On Attitude Synchronization of Multiple Rigid Bodies with Time Delays

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Abstract: This paper investigates the attitude synchronization problem under communication time delays. The encountered difficulty in synchronizing rigid body attitudes is non-convexity of the rotation group. Through exploiting the geometrical structure of the rigid body attitude kinematics, we propose a PD-like synchronization control law. Using Lyapunov stability analysis, it is shown that the attitude synchronization errors converge asymptotically to zero, and that the proposed controller achieves robust stability for communication time delays. Simulation results are presented to show the performance of the proposed synchronization control algorithm.

Keywords: Attitude synchronization; time delay; PD controller.

1. INTRODUCTION

Synchronization of multi-agent systems has received great attention in recent years and it has important applications in various areas such as spacecraft formation control (Wang, Hadaegh, and Lau, 1999; Ren, 2007; Dimarogonas, Tsiotras, and Kyriakopoulos, 2006), robot telemanipulation (Lee and Spong, 2006; Chopra, Spong, and Lozano, 2008), cooperative control of multiple robot manipulators (Rodrigues-Angeles and Nijmeijer, 2004; Chung and Slotine, 2009), unmanned air vehicles (UAVs) and mobile robots (Cheah, Hou, and Slotine, 2009), and attitude synchronization of multiple rigid bodies (Sarlette, Sepulchre, and Leonard, 2009; Meng, Ren, and You, 2009; Dimarogonas, Tsiotras, and Kyriakopoulos, 2006; Ren, 2010; Chung, Ahsun, and Slotine, 2009).

Attitude synchronization of rigid bodies or multiple spacecraft, as a special control problem, has some distinct features incurred by its complex rigid body attitude kinematics. Moreover, it is impossible to obtain a rotation matrix via adding two different rotation matrices, that is, the space formed by rotation matrices is non-convex, which is well-known in Lie group literature. To tackle this problem, several approaches via exploiting the geometric structure of the attitude kinematics have been proposed (Sarlette, Sepulchre, and Leonard, 2009; Smith, Hausmann, and Leonard, 2001). And other control schemes have also been presented, which are based on various representations of the attitude synchronization error (Meng, Ren, and You, 2009; Dimarogonas, Tsiotras, and Kyriakopoulos, 2006; Ren, 2010; Chung, Ahsun, and Slotine, 2009; VanDyke and Hall, 2006; Abdessameud and Tayebi, 2009). However, most of these control schemes cannot tolerate communication delays, which is thought to be ubiquitous in the attitude synchronization of multiple rigid bodies or master-slave teleoperator end-effectors.

One attitude synchronization scheme that provides robust stability for constant time delay is from Igarashi, Hatanaka, Fujita, and Spong (2007; 2009). Assuming the positive definiteness of the rotational matrix, it achieves asymptotic convergence of attitude synchronization errors at the kinematic level. Nevertheless, the assumption of positive definite rotational matrix implies that the controller can only work for the case of attitude deviation that is below 90 degree rotation. In addition, Igarashi, Hatanaka, Fujita, and Spong (2007; 2009) did not take the attitude dynamics into consideration, and thus it is unclear about the total system’s behavior if taking both the attitude kinematics and dynamics into account. Although there is only limited work in delayed attitude synchronization, many synchronization-based control approaches have been proposed to deal with communication time delays in bilateral teleoperation (Lee and Spong, 2006; Chopra, Spong, and Lozano, 2008; Chopra, Spong, Ortega, and Barabanov, 2006; Nuno, Ortega, Basanez, and Barabanov, 2008) and synchronization of multi-robot networks (Chung and Slotine, 2009). Master-slave joint position synchronization (joint space of the manipulator $R^n$ is Euclidean and convex) is achieved via injecting joint dissipation both at the master and the slave sides. However, in attitude synchronization problem, the rotational group $SO(3)$ is non-Euclidean and non-convex, which presents a major obstacle to accommodate the time delays.

