Abstract: This paper presents an adaptive control approach for Micro-Electro-Mechanical Systems (MEMS) z-axis gyroscope sensor. The dynamical model of MEMS gyroscope sensor is developed and established. The proposed adaptive control approaches can estimate the angular velocity and the damping and stiffness coefficients including the coupling terms due to the fabrication imperfection. The stability of the closed-loop systems are established with the proposed adaptive control strategies. Numerical simulations are investigated to verify the effectiveness of the proposed control schemes.

Keywords: Adaptive control, sliding mode control, MEMS gyroscope.

1. INTRODUCTION

Gyrosopes are commonly used sensors for measuring angular velocity in many areas of applications such as navigation, homing, and control stabilization. Vibratory gyroscopes are the devices that transfer energy from one axis to other axis through Coriolis forces. The conventional mode of operation drives one of the modes of the gyroscope into a known oscillatory motion and then detects the Coriolis acceleration coupling along the sense mode of vibration, which is orthogonal to the driven mode. The response of the sense mode provides information about the applied angular velocity. Fabrication imperfections result in some cross stiffness and cross damping effects that may hinder the measurement of angular velocity of MEMS gyroscope. Therefore the angular velocity measurement and minimization of the cross coupling between two axes are challenging problems in gyroscopes that need to be solved using advanced control methods.

Adaptive control is an effective approach to handle parameter variations. In the presence of model uncertainties and external disturbances, sliding mode control is necessary to be incorporated into the adaptive control to improve the robust performance of control system. Sliding mode control is a robust control technique which has many attractive features such as robustness to parameter variations and insensitivity to disturbances. Adaptive sliding mode control has the advantages of combining the robustness of variable structure methods with the tracking capability of adaptive control. In the last few years, many applications have been developed using sliding mode control and adaptive control. Utken (1977) showed that variable structure control is insensitive to parameters perturbations and external disturbances. Ioannou et al. (1996) described the model reference adaptive control. Chou et al. (2003) proposed an integral sliding surface and derived an adaptive law to estimate the upper bound of uncertainties. Sam et al. (2004) presented a class of proportional and integral sliding mode control with application to active suspension system. Some control algorithms have been proposed to control the MEMS gyroscope. Batur et al. (2006) developed a sliding mode control for a MEMS gyroscope system. Leland (2006) presented an adaptive force balanced controller for tuning the natural frequency of the drive axis of a vibratory gyroscope. Novel robust adaptive controllers are proposed by Fei et al. (2009 a, b) to control the vibration of MEMS gyroscope. Sun et al. (2009) developed a phase-domain design approach to study the mode-matched control of MEMS vibratory gyroscope. Zheng et al. (2009) described an active disturbance rejection control for MEMS gyroscopes. Antonelli et al. (2009) used extremum-seeking control to automatically match the vibration mode in MEMS vibrating gyroscopes. Feng et al. (2007) presented an adaptive estimator-based technique to estimate the angular motion by providing the Coriolis force as the input to the adaptive estimator and to improve the bandwidth of microgyroscope. Tasi et al. (2010) proposed integrated model reference adaptive control and time-varying angular rate estimation algorithm for micro-machined gyroscopes. Raman et al. (2009) developed a closed-loop digitally controlled MEMS gyroscope using unconstrained sigma-delta force balanced feedback control. Park et al. (2004, 2007) presented an adaptive controller for a MEMS gyroscope which drives both axes of vibration and controls the entire operation of the gyroscope. In this paper, the proposed adaptive control is different from the adaptive controller developed by Park et al. (2004) in that an addition controller is incorporated into the state feedback controller to give more freedom in designing the adaptive controller. Therefore the error dynamics is determined by the reference model dynamics and addition control.

This paper investigates the adaptive control approach to identify the angular velocity of MEMS gyroscope using state tracking controllers. The contribution of this paper is that novel adaptive control is proposed to control the MEMS gyroscope and to estimate the angular velocity and all
unknown gyroscope parameters. The paper is organized as follows. In section II, the dynamics of MEMS gyroscope is described. The adaptive controller is derived in section III. Simulation results are presented in section IV. The Conclusion is provided in section V.

2. DYNAMICS OF MEMS GYROSCOPE

The typical MEMS vibratory gyroscope includes a proof mass suspended by springs, an electrostatic actuation and sensing mechanisms for forcing an oscillatory motion and sensing the position and velocity of the proof mass.

In the Fig. 1, \( r_a \) is position vector of an arbitrary point \( A \) as measured against the axes of the inertial frame. \( r_{\beta} \) is an arbitrary point \( A \) relative to the origin of the rotating axes, \( r_{\beta} \) is position vector of the origin of the rotating frame relative to the origin of the inertial frame \( B \). Their relation can be expressed as

\[
r_{\beta} = r_a + r_{\beta_a}
\]

(1)

The velocity vector of an arbitrary point \( A \) as measured against the axes of the inertial frame can be derived as

\[
v_a = v_{\beta} + \frac{dx}{dt}i + \frac{dy}{dt}j + \frac{dz}{dt}k + x\frac{di}{dt} + y\frac{dj}{dt} + z\frac{dk}{dt}.
\]

(2)

