Assessing Hybrid Policy of Carbon Tax and Emissions Trading under Uncertainty for Preserving Global Environment

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Abstract: In this paper we show the effectiveness of a hybrid policy of carbon tax and emissions trading under uncertainty compared with the use of each policy independently. A mathematical model is described for assessing quantitatively how the hybrid policy of carbon tax and emissions trading would be effective to achieve the targeted reduction of the post-Kyoto target. We use a method of decision analysis called Prospect Theory under Uncertainty.

Keywords: decision theory, utility functions, uncertainty, policy evaluation, global environment.

1. INTRODUCTION

The Kyoto protocol, which is due to expire at the end of 2012, is an agreement made under the United Nations Framework Convention on Climate Change (UNFCCC, 2003). Countries that ratify this protocol commit to reduce their emissions of CO₂ and five other greenhouse gases (GHG), or engage in emissions trading if they maintain or increase emissions of these gases (Hertel, ed., 1993).

Post-Kyoto negotiation refers to high level talks and major emitting countries including United States, China and India declared their mid-term targets just before the UNFCCC COP15 starting on December 7, 2009, in Copenhagen (RITE 2009).

Japanese government is conservative to introduce carbon tax, emissions trading between various industries in Japan and also between countries in the world, to feedback the carbon tax revenue for reducing the greenhouse gas emissions. This is because the industrial circles in Japan are opposing against these policies for fear that the economic competitiveness is decreasing.

In this paper, we consider the carbon tax and the emissions trading among 5 developed countries and Russia as an emissions permit supplier in order to meet the CO₂ emissions reduction target of post-Kyoto protocol. There exists various kinds of uncertainty when we evaluate the effectiveness of various policies for preserving global environment.

In this paper we try to take into account uncertainty as much as possible even for the case that we do not know the probability of occurring each event. In such case we cannot use so-called Bayes’ probability. Instead we need to use Dempster-Shafer (1976) probability theory where we could postulate basic probability for a set of events, and we use a method of decision analysis called prospect theory under uncertainty (Tamura 2009) which was developed based on the prospect theory developed by Kahneman and Tversky (1979).

2. MODELING FOR POLICY ASSESSMENT

Taking into account the uncertainty we use a modified version of the mathematical model described in Tamura and Kimura (2008). The objective countries are Japan, United States, United Kingdom, France and Germany. Russia is added to these five countries as an emissions permits supplier in the emissions trading.

This model consists of a profit maximization problem and sub-problems. The former problem is based on the dynamic input-output analysis and expresses the profit maximization behavior of each country, which introduces the carbon tax and the emissions trading to meet the emissions reduction target. The latter problems express domestic transaction, international trades, and the CO₂ cost, which describes the cost to reduce CO₂ plus the cost for paying carbon tax.

Figure 1 shows CO₂ marginal reduction cost with respect to CO₂ reduction rate. In general, the more energy efficiency is improved, the more marginal reduction cost of CO₂ emissions is increased (Energy Information Administration, 2003). In Fig. 1, \( t_0 \) denotes the CO₂ marginal reduction cost at the present time. If the carbon tax rate of \( t_1 \) was imposed,
each economic unit would reduce CO₂ up to the reduction rate of \( s_1 \). At this point CO₂ marginal reduction cost would be equal to the carbon tax rate, since each economic unit would minimize the CO₂ cost that is composed of the cost for reducing CO₂ and the cost for carbon tax.

![Fig. 1. CO₂ cost composed of cost for CO₂ reduction and carbon tax.](image)

Hence, the total CO₂ cost is described as

\[
k = \min_{s_1} \left[ \int_0^{s_1} f(s) \, ds + (1 - s_1) t_1 \right]
\]

subject to \( (1 - s_1) E \leq T \)

where \( t_1 \) denotes carbon tax rate, \( s_1 \) denotes CO₂ reduction rate, \( f(*) \) denotes cost function for CO₂ reduction, \( E \) denotes total amount of CO₂ emission before reduction and \( T \) denotes CO₂ emission target. The first term of eqn. (1a) shows the cost for reducing CO₂ emission and the second term shows the carbon tax to be paid. Equation (1b) shows the constraint to meet that the total amount of CO₂ emission must be less than or equal to the emission target \( T \). CO₂ reduction rate \( s_1 \) in Fig. 1 denotes an optimal solution to eqns. (1a) and (1b).