Inspired by the results for bilateral teleoperation (Lee and Spong, 2006; Chopra, Spong, Ortega, and Barabanov, 2006; Nuno, Ortega, Basanez, and Barabanov, 2008; Wang and Xie, 2011), this paper attempts to investigate a new attitude synchronization strategy under communication delays, and both the attitude dynamics and kinematics are taken into consideration. It is shown that the proposed new approach can ensure robust stability for constant time delays provided the dissipation gains satisfy some inequality. The proposed delay-robust controller bears simple PD structure, and furthermore it is singularity-free...
2. RIGID BODY NETWORKS AND ITS KINETICS AND DYNAMICS

2.1 Network topology

This paper considers a rigid body network that consists of \( n \) rigid bodies. We assume that these \( n \) rigid bodies form a connected undirected graph, that is, every two different bodies is connected by an undirected path (Jungnichel, 2008). The vertex of the network \( i (i = 1, 2, \ldots, n) \) represents the rigid body \( i \) and its edge set \( E \) denotes the corresponding information flow between rigid bodies. The set formed by the neighbours of node \( i \) is denoted by \( \mathcal{N}_i = \{ j \in \nu \mid (i,j) \in E \} \), where \( \nu \) is the vertex set of the network. In an undirected network, the communicating interaction between rigid body agents bears symmetric features, that is, if \( j \in \mathcal{N}_i \), then \( i \in \mathcal{N}_j \).

2.2 Rigid body kinematics and dynamics

The \( i \)-th rigid body kinematics can be written as (Engel and Godhavn, 1994),

\[
\dot{R}_i = R_i S(\omega_i)
\]

where \( R_i \in \text{SO}(3) \) denotes the orientation matrix of the \( i \)-th rigid body with respect to some inertial frame, \( \omega_i \) is its angular velocity expressed in body-fixed frame, and \( S(\omega_i) = \begin{bmatrix} 0 & -\omega_i,z & \omega_i,y \\ \omega_i,z & 0 & -\omega_i,x \\ -\omega_i,y & \omega_i,x & 0 \end{bmatrix} \).

The equations of motion of the attitude dynamics can be expressed as (Engel and Godhavn, 1994),

\[
H_i \dot{\omega}_i - S(H_i \omega_i) \omega_i = \tau_i
\]

where \( H_i \in \mathbb{R}^{3 \times 3} \) is the constant, positive definite, and symmetric inertia matrix, \( \tau_i \in \mathbb{R}^3 \) is the applied control torque vector.

2.3 Attitude synchronization error representation and its basic properties

Consider two different rigid bodies \( i, j \in \nu \), and their error attitude matrix can be written as \( R_i^T R_j \). Differentiating the error attitude matrix yields (Engel and Godhavn, 1994),

\[
\frac{d}{dt} [R_i^T R_j] = R_i^T \dot{R}_i (\omega_i - R_i^T \dot{R}_j \omega_j)
\]

Using the representation from Bullo and Murray (1999), we obtain the following attitude error representation,

\[
\text{skew} (R_i^T R_j) = \frac{R_i^T \dot{R}_i - R_i^T \dot{R}_j}{2}
\]

With the operator \( \forall : \text{so}(3) \rightarrow \mathbb{R}^3 \), we obtain the attitude error \( \text{skew}(R_i^T R_i) \) from the skew-symmetric matrix \( \text{skew}(R_i^T R_j) \), where \( \forall \) is the inverse of the cross product operator \( S(\cdot) \).

The passivity property related to the attitude error representation \( \text{skew}(R_i^T R_i) \) is given by the following lemma.

\[
\text{Lemma 1.} \quad \forall [\int_0^t A(r) dr] \leq t \cdot \text{tr} \left[ \int_0^t A(r) A(r) dr \right] \]

where \( \text{tr}(\cdot) \) denotes the trace of the matrix \( \cdot \).

3. ATTITUDE SYNCHRONIZATION CONTROL

In this section, we attempt to propose control laws for attitude synchronization of multiple rigid bodies. To say attitude synchronization, we mean that \( \text{skew}(R_i^T R_i) \rightarrow 0 \) for \( \forall i, j \in \nu \) as \( t \to \infty \).

To achieve the goal of attitude synchronization, we propose the following delay-robust synchronization control law,

\[
\tau_i = - \sum_{j \in \mathcal{N}_i} k_v (\omega_i - R_i^T R_j (t-T) \omega_j(t-T)) - k_p \sum_{j \in \mathcal{N}_i} \text{skew}(R_i^T R_j (t-T)) - (K_{D,i} + P_i) \omega_i
\]

where \( T \) is the communication delay, which is assumed to be constant and bounded, and for simplicity we assume that all rigid body agents suffer the same time delay. \( k_v, k_p \) are positive scalar feedback gains, \( -P_i \omega_i \) is the \( i \)-th rigid body’s self-dissipation for achieving the asymptotic stabilization purpose, and the dissipation term \( -K_{D,i} \omega_i \) is included to compensate for the effects of the communication time delay, where \( P_i, K_{D,i} \) are both symmetric positive definite matrices.

