Boundary Control of Nonlinear Distributed Parameter Systems by Input-Output Linearization

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Abstract: The present study focuses on boundary control of nonlinear distributed parameter systems and deals with Dirichlet actuation. Thus, a design approach of a geometric control law that enforces stability and output tracking of a given punctual output is developed based on the notion of the characteristic index. The control performance of the proposed strategy is evaluated through numerical simulation by considering two control problems. The former concerns the control of the temperature of a thin metal rod modelled by a heat equation with a nonlinear source, and the later concerns the control of concentration of a dye in liquid medium modeled by Fick law with nonconstant diffusivity.

Keywords: Distributed parameter systems, nonlinear partial differential equation, geometric control, input-output linearization.

1. INTRODUCTION

Boundary control of distributed parameter systems (DPS) occupies an important place in control theory and constitutes an active research area. For DPS described by nonlinear partial differential equations (PDEs), boundary control remains an open problem (Vazquez and Krstic, 2004). In practice, boundary control is actually easy and convenient for real implementation. In addition, it is an economic approach since it does not need a series of actuators attached to the inner spatial domain of the DPS but requires only one actuator and one sensor placed at the boundaries.

Many authors addressed the boundary control of DPS. For a review, see e.g Maidi et al. (2009b); Padhi and Faruque Ali (2009). Design methodologies of boundary control of a system described by PDE can be split into two approaches (Christofides, 2001; Ray, 1989). The first one called early lumping represents the conventional approach. It consists in performing a spatial discretization of the PDE to derive a set of ordinary differential equations (ODE) that constitute an approximation of the original PDE model, and the controller design is performed in the framework of the classical control theory of lumped parameter systems (LPS). It must be noted that through early lumping, the fundamental control theoretical properties (controllability, observability and stability) are lost (Christofides, 2001; Ray, 1989). This generally leads to high dimension controllers which are difficult to implement (Christofides, 2001). The second approach, termed as late lumping, uses the PDE model without approximation for the controller design. The approximation is performed only for implementation purposes of the controller. Late lumping allows the control designer to avoid losing the distributed nature of the PDE system and to take full advantage of their natural properties. However, direct handling of PDEs is difficult and generally leads to a state feedback control law, which requires the design of an observer for practical implementation. In recent years, several control methods that directly take into account the distributed nature of the processes have been developed especially for linear and quasi-linear system, and deal with distributed control rather than boundary control. (Christofides, 2001; Shang et al., 2005; Wu and Liou, 2001).

Geometric control has proved to be very successful as a control approach of PDE system and successful applications are reported in literature (Christofides and Daoutidis, 1996; Wu and Liou, 2001; Shang et al., 2005; Maidi et al., 2009a, 2010). Designing a control law based on geometric control theory presents the advantage that the PDE model can be used in control design without any approximation, which allows to preserve the fundamental control theoretical properties as the distributed nature of the system is taken into account (Ray, 1989; Christofides, 2001). In addition, control design based on geometric control requires simple calculations (derivation and integration), which are easy to perform.

In this paper, a design approach of a boundary control law for nonlinear distributed parameter systems is proposed. In Section 2, the general boundary control problem is formulated. Then the design approach of a boundary geometric control law is developed in Section 3, which gives also the proposed control strategy. The developed design approach is illustrated by application to two nonlinear
PDEs reported in Section 4 whereas Section 5 is devoted to the conclusion.

2. CONTROL PROBLEM FORMULATION

Consider a dynamical system modeled by nonlinear PDEs with respect to one dimension according to the following state-space representation

$$\frac{\partial x(z,t)}{\partial t} = \mathcal{F} \left( x(z,t), \frac{\partial x(z,t)}{\partial z}, \ldots, \frac{\partial^{(n)} x(z,t)}{\partial z^{(n)}} \right), \quad z \in \Omega$$

(1)

$$y(t) = \mathcal{C} x(z,t) = \int_0^L \delta(z - z_y) x(z,t) \, dz = x(z_y,t), \quad z_y \in [0, L]$$

(2)

the boundary condition at $z = 0$ is assumed to be of Dirichlet type and the manipulated variable $u(t)$ is applied at the left-hand end ($z = 0$), which leads to nonhomogeneous boundary condition at $z = 0$

$$x(0,t) = u(t), \quad (3)$$

The boundary condition at $z = L$ can be either of Dirichlet or Neumann type. The initial condition is given as

$$x(z,0) = x^*(z)$$

(4)

where $x(z,t)$ is the vector of state variables, $u(t)$ is the manipulated variable, $y(t)$ is both the controlled and measured variable, $\mathcal{F}$ and $\mathcal{C}$ are bounded nonlinear and linear operators, respectively. Variables $z$ and $t$ denote position and time respectively. $\Omega = [0, L]$ is the spatial domain. $z_y$ denotes the position of the sensor, i.e. the controlled output. $\delta(z)$ is the Dirac function. In the following, all used functions and operators are assumed to be defined in appropriate functional spaces.