When we take into account the emissions trading besides the carbon tax, total CO₂ cost is described as

\[
k = \int_0^{s_2} f(s) \, ds + (1 - 2s_2 + s_1) t_2
\]

where if the carbon tax rate of \( t_2 \) was imposed, each economic unit would reduce CO₂ up to the reduction rate of \( s_2 \) as shown in Fig. 2. Price for emissions trading is also \( t_2 \). In this case the economic unit (buyer) of emissions right would buy \((s_1-s_2) t_2 \) and carbon tax of \((1-s_2) t_2 \). As shown in Fig. 3 when \( s_1>s_2 \) the economic unit would buy the emissions right, on the contrary, when \( s_1<s_2 \) and \( t_1<t_2 \) the economic unit would sell emissions right.

![Fig. 2. CO₂ cost for emissions right buyer composed of cost for CO₂ reduction, cost for buying emissions right and carbon tax.](image)

3. EVALUATION BY USING PROSPECT THEORY UNDER UNCERTAINTY

3.1 Prospect Theory under Risk

Kahneman and Tversky (1979) proposed prospect theory in order to explain people’s decision making such that

(a) People’s value judgment is highly dependent on the reference point, that is, people are more focused on changes in their value (utility) states than the states themselves.

(b) People’s marginal value (utility) is diminishing both in gain domain and in loss domain.

(c) Value function in loss domain is steeper than in gain domain, that is, losses looms larger than gains and people have a tendency of loss aversion.

(d) People feel that weight for very small probability is disproportionate and they have a tendency to overestimate for low probability and underestimate for higher probability, that is, subjective probability is severely biased by anchoring.

Let \( X \) be a set of all outcomes, \( x \in X \), and \( A \) be a set of all risky alternatives; a prospect (risky alternative) \( \ell \in A \) is written as

\[
\ell = (x_1, x_2, \ldots, x_n; p_1, p_2, \ldots, p_n)
\]

that yields outcome \( x_i \in X \) with probability \( p_i, i = 1, 2, \ldots, n \), where \( \sum p_i = 1 \).

In prospect theory (PT), the value \( V \) for this prospect is
evaluated using the evaluation function

\[ V = \sum_{j=1}^{n} \pi(p_j) w(x_j) \]  \hspace{1cm} (4)

where the value function \( V \) is convex with a gentle curve in the gain domain, while it is concave and its curve is steeper in the loss domain, as shown in Fig. 3. This shows that people, in general, are loss averse. Tversky and Kahneman (1992) proposed a value function

\[ v(x) = \begin{cases} 
\lambda x^\theta & \text{if } x > 0 \\
-\lambda (-x)^\theta & \text{if } x < 0 
\end{cases} \]  \hspace{1cm} (5)

The weighting function for prospect theory (PT) is a convex function as shown in Fig. 4, so a small probability is weighted higher and middle or large probabilities are weighted lower. However, this weighting function is not defined near the end points 0 and 1. The dotted line in Fig. 4 shows the case for the expected utility (EU) model.

The weighting function \( \pi \) for prospect theory (PT) is a convex function as shown in Fig. 4, so a small probability is weighted higher and middle or large probabilities are weighted lower. However, this weighting function is not defined near the end points 0 and 1. The dotted line in Fig. 4 shows the case for the expected utility (EU) model.

The weighting function \( w \) shown in Fig. 5 was originally proposed for cumulative prospect theory (CPT) by Tversky and Kahneman (1992). But this weighting function could be also used for prospect theory as well. Derivative of this weighting function is larger for the region that probability is nearly equal to 0 or 1.

![Fig. 3. Value function.](image)

![Fig. 4. Weighting function used in PT.](image)

![Fig. 5. Weighting function used in CPT.](image)

3.2 Prospect Theory under Uncertainty

A. Basic principle

In this section we deal with the case where the probability of occurrence for each event is unknown. When we describe the degree of ignorance and uncertainty by the basic probability of Dempster and Shafer (1976) theory, the problem is how to represent the value of a set element in constructing a measurable value function under uncertainty based on this concept.

