0.5$, $T_{12} = 10$, $T_{d2} = 1$ have been fixed. With these parameters, after a setpoint step change in the first loop, the value of the integrated absolute errors are $IAE_1 = 44.18$ and $IAE_2 = 16.50$, while, after a setpoint step change in the second loop, it results $IAE_1 = 21.71$ and $IAE_2 = 32.98$. The corresponding process variables are plotted in Figures 4 and 5 as dashed lines. By applying the proposed identification procedure, the following process parameters are determined:

$$T_{11}^0 = 24.10, \quad T_{12}^0 = 116.45, \quad T_{21}^0 = 68.87, \quad T_{22}^0 = 10.19$$

$$\theta_{11} = 7.33, \quad \theta_{12} = 48.12, \quad \theta_{21} = 26.82, \quad \theta_{22} = 2.82$$

$$\mu_{11} = 1.2, \quad \mu_{12} = 0.39, \quad \mu_{21} = 0.60, \quad \mu_{22} = 0.80$$

and therefore (see (31)):

$$\tilde{\tau}_{11} = 16.77, \quad \tilde{\tau}_{22} = 7.37$$

Hence, by applying the tuning formulae (35), the following PID parameters can be used for retuning the two controllers:

$$K_{p1} = 1.06, \quad T_{11} = 14.04, \quad T_{d1} = 1.59$$

$$K_{p2} = 1.78, \quad T_{12} = 6.09, \quad T_{d2} = 0.62$$

With these values of the parameters, the performance of the controllers have been improved to $IAE_1 = 24.45$ and $IAE_2 = 8.94$ when the set-point unit step change is applied to the set-point of the first loop and to $IAE_1 = 7.32$ and $IAE_2 = 6.87$ when the unit step signal is applied to the set-point of the second loop.

The step responses obtained with the returned controller
In this paper we have proposed a new autotuning methodology for multiloop PID controllers. The closed-loop identification procedure is based on the evaluation of a sequence of standard set-point responses and therefore routine operating data can be effectively employed for retuning the controllers and improve the performance. Being based on the integral of signals, the technique is inherently robust to measurement noise. The rationale of the method relies in the “half-rule” model reduction technique and the tuning rules suggested are appropriate in this context. Simulation results have shown the effectiveness of the method also in the presence of high-order processes. The methodology can be implemented by means of standard control instrumentation.

6. CONCLUSIONS

REFERENCES