As the manipulated variable appears only in the boundary condition, the boundary condition is inhomogeneous and the control problem is termed boundary control with Dirichlet actuation. In addition, as $0 < z_y < L$, the output (2) defined by the operator $\mathcal{C}$ is punctual. Another type of controlled variable that will be used in this study, is defined as the spatial weighted average given as

$$\bar{y}(t) = \mathcal{L} x(z,t) = \int_0^L c(z) x(z,t) \, dz$$

(5)

where $\mathcal{L}$ is a linear operator and $c(z)$ is a smooth shaping function.

The two considered outputs are illustrated by Figure 1 in the case of the problem of temperature control at a given position $z_y$ in a rod by manipulating the heating source $u(t)$.

The control problem consists to design a boundary control law $u(t)$ that achieves a desired performance of the output $y(t)$ using directly the PDE model to enhance the performances (Christofides, 2001). In this work, a design approach of such a control law is proposed and our presentation is based on the design approach for boundary control of linear PDE systems proposed by Māidi et al. (2010), which will be summarized in Section 3.1.

3. CONTROL LAW DESIGN AND STRATEGY

To solve the boundary control problem of nonlinear PDE systems, an extension of the design approach proposed by Māidi et al. (2010) for linear PDE systems that uses some concepts of geometric control (Isidori, 1995) is proposed. The control law makes use of the characteristic index (Christofides and Daoutidis, 1996; Christofides, 2001), which is a generalization of the concept of relative degree (Isidori, 1995) used in lumped parameter systems (ODE systems) for PDE systems. The characteristic index is the smallest order of the time derivative of a given controlled variable which explicitly depends on the manipulated variable. We begin by summarizing the design approach in the case of linear PDE systems, then an extension for nonlinear PDE systems is proposed.

3.1 Linear PDE system

In the case of linear PDE systems, the model of the dynamical system is described by the following state-space representation

$$\frac{\partial x(z,t)}{\partial t} = \mathcal{A} x(z,t), \quad z \in \Omega$$

(6)

with the following boundary conditions

$$x(0,t) = u(t)$$

(7)

where $\mathcal{A}$ is a linear bounded operator.

According to (Māidi et al., 2009b, 2010) by considering the punctual output $y(t)$ directly in control law design, based on geometric control theory, the problem of system controllability occurs , i.e. the characteristic index does not exist ($\sigma \rightarrow \infty$) and this can be justified by the fact that the system is infinite dimensional. Thus to overcome this difficulty and design a control law, the manipulated input must be inserted in the state equation by means of Dirac function $\delta$, to make the boundary condition homogeneous. Furthermore, the spatial weighted average output $\bar{y}(t)$ must be taken into account instead of the output $y(t)$ in the design approach. This allows us to obtain a finite characteristic index (Māidi et al., 2009b, 2010). The different steps of the design approach are summarized as follows:

Step 1. Make the boundary condition (7) homogeneous by inserting the manipulated variable $u(t)$ in the state equation (6) using the Dirac function $\delta(z)$.

Step 2. Define the spatial weighted average output $\bar{y}(t)$ as in (5).

Step 3. Calculate the successive derivatives of the output $\bar{y}(t)$ with respect to time $t$. 

Fig. 1. Controlled and measured outputs.
\[
\frac{dy(t)}{dt}, \frac{d^2 y(t)}{dt^2}, \ldots, \frac{d^\sigma y(t)}{dt^\sigma} \tag{8}
\]
and choose the shaping function \(c(z)\) in (5) so that the condition of controllability is ensured (existence of a finite characteristic index) (Maidi et al., 2009b, 2010).