Let the value function under uncertainty based on this basic probability be

\[ f^*(B, \mu) = \pi'(\mu)v^*(B) \]  \hspace{1cm} (6)

where \( B \) denotes a set element, \( \mu \) denotes the basic probability, \( \pi' \) denotes the weighting function for the basic probability, and \( v^* \) denotes the value function with respect to a set element. The set element \( B \) is a subset of \( \Lambda = 2^\Theta \) where \( \Theta \) denotes a set containing every possible element. Equation (6) is an extended version of the value function, eqn. (4), where an element is extended to a set element and the Bayes’ probability is extended to the Dempster-Shafer basic probability.

For identifying \( v^* \), we need to find the preference relations among set elements, which is not an easy task. If the
number of elements contained in the set $\Theta$ is getting larger, and the set element $B$ contains a considerable number of elements, it is not practical to find $v^*$ as a function of $B$. To cope with this difficulty we could use some appropriate axiom of dominance as follows:

**Axiom of Dominance 1:**
In the set element $B$ let the worst outcome be $m_B$ and the best outcome be $M_B$. For any $B_1, B_2 \subset \Lambda$, $2^\Theta$

\[
\begin{align*}
M_{B_1} &\leq M_{B_2}, \quad M_{B_1} \leq M_{B_2} \Rightarrow B_1 \leq B_2 \\
m_{B_1} &\leq m_{B_2}, \quad M_{B_1} \sim M_{B_2} \Rightarrow B_1 \sim B_2.
\end{align*}
\]

Our descriptive model $f^*(B, \mu)$ could resolve the so-called Ellsberg (1961) paradox by restricting a set element $B$ to

\[
\Omega = \left\{ (m, M) \in \Theta \times \Theta : m \leq M \right\} 
\]

(7)

where $m$ and $M$ denote the worst and the best outcome in the set element $B$, respectively. In this case eqn. (6) is reduced to

\[
f^*(\Omega, \mu) = \pi'(\mu)v^*(\Omega).
\]

(8)

Suppose we look at an index of pessimism $\alpha(m, M)$ such that the following two alternatives are indifferent. (The index of optimism $\beta(m, M) = 1 - \alpha(m, M)$ may be defined instead of the index of pessimism depending upon the situation.)

**Alternative 1:** One can receive $m$ for the worst case and $M$ for the best case. There exists no other information.

**Alternative 2:** One receives $m$ with probability $\alpha(m, M)$ and receives $M$ with probability $1 - \alpha(m, M)$, where $0 < \alpha(m, M) < 1$.

If one is quite pessimistic, $\alpha(m, M)$ becomes nearly equal to 1, or if one is quite optimistic, $\alpha(m, M)$ becomes nearly equal to zero. If we incorporate this pessimism index $\alpha(m, M)$ into eqn. (7), the value function is described as

\[
v^*(\Omega) = v^*((m, M)) = \alpha(m, M)v'(m) + (1 - \alpha(m, M))v'(M)
\]

(9)

where $v'$ denotes a value function for a single element.

Incorporating Dempster-Shafer probability theory in the descriptive model $f^*(\Omega, \mu)$ of a value function under uncertainty, we could model the lack of belief which cannot be modeled by Bayes’ probability theory.

B. Further Axiom of Dominance

There exist some cases for which Axiom of Dominance 1 is unsuitable.

**Definition:**
Let the elements in the set element $B$ be $a_1, a_2, \ldots, a_n$ such that $a_i < a_{i+1}, \ i = 1, 2, \ldots, n-1$, the value of element $a_i, \ i = 1, 2, \ldots, n$ be $v(a_i)$ and the average value of elements be

\[
v(g) = \frac{\sum_{i=1}^{n} v(a_i)}{n}
\]

Further, let the pessimism index decided by the question for the worst element $a_1 = m$ and the best element $a_n = M$ be $\alpha(m, M)$. We assume the value $h$ of the set element $B$ to be

\[
h(B \mid \alpha) = a + b e^{-\alpha(m, M)} \quad \text{if} \quad v(g) = \frac{v(M) + v(m)}{2}
\]

\[
h(B \mid \alpha) = a + b \alpha(m, M) \quad \text{if} \quad v(g) = \frac{v(M) + v(m)}{2}
\]

where unknown parameters $a, b, c$ are decided by

\[
h(B \mid 0) = v(M), \ h(B \mid 0.5) = v(g), \ h(B \mid 1) = v(m).
\]