Now we present the main result of this paper.

\[\text{Theorem 1.} \quad \text{If the rigid bodies form a tree network, and furthermore the controller gains are chosen such that } K_{D,i} \geq \sum_{j \in \mathcal{N}_i} T k_{p} I \text{, then, the control (6) results in attitude synchronization of the rigid body networks (1) (2), i.e., } \text{skew}(R_i^T R_i) \rightarrow 0 \text{ for } \forall i, j \in \nu \text{ as } t \to \infty.\]

Before proving Theorem 1, we first give a useful lemma.

\[\text{Lemma 2. Let } A(t) \text{ and } B(t) \text{ be } n \times n \text{ matrices, for } \forall t \geq 0, \]

\[
\int_0^t \text{tr} (A^T B) dt \leq \left( \int_0^t \text{tr} (A^T A) dt \right)^{\frac{1}{2}} \left( \int_0^t \text{tr} (B^T B) dt \right)^{\frac{1}{2}}
\]

\[
\text{tr} \left\{ \left[ \int_0^t A(r) dr \right]^T \left[ \int_0^t A(r) dr \right] \right\} \leq t \cdot \text{tr} \left[ \int_0^t A^T (r) A(r) dr \right]
\]
Proof. The matrices $A(t), B(t)$ can be expressed as the cluster of column vectors,
\begin{align}
A(t) &= \begin{bmatrix} f_1(t) & f_2(t) & \cdots & f_n(t) \end{bmatrix} \\
B(t) &= \begin{bmatrix} g_1(t) & g_2(t) & \cdots & g_n(t) \end{bmatrix}
\end{align}
Then, using Schwarz inequality, we have,
\begin{align}
\int_0^t (A(t)B) \, dt &= \int_0^t \sum_{k=1}^n f_k(t) g_k \, dt \\
&= \left[ \int_0^t f^T g \, dt \right] + \int_0^t f^T g \
&= \left[ \int_0^t \left( \int_0^r \omega_i^T \omega_i \, dr \right) \left( \int_0^r \omega_j^T \omega_j \, dr \right) \right]^{\frac{1}{2}} \\
&= \left[ \int_0^t \left( \int_0^r \omega_i^T \omega_i \, dr \right) \left( \int_0^r \omega_j^T \omega_j \, dr \right) \right]^{\frac{1}{2}} \\
&= \left[ \int_0^t \left( \int_0^r \omega_i^T \omega_i \, dr \right) \left( \int_0^r \omega_j^T \omega_j \, dr \right) \right]^{\frac{1}{2}}
\end{align}
where $\hat{f} = \begin{bmatrix} f_1^T & f_2^T & \cdots & f_n^T \end{bmatrix}^T, \hat{g} = \begin{bmatrix} g_1^T & g_2^T & \cdots & g_n^T \end{bmatrix}^T$.
This completes the proof of the inequality (7). Next, we show the inequality (8). In fact, from Schwarz inequality, we obtain,
\begin{align}
\text{tr} \left\{ \left[ \int_0^t A(r) \, dr \right]^T \left[ \int_0^t A(r) \, dr \right] \right\} \\
&= \sum_{i=1}^n \left[ \int_0^t f_i(r) \, dr \right]^T \int_0^t f_i(r) \, dr \\
&\leq \sum_{i=1}^n t \cdot \left[ \int_0^t f_i^2(r) \, dr \right] \\
&= t \cdot \text{tr} \left[ \int_0^t (A(t)) \, dr \right]
\end{align}
Proof of Theorem 1. We consider the following Lyapunov-Krasovskii function candidate,
\begin{align}
V &= \sum_{i=1}^n \frac{1}{2} \omega_i^T H_i \omega_i + \frac{k_p}{2} \sum_{i=1}^n \sum_{j \in N_i} \varphi \left( R_j^T R_i \right) \\
&+ \frac{k_p}{2} \sum_{i=1}^n \sum_{j \in N_i} \int_{t-T}^t \omega_j^T \omega_j \, dr
\end{align}
Differentiating $V$ with respect to time and using the skew-symmetric property of $S(R_i \omega_i)$ and Lemma 1, we have,
\begin{align}
\dot{V} &= -\frac{k_v}{2} \sum_{i=1}^n \sum_{j \in N_i} \left[ \omega_i^T (R_j^T (r) R_j (r-T) \omega_j (r-T) \right] \\
&+ \sum_{i=1}^n \omega_i^T (K_{D,i} + P_i) \omega_i - \frac{k_p}{2} \sum_{i=1}^n \sum_{j \in N_i} \omega_i^T \text{skew}^\omega \left( R_j^T (t-T) R_i \right) \\
&+ \sum_{i=1}^n \frac{k_p}{2} \sum_{j \in N_i} \left( \omega_i - R_j^T R_j \omega_j \right)^T \text{skew}^\omega \left( R_j^T R_i \right) \\
&= \frac{1}{2} \sum_{i=1}^n \sum_{j \in N_i} k_p \left( \omega_i - R_j^T R_j \omega_j \right)^T \text{skew}^\omega \left( R_j^T R_i \right)
\end{align}
Using (15), we can rewrite (14) as,
\begin{align}
\dot{V} &= \frac{k_v}{2} \sum_{i=1}^n \sum_{j \in N_i} \left[ \omega_i (r) - R_j^T (r) R_j (r-T) \omega_j (r-T) \right]^T \\
&\cdot \left[ \omega_i (r) - R_j^T (r) R_j (r-T) \omega_j (r-T) \right] \\
&- \sum_{i=1}^n \omega_i^T (K_{D,i} + P_i) \omega_i + k_p \sum_{i=1}^n \sum_{j \in N_i} \omega_i^T \\
&\cdot \left[ \text{skew}^\omega \left( R_j^T R_i \right) - \text{skew}^\omega \left( R_j^T (t-T) R_i \right) \right]
\end{align}
Now integrating $\dot{V}$ in (16) with respect to time and using the result (18), we have,
\[
V(t) - V(0) \leq -\frac{k_e}{2} \sum_{i=1}^{n} \sum_{j \in N_i, j \neq i} \int_0^t \left[ \omega_i(r) - R_i^T r_j(r - T) \omega_j(r - T) \right]^T \cdot \left[ \omega_i(r) - R_i^T r_j(r - T) \omega_j(r - T) \right] dr \\
- \sum_{i=1}^{n} \int_0^t \omega_i^T \left( K_{D,i} - k_p \sum_{j \in N_i} T_i \cdot I \right) \omega_i dr \\
- \sum_{i=1}^{n} \int_0^t \omega_i^T P_i \omega_i dr
\]
(19)
where $D_i = K_{D,i} - k_p \sum_{j \in N_i} T_i \cdot I$ is positive semi-definite due to the controller gain choice given in Theorem 1.