**Step 4.** The characteristic index \(\sigma\) suggests requesting a dynamics between an external input \(v(t)\) and the output \(\bar{y}(t)\) characterized by the following equation

\[
\tau_\sigma \frac{d^\sigma y(t)}{dt^\sigma} + \tau_{\sigma-1} \frac{d^{\sigma-1} y(t)}{dt^{\sigma-1}} + \cdots + y(t) = v(t) \tag{9}
\]
Choose the adjustable parameters \(\tau_\sigma, \tau_{\sigma-1}, \ldots, \tau_1\) in order to ensure the stability and to enforce the desired performance of the closed-loop system described by (9).

**Step 5.** Substitute \(\frac{d^\sigma y(t)}{dt^\sigma}, \frac{d^{\sigma-1} y(t)}{dt^{\sigma-1}}, \ldots, \frac{d y(t)}{dt}\) by their expressions calculated in Step 3, and deduce the control law \(u(t)\) under the following state-feedback

\[
u(t) = K \bar{x}(z, t) + S \bar{v}(t) \tag{10}\]
where \(K\) and \(S\) are bounded operators.

**Step 6.** Define the external input \(v(t)\) by an internal robust controller

\[
v(t) = \int_0^t G_1(t - \xi) \{\bar{y}^d(\xi) - \bar{y}(\xi)\} \, d\xi \tag{11}\]
where \(\bar{y}^d(\xi)\) is the desired set point of the output \(\bar{y}(\xi)\) and the function \(G_1(\xi)\), for example, can be chosen as the inverse of an appropriate transfer function.

**Step 7.** Define the desired set point \(\bar{y}^d(t)\) of the output \(\bar{y}(t)\) by an external robust controller

\[
\bar{y}^d(t) = \int_0^t G_2(t - \xi) \{\bar{y}^d(\xi) - \bar{y}(\xi)\} \, d\xi\tag{12}\]
with respect to \(\bar{y}^d(t)\) desired set point of the punctual controlled output \(y(t)\).

**Remark 1.** For practical implementation, the control law (10) requires that the complete state \(x(t, z)\) must be available. From a practical point of view, this is impossible since the state \(x(t, z)\) is infinite thus designing a state estimator is necessary. In this work, it is assumed that the vector of state is fully available to clearly show the effectiveness and the contribution of the proposed design approach. However, for practical implementation issue of the presented control strategies, using a Kalman filter, the reader can refer to the works by Maidi et al. (2009a, 2010).

The global control strategy is summarized in Fig. 2

### 3.2 Nonlinear PDE system

The main idea in designing a geometric control law for PDE linear systems, consists to insert the manipulated variable \(u(t)\) in the state equation of the system by means of a Dirac delta function \(\delta(z)\) to make the boundary condition homogeneous. Thus the boundary control is transformed into a punctual control at \(z = 0\). This is achieved using the Laplace transformation wrt. space.

For PDE nonlinear systems, we propose to perform a linearization around an operating point defined by a uniform profile \(\bar{x}(z)\) and \(\bar{u}\), then to insert the control of the linearized system in the resulting linear state space representation. Following this approach, the punctual control form can be deduced correspondingly for the boundary control of the nonlinear PDE systems (1), which allows the design of the control law following the same steps as for PDE linear systems (see Section 3.1).

For instance, consider the following boundary control

\[
\frac{\partial x(z, t)}{\partial t} = x(z, t) \frac{\partial x(z, t)}{\partial z} \tag{13}\]
\[x(0, t) = u(t), \quad x(L, t) = x_L \tag{14}\]
To insert the control law in the state space (13), one may linearize the system around a uniform spatial profile \(\bar{x}\) and its corresponding control \(\bar{u}\). Thus, defining the perturbation variables \(X\) and \(U\) as

\[X(z, t) = x(z, t) - \bar{x}, \quad U(t) = u(t) - \bar{u} \tag{15}\]
and inserting into (13) and (14), yields the perturbation equations

\[
\frac{\partial X(z, t)}{\partial t} = X(z, t) \frac{\partial X(z, t)}{\partial z} + \bar{x} \frac{\partial X(z, t)}{\partial z} \tag{16}\]
\[X(0, t) = U(t) \tag{17}\]
\[X(L, t) = x_L \tag{18}\]
The linearized equations are now obtained by omitting second order terms in the perturbation equations. Thus, the linearized model results

\[
\frac{\partial X(z, t)}{\partial t} = \bar{x} \frac{\partial X(z, t)}{\partial z} \tag{19}\]
with boundary conditions (17) and (18).