We introduce Axiom of Dominance 2 in order to evaluate values based on the above definition as follows:

**Axiom of Dominance 2:**

\[
h(B_1 \mid \alpha) < h(B_2 \mid \alpha) \Rightarrow B_1 < B_2
\]

\[
h(B_1 \mid \alpha) = h(B_2 \mid \alpha) \Rightarrow B_1 \sim B_2
\]

By using Axiom of Dominance 2, we are able to write the value function in the prospect theory under uncertainty as

\[
f^*(h(B \mid \alpha), \mu) = \pi'(\mu)h(B \mid \alpha).
\]

(10)

We could properly describe the value judgment of pessimistic people and optimistic people, respectively, by using eqn. (10).

We are able to evaluate the value $V$ of the prospect that includes the case where the probability of occurrence for each element is unknown but the basic probability of occurrence for each set element is known through the evaluation function

\[
V = \sum_{j=1}^{n} \pi(\mu_j)v^*(B_j)
\]

(11)

where $\pi$ denotes the weighting function of prospect theory and $v^*$ denotes the value function with respect to a set element $B_j$. 

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4. DATA USED FOR ANALYSIS

In this Section we show quantitatively how the hybrid policy of carbon tax and emissions trading would be effective to achieve the targeted reduction of the post-Kyoto Protocol shown in Table 1.

Table 1. Mid-term target of CO₂ reduction for the post-Kyoto protocol (RITE, 2009)

<table>
<thead>
<tr>
<th></th>
<th>Japan</th>
<th>U.S.</th>
<th>U.K.</th>
<th>France</th>
<th>Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td>Emissions in 1990 (Mt-C)</td>
<td>311.8</td>
<td>1380.4</td>
<td>161.1</td>
<td>107.9</td>
<td>281.5</td>
</tr>
<tr>
<td>Emissions in 2020 (Mt-C)</td>
<td>338.6</td>
<td>1650.3</td>
<td>152.9</td>
<td>112.0</td>
<td>241.2</td>
</tr>
<tr>
<td>Targeted reduction from 1990</td>
<td>-25%</td>
<td>-0.50%</td>
<td>-26%</td>
<td>-20%</td>
<td>-40%</td>
</tr>
<tr>
<td>Upper limit in 2020 (Mt-C)</td>
<td>233.8</td>
<td>1373.2</td>
<td>119.2</td>
<td>86.3</td>
<td>168.9</td>
</tr>
<tr>
<td>Surplus in 2020 (Mt-C)</td>
<td>-104.8</td>
<td>-277.0</td>
<td>-33.7</td>
<td>-25.7</td>
<td>-72.3</td>
</tr>
</tbody>
</table>

Input-output coefficients are obtained from Japan, US, EU and ASEAN International Input-Output Table (1990). It is assumed that carbon tax is imposed on the consumption of coals, natural gas and oil products. It is assumed that CO₂ emissions coefficients in Japan, that is, the amount of CO₂ emissions per unit production in each sector is estimated from the basic unit of CO₂ emissions based on Basic Unit of CO₂ emissions. Based on the Input-Output Table (1997) CO₂ emissions coefficients in other countries are estimated from Japanese emissions coefficients and the amount of consumption of coals, natural gas and oil products in each country.

Marginal cost function for reducing CO₂ is estimated for each country based on the efficiency in energy consumption as follows:

**Japan:**

\[ f_i(s_i) = \begin{cases} 
297(s_i + 0.262)^{2.08} & f_i(s_i) \leq 100 \\
3595(s_i - 0.046)^{2.85} & f_i(s_i) > 100 
\end{cases} \]

**U.S.:**

\[ f_i(s_i) = \begin{cases} 
297(s_i + 0.492)^{2.08} & f_i(s_i) \leq 100 \\
3595(s_i + 0.184)^{2.85} & f_i(s_i) > 100 
\end{cases} \]

**U.K.:**

\[ f_i(s_i) = \begin{cases} 
297(s_i + 0.291)^{2.08} & f_i(s_i) \leq 100 \\
3595(s_i - 0.046)^{2.85} & f_i(s_i) > 100 
\end{cases} \]