Thus, from (19), we obtain,
\[
V(t) + \frac{k_e}{2} \sum_{i=1}^{n} \sum_{j \in N_i, j \neq i} \int_0^t \left[ \omega_i(r) - R_i^T r_j(r - T) \omega_j(r - T) \right]^T \cdot \left[ \omega_i(r) - R_i^T r_j(r - T) \omega_j(r - T) \right] dr \\
+ \sum_{i=1}^{n} \int_0^t \omega_i^T P_i \omega_i dr + \sum_{i=1}^{n} \int_0^t \omega_i^T D_i \omega_i dr \leq V(0)
\]
(20)

Equation (20) means that $V(t) \in L_{\infty}$, implying the boundedness of $\omega_i$ for $i = 1, 2, \ldots, n$. And hence $\omega_i - R_i^T r_j(t - T) \omega_j(t - T)$ must be bounded. Then the control input $\tau_i$ is bounded. Using the rigid body attitude dynamics (2), we obtain the boundedness of $\dot{\omega}_i$. The boundedness of $\dot{\omega}_i(t)$ gives rise to the uniform continuity of $\omega_i - R_i^T r_j(t - T) \omega_j(t - T)$ and $\omega_i$. Moreover, from (20), we have $\omega_i - R_i^T r_j(t - T) \omega_j(t - T) \in L_2, \omega_i \in L_2$. Therefore, $\omega_i - R_i^T r_j(t - T) \omega_j(t - T) \to 0$ and $\omega_i \to 0$ as $t \to \infty$.