The Laplace transform in space domain of (19) gives

\[
\frac{d\bar{X}(s, t)}{dt} = \bar{x} \left( \bar{X}(z, t) - \bar{X}(0, t) \right) \tag{20}\]
using the Laplace transform of a spatial derivative

\[
\mathcal{L} \left( \frac{df(z)}{dz} \right) = s \bar{f}(s) - f(0) \tag{21}\]
where in fact \(f(0)\) refers to the value of \(f\) for strictly negative values of \(z\).

Equation (20) can be written as

\[
\frac{d\bar{X}(s, t)}{dt} = \bar{x} \bar{s} \bar{X}(z, t) - \bar{x} \bar{X}(0, t) \tag{22}\]
or
\[
\frac{d\bar{X}(s, t)}{dt} = \bar{x} \bar{s} \bar{X}(z, t) - \bar{x} \bar{U}(t) \tag{23}\]
using boundary condition (17).

Now, the inverse Laplace transform can be used based on the following property

\[
\mathcal{L}^{-1} \left( \bar{f}(s) \right) = \delta f(z, t) \tag{24}\]
where \(\delta f(z, t)\) is the deviation variable defined as

\[\delta f(z, t) = f(z, t) - f(0^-, t) \tag{25}\]
with \(f(0^-, t)\) referring to the value of \(f\) for negative values of \(z\).

Thus the inverse Laplace transform of equation (23) yields

\[
\frac{\partial \delta X(z, t)}{\partial t} = \bar{x} \frac{\partial \delta X(z, t)}{\partial z} - \bar{x} \bar{U}(t) \delta(z) \tag{26}\]
Fig. 2. Control strategy for PDE systems.

Defining the initial value of the variable as
\[ X(0^+, t) = 0 \]
the final equation results
\[ \frac{\partial X(z, t)}{\partial t} = \ddot{x} \frac{\partial X(z, t)}{\partial z} - \ddot{x} \delta(z) U(t) \]  
Equation (28) together with the condition (27) is the final form of the punctual control. Note that the boundary condition (27) is homogeneous and the manipulated variable \( U(t) \) appears in state-space representation (26) for any \( \ddot{x} \). Therefore, the certainty equivalence nonlinear punctual control form follows
\[ \frac{\partial x(z, t)}{\partial t} = x(z, t) \frac{\partial x(z, t)}{\partial z} - \ddot{x} \delta(z) u(t) \]
\[ x(0^-, t) = 0, \quad x(L, t) = x_L \]  
Thus, the corresponding punctual control form of (1) and (3) is
\[ \frac{\partial x(z, t)}{\partial t} = F(x(z, t), \frac{\partial x(z, t)}{\partial z}, \ldots, \frac{\partial^{(n)} x(z, t)}{\partial z^{(n)}}) \]
\[ + f(\ddot{x}) \delta^{(r)}(z) u(t), \quad r \geq 0 \]
\[ x(0, t) = 0 \]
where the function \( f(.) \) is derived by the linearization procedure as explained according to the example above. The integer \( r \) is determined from the inverse Laplace transform in space. For instance, for the example presented above, from (26) one gets \( r = 0 \).

To design the control law \( u(t) \), a model uncertainty is assumed by considering \( f(\ddot{x}) = 1 \), which will be handled by a robust controller. Thus, the punctual control form that will be considered for control law design is
\[ \frac{\partial x(z, t)}{\partial t} = F(x(z, t), \frac{\partial x(z, t)}{\partial z}, \ldots, \frac{\partial^{(n)} x(z, t)}{\partial z^{(n)}}) \]
\[ + \delta^{(r)}(z) u(t) \]
with the boundary condition (32).

Once the punctual control form is obtained, the same steps of the design approach as for linear PDE systems summarized in Section 3.1 are to be followed.

4. APPLICATION EXAMPLES

4.1 Heated rod with nonlinear source

Control problem: consider the control of temperature of a thin metal rod with a nonlinear source (Fig. 3). The temperature at \( z = 0 \) can be manipulated by adjusting the steam pressure. The right-hand end is insulated, which leads to the Neumann type boundary condition at \( z = L \).