**France:**

\[ f_i(s_i) = \begin{cases} 
297(s_i + 0.275)^{2.08} & f_i(s_i) \leq 100 \\
3595(s_i - 0.035)^{2.85} & f_i(s_i) > 100 
\end{cases} \]

**Germany:**

\[ f_i(s_i) = \begin{cases} 
297(s_i + 0.263)^{2.08} & f_i(s_i) \leq 100 \\
3595(s_i - 0.045)^{2.85} & f_i(s_i) > 100 
\end{cases} \]

**Russia:**

\[ f_i(s_i) = \begin{cases} 
297(s_i + 0.107)^{2.08} & f_i(s_i) \leq 100 \\
3595(s_i - 0.201)^{2.85} & f_i(s_i) > 100 
\end{cases} \]

5. SCENARIO SETTING

We will consider carbon tax and emissions trading as the national policy. To evaluate the influence on the revenue, the price of emissions trading, and the amount of CO₂ emissions, etc., we postulate three scenarios as follows:

**Scenario 1:** Each country reduces its emissions by taking into account the carbon tax only without the emissions trading.

**Scenario 2:** Each country reduces its emissions by taking into account the emissions trading only without the carbon tax.

**Scenario 3:** Each country reduces its emissions by taking into account both the carbon tax and the emissions trading.

6. POLICY ASSESSMENT

Table 2 shows the carbon tax rate in Scenario 1 and in Scenario 3 to achieve the targeted reduction of CO₂ in the post-Kyoto protocol in 2020.

Table 2. Carbon tax rate in Scenario 1 and in Scenario 3 to achieve the targeted reduction in 2020 ($/ton carbon).

<table>
<thead>
<tr>
<th></th>
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<th>Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1</td>
<td>480.3</td>
<td>51.3</td>
<td>73.6</td>
<td>71.5</td>
<td>89.8</td>
</tr>
<tr>
<td>Scenario 3</td>
<td>68.4</td>
<td>68.4</td>
<td>68.4</td>
<td>68.4</td>
<td>68.4</td>
</tr>
</tbody>
</table>

In Scenario 1 the carbon tax rate in Japan is especially high. Since the energy efficiency in Japan is already quite high, it is not possible to reduce CO₂ emissions further without
imposing high carbon tax rate. In case of Scenario 3 we could get much lower carbon tax rate with the same level for each country compared with in case of Scenario 1. This is because in Scenario 3 we could get the same level of the carbon tax rate with the price for emissions trading.

Based on eqn. (10) of the prospect theory under uncertainty we evaluate the achievement ratio of CO2 reduction target and cost saving for unit CO2 reduction where we use $\beta = 0.88$, $\lambda = 2.25$ in eqn.(5) of value function and we use three different values 0.2, 0.5 and 0.8 for the pessimism index.

Figure 6 shows the degree of achieving CO2 reduction target. Figure 7 shows cost saving for unit CO2 reduction.

As seen from Fig. 6, CO2 reduction target of post-Kyoto protocol is not attained at all by Scenario 2. CO2 reduction target is attained both in Scenario 1 and in Scenario 3 for any value of pessimism index. This result implies that the emissions trading without carbon tax is not effective for preserving global environment.

Figure 7 shows that Scenario 3 is the most effective policy that it provides the highest cost saving for unit CO2 reduction. This implies that the policy mix of carbon tax and emissions trading is the most effective for preserving global environment.

7. CONCLUDING REMARKS

Since Japanese industries have already improved the efficiency of consuming energy extensively, it is not rational to achieve the emissions reduction target with the carbon tax only. Using the policy mix of carbon tax and emissions trading, it is found that much more cost saving is achieved for both CO2 cost and unit CO2 reduction cost.

In this paper it is assumed that the revenue of the carbon tax is assigned to the general funds. But if we assume that the revenue of the carbon tax is assigned to the earmarked funds, all of the revenue can be used for the investment of the CO2 reduction behavior or the purchase of the CO2 emissions permit. Then, the sum of the cost for CO2 reduction and expenses for the carbon tax could be much less than the general funds case. We need further research for this aspect.

REFERENCES