From the rigid body attitude dynamics, we get $\dot{\omega}_i = H_{\dot{\omega}}^{-1} [\tau_i + S(H_\omega \omega) \omega_i]$. Due to the uniform continuity of $\tau_i$ and $S(H_\omega \omega), \dot{\omega}_i$ must also be uniformly continuous. Using Barbalat Lemma (Slotine and Li, 1991), we have $\dot{\omega}_i \to 0$ as $t \to \infty$. Based on the closed-loop attitude dynamics,
\[
H_\omega \dot{\omega}_i = S(H_\omega \omega) \omega_i = \\
- \sum_{j \in N_i, j \neq i} k_e (\omega_i - R_i^T r_j(t - T) \omega_j(t - T)) \\
- k_p \sum_{j \in N_i} \text{skew}^\omega (R_i^T(t - T)R_i) - (K_{D,i} + P_i) \omega_i
\]
(21)
we arrive at the conclusion that, as $t \to \infty$,
\[
\sum_{j \in N_i} \text{skew}^\omega (R_i^T(t - T)R_i) \to 0
\]
(22)
Using finite increment theorem, we have,
\[
R_i^T(t - T)R_j - I = R_i^T(t - T)R_j(\xi) T \xi, \xi \in [t - T, t]
\]
(23)
Since $\dot{R}_j(\xi) = R_j(\xi) S(\omega_j(\xi)) \to 0$ as $t \to \infty$, thus, $R_i^T(t - T)R_j(t) \to I$ as $t \to \infty$. Then, equation (22) can be further written as (at the extreme state $t = \infty$),
\[
\sum_{j \in N_i} \text{skew}^\omega (R_i^T R_j) = 0
\]
(24)
For a tree network topology, i.e., an undirected graph in which every two different bodies is connected by an
where $\lambda_{\min}(\cdot)$ denotes the minimum eigenvalue of $\cdot$. From (25) and (28), we have $tr(I - RT_j R_i) \to 0$ for $\forall i \in \nu, j \in N_i$ as $t \to \infty$, which further results in the conclusion that $skew^\nu(R_j R_i) \to 0$ for $\forall i, j \in \nu$ as $t \to \infty$ due to the connectedness of the rigid body network. Thus, delay-robust attitude synchronization is achieved for general connected graph case.

4. SIMULATION RESULTS

In this section, we present simulation results to examine the performance of the proposed attitude synchronization strategy under communication time delays. In simulation, we employ six rigid body agents and they are required to synchronize their attitudes asymptotically. The network topology of the six rigid bodies is assumed to be a tree for simplicity, as shown in Fig. 1. Due to the limited space, we did not perform simulation study for the general network topology case. The communication delay between rigid body agents is $T = 1s$.

For simplicity, we assume that the six rigid bodies have the same inertia matrix $H_i = \begin{bmatrix} 30 & 8 & 9 \\ 8 & 21 & 6 \\ 9 & 6 & 25 \end{bmatrix}$, $i = 1, 2, \cdots, 6$.

In the first simulation, the controller parameters are chosen as $k_v = 5, K_{Di} = 0, P_1 = 5I, k_p = 20$. The element $sk_x$ of the $i$-th agent attitude representation $skew^\nu(R_i) = [sk_x \ sk_y \ sk_z]^T, i = 1, 2, \cdots, 6$ is shown in Fig. 2. It can be seen that without additional dissipation term, i.e., $K_{Di} = 0$, attitude synchronization cannot be achieved and instability phenomenon is observed.

In the second simulation, we choose the dissipation gains as $K_{Di} = \sum_{i \in \nu} T k_p \cdot I$, the damping gain $P_1 = 5I, i = 1, 2, \cdots, 6$, and $k_v, k_p$ are chosen to be same as the first simulation. The time history of the six agents’ attitudes is shown in Fig. 3 to Fig. 5, from which we can see that attitude synchronization with communication time delay is achieved via adding dissipation $-K_{Di}\omega_i$ according to Theorem 1.

5. CONCLUSION

In this paper, we studied the time-delayed attitude synchronization problem of the rigid body networks with undirected graph topology. Via exploitation of the geometrical structure of rigid body attitude kinematics, we proposed a synchronization control law which is composed of angular velocity and attitude synchronization terms and an additional dissipation term. It is shown that if the dissipation gain satisfies some inequality, the closed-loop networked system is stable and the attitude synchronization errors converge to zero. And the convergence of the attitude synchronization errors holds with any bounded known constant communication time delay. A six-rigid-body network with tree topology is employed as a simulation example to show the performance of the proposed attitude synchronization controller.

For general connected graphs, we just obtain a sufficient condition for ensuring attitude synchronization, which, however, seems somewhat restrictive. Our future research will focus on how to relax or remove this condition.

REFERENCES


Fig. 4. The attitude coordinate $s_{ky}$ of the six rigid body agents (with additional dissipation)

Fig. 5. The attitude coordinate $s_{kz}$ of the six rigid body agents (with additional dissipation)