The model of the system is given as
\[ \frac{\partial T(z, t)}{\partial t} = \alpha \frac{\partial^2 T(z, t)}{\partial z^2} - \beta T^2(z, t), \quad 0 < z < L \]  
\[ T(0, t) = u(t), \quad \frac{\partial T(z, t)}{\partial z} \bigg|_{z=L} = 0 \]  
\[ y(t) = T(L/4, t) \]  
with \( \alpha, \beta > 0 \). The objective is to design a control law \( u(t) \) that achieves a desired set point \( y_d(t) \) for the controlled output \( y(t) \). The initial condition \( T(z, 0) \) is the steady-state profile defined by \( u(0) = 25^\circ C \).

Control law: the design approach of Section 3.1 gives:

Step 1. Following the procedure presented in Section 3.2, the punctual control form is
\[ \frac{\partial T(z, t)}{\partial t} = \alpha \frac{\partial^2 T(z, t)}{\partial z^2} - \beta T^2(z, t) - \alpha \dot{c}(z) u(t) \]  
\[ T(0^-, t) = 0, \quad \frac{\partial T(z, t)}{\partial z} \bigg|_{z=L} = 0 \]  

Step 2. The weighted output is \( y(t) = \int_0^L c(z) T(z, t) \, dz \),

Step 3. The first derivative of \( y(t) \) with respect to \( t \) is:
\[ \frac{dy(t)}{dt} = \int_0^L c(z) \frac{\partial T(z, t)}{\partial t} \, dz = \int_0^L c(z) \left[ \alpha \frac{\partial^2 T(z, t)}{\partial z^2} - \beta T^2(z, t) \right] \, dz - \alpha \int_0^L c(z) \dot{c}(z) \, dz \, u(t) \]

Step 4. The desired dynamics between the external input \( v(t) \) and the output \( y(t) \) is
\[ \tau_1 \frac{dy(t)}{dt} + \bar{y}(t) = v(t) \]  

Step 5. Substituting \( \frac{dy(t)}{dt} \) by its expression given by (39) in (40), the control law results

\[
u(t) = \frac{1}{\tau_1} \left[ v(t) - \int_0^L zT(z, t) \, dz - \tau_1 \alpha \int_0^L z \frac{\partial^2 T(z, t)}{\partial z^2} \, dz \right] + \tau_1 \beta \int_0^L zT^2(z, t) \, dz
\]  

Step 6. The external input \( v(t) \) is defined by a PI controller

\[ v(t) = K^i \left[ \left( \bar{y}^d(t) - \bar{y}(t) \right) + \frac{1}{T^i} \int_0^t \left( \bar{y}^d(\xi) - \bar{y}(\xi) \right) \, d\xi \right]
\]

where \( K^i \) and \( T^i \) are respectively the proportional gain and integral time constant of the PI controller.

Step 7. The desired set point \( \bar{y}^d(t) \) is also defined by a PI controller

\[ \bar{y}^d(t) = K^e \left[ \left( y^d(t) - y(t) \right) + \frac{1}{T^e} \int_0^t \left( y^d(\xi) - y(\xi) \right) \, d\xi \right]
\]

where \( K^e, T^e \) and \( y^d(t) \) are respectively the proportional gain, integral time constant and the desired set point of the controlled output \( y(t) \).

Simulation results: for simulation purpose of the closed-loop system, the method of lines (Wouwer et al., 2001) is used by considering \( N = 100 \) discretization points. The control is held constant over the sampling period equal to 0.02 s in all simulation runs performed. All integral terms in the control law are evaluated numerically using the trapezoidal method. The terms involving differentiation according to the space variable \( z \) are evaluated by means of the finite differences. To avoid the consequences due to brutal set point steps, the set point \( y^d(t) \) has been filtered by a first order filter with a time constant \( \tau_f = 0.2 \) s. The system parameters are \( \alpha = 1 \, \text{m}^2 \, \text{s}^{-1}, \beta = 1 \, \text{s}^{-1} \) and \( L = 1 \, \text{m} \). The desired constant time \( \tau_1 \) is taken equal to 0.2 s. The parameters of the internal PI controller are tuned so that the denominator of the transfer function \( \frac{Y(s)}{Y^d(s)} \) is close to a polynomial minimizing an ITAE criterion (Corrion, 2004), the obtained values are \( K^i = 7.8 \) and \( T^i = 1.03 \) s. The tuning of the external PI controller has been performed by trial and error since the system \( \bar{y}^d - y(t) \) is nonlinear infinite dimensional, the retained parameters are \( K^i = 0.2 \) and \( T^i = 0.25 \) s.

In order to evaluate the performance of the control strategy, it is assumed that the whole vector of state variables is available. Thus, a set point step corresponding to \( y^d(t) = 8^\circ \text{C} \) of the temperature is specified at \( t = 1 \) s followed by a variation of \( \pm 20\% \) of the diffusivity constant \( \alpha \). From Fig. 4, it is clear that the controller behaves adequately and tracks perfectly the set point in spite of these significant parameter uncertainties. Also, the control moves of \( u(t) \) are smooth and acceptable.

4.2 Dye diffusion

Control problem: consider the example of the diffusion of a dye, in liquid medium, where the diffusivity depends on the species concentration (Fig. 5). The spatio-temporal evolution of the concentration of the dye \( C(z, t) \) is modeled by the nonlinear Fick law

\[ \frac{\partial C(z, t)}{\partial t} = \frac{\partial}{\partial z} \left[ D(C(z, t)) \frac{\partial C(z, t)}{\partial z} \right] \quad (42) \]

where \( C(z, t) \) is the concentration of the diffusing species and \( D(\cdot) \) is a nonconstant diffusion coefficient. This equation similar to Fourier law in heat transfer occurs in nonlinear problems of mass transfer and flows in porous media. Expanding the right-hand side will give

\[ \frac{\partial C(z, t)}{\partial t} = \frac{\partial D(C(z, t))}{\partial C(z, t)} \left( \frac{\partial C(z, t)}{\partial z} \right)^2 + D(C(z, t)) \frac{\partial^2 C(z, t)}{\partial z^2} \]  

(43)

Initially, the dye is concentrated at the left-hand end \((z = 0)\) and moves from the position of higher concentration \((z = 0)\) to position of lower concentration \((z = L)\). The diffusivity is assumed to be linear, i.e. \( D(C(z, t)) = \gamma C(z, t) \) with a constant \( \gamma \) parameter. The boundary conditions are
\[ C(0, t) = u(t), \quad C(z, t)|_{z=L} = 0 \quad (44) \]

and the initial condition \( C(z, 0) \) is the steady-state profile defined for \( u(t) = 0.1 \text{ mol. m}^{-3} \).

The problem consists to design a control law that allows to have a desired concentration of the dye at location \( z = L/2 \)
\[ y(t) = C(L/2, t) \quad (45) \]

**Control law:** the system to be controlled is nonlinear and the control variable \( u(t) \) is applied at \( z = 0 \), so by taking \( c(z) = z \), the proposed design approach yields the following control law
\[
\begin{align*}
  u(t) &= \frac{1}{\tau \gamma} \left[ v(t) - \int_0^L z C(z, t) dz - \tau \gamma \left( \int_0^L z \left( \frac{\partial C(z, t)}{\partial z} \right) dz \right)^2 \right. \\
  &\quad + \left. \int_0^L z C(z, t) \frac{\partial^2 C(z, t)}{\partial z^2} dz \right] \\
  &\quad (46)
\end{align*}
\]

**Simulation results:** the simulation run concerns a set point tracking problem and the previous simulation conditions are kept. \( \gamma = 2 \). The PI controllers are tuned using the same approaches as for the heated rod. The obtained parameters are: \( K^i = 6.7 \), \( K^f = 0.93s \), \( K^c = 0.05 \) and \( K^e = 0.2s \). Thus, a step set point corresponding to \( y^s(t) = 0.1 \text{ mol. m}^{-3} \) is specified at \( t = 2s \). For the state-feedback, the adjustable parameter \( \tau = 0.8s \). In Fig. 5, the resulting dynamic behavior of the characteristic variables is shown. The results show that the imposed set point is reached perfectly (Fig.6a) with acceptable variation of the manipulated variable (Fig.6b).

5. CONCLUSION

In this paper, a design approach of a boundary control, by input-output linearization, for a nonlinear PDE system is developed and its validity is demonstrated by two applications. The control performances are evaluated through numerical simulation by considering both tracking and sudden fluctuations of system parameters. The obtained simulation results show the effectiveness of the developed control strategy. This study demonstrates that the design of the boundary control of PDE system, based on input-output linearization, is a very successful control approach since it leads to a control law that enhances the control performance by preserving the fundamental control properties, consequently the distributed nature of the PDE system.

REFERENCES


Fig. 6. Dye diffusion: set point tracking.