Since \( \frac{di}{dt} = \Omega \times x \), \( \frac{dj}{dt} = \Omega \times j \), \( \frac{dk}{dt} = \Omega \times k \), substituting these properties into (2) yields

\[
v_A = v_{\beta} + \frac{dx}{dt}i + \frac{dy}{dt}j + \frac{dz}{dt}k + x\frac{di}{dt} + y\frac{dj}{dt} + z\frac{dk}{dt} + x\Omega \times i + y\Omega \times j + z\Omega \times k
\]

(3)

where \( v_A \) and \( v_{\beta} \) are the velocities with respect to the inertial frame, \( v_{\beta_a} = \frac{dx}{dt}i + \frac{dy}{dt}j + \frac{dz}{dt}k \) is the relative velocity vector of an arbitrary point \( A \) as measured against the axes of the rotating system.

The velocity of \( A \) relative to \( B \) is therefore made up of four terms - the acceleration measured against the rotating axes and three components that result from the rotation of the axes, and are thus invisible to the observer in the rotating frame.

\[
dv/\gamma/\beta = a_\beta + 2\Omega \times v_{\gamma/\beta} + \frac{d\Omega}{dt} \times r_{\gamma/\beta} + \Omega \times (\Omega \times r_{\gamma/\beta})
\]

(6)

Differentiating (3) and using (4) and (5) yields

\[
a_\beta = a_\gamma + 2\Omega \times v_{\gamma/\beta} + \frac{d\Omega}{dt} \times r_{\gamma/\beta} + \Omega \times (\Omega \times r_{\gamma/\beta})
\]

(7)

where \( a_\beta \) and \( a_\gamma \) are the accelerations with respect to the inertial frame, \( a_{\beta/\gamma} = \frac{dx^2}{dt^2}i + \frac{dy^2}{dt^2}j + \frac{dz^2}{dt^2}k \) is the acceleration vector of an arbitrary point \( A \) as measured against the axes of the rotating system.

The acceleration of \( A \) relative to \( B \) is therefore made up of four terms - the acceleration measured against the rotating axes and three components that result from the rotation of the axes, and are thus invisible to the observer in the rotating frame. These are the Euler (tangential), Centripetal and Coriolis accelerations respectively.

Multiplying (6) by mass \( m \) gives

\[
ma_\beta = ma_\gamma + ma_{\beta/\gamma} + 2m\Omega \times v_{\gamma/\beta} + m\frac{d\Omega}{dt} \times r_{\gamma/\beta} + m\Omega \times (\Omega \times r_{\gamma/\beta})
\]

(8)

where \( 2m\Omega \times v_{\gamma/\beta} \) is the Coriolis force and

\[
m\Omega \times (\Omega \times r_{\gamma/\beta}) \]

(9)

is the Centrifugal force.

The Coriolis force acting on the proof mass along \( x \) direction is derived as

\[
F_{\text{coriolis}-x} = 2m\Omega \times (x \times j) = 2m\Omega \times j
\]

(10)

By using the property of \( k \times (k \times i) = -i \), the Centripetal force acting on the proof mass along \( x \) direction can be derived as

\[
F_{\text{centripetal}-x} = m\Omega \times (\Omega \times k \times i) = -m\Omega^2 \times i
\]
By using the property of $k \times (k \times j) = -j$, the Centripetal forces acting on the proof mass along $y$ direction can be derived as
\[ F_{\text{centripetal}}(y) = m \Omega^2 y = -m \Omega^2 y^2. \] (11)

We assume that the table where the proof mass is mounted is moving with a constant velocity; the gyroscope is rotating at a constant angular velocity $\Omega$, over a sufficiently long time interval; the centripetal forces $m \Omega^2 x$ and $m \Omega^2 y$ are assumed to be negligible; gyroscopes undergo rotation about the $z$ axis only, and thereby Coriolis force is generated in a direction perpendicular to the drive and rotational axes. A $z$-axis MEMS gyroscope is depicted in Fig. 2.

With the assumptions the dynamics of gyroscope become
\[ m \ddot{x} + d_{xx} \dot{x} + k_{xx}, x + k_{xy}, y = u_x + 2m \Omega, \dot{y} \] (12)
\[ m \ddot{y} + d_{yy} \dot{y} + k_{yy}, x + k_{yy}, y = u_y - 2m \Omega, \dot{x}. \] (13)

Fabrication imperfections contribute mainly to the symmetric spring and damping terms, $k_{xx}$ and $d_{xx}$. The $x$ and $y$ axes spring and damping terms $k_{xy}$, $d_{xy}$, $k_{yy}$, and $d_{yy}$ are mostly known, but have small unknown variations from their nominal values. The mass of proof mass can be determined very accurately, and $u_x, u_y$ are the control forces in the $x$ and $y$ direction.

![Fig. 2. A simple model of a MEMS Z axis gyroscope](image)

Dividing (12) and (13) by the reference mass and rewriting the gyroscope dynamics in vector forms result in
\[ q^* = \frac{q}{q_0}, \quad D^* = \frac{D}{w_0}, \quad \Omega^* = \frac{\Omega}{w_0}, \] (16)
\[ u^* = \frac{u}{w_0}, \quad w_x = \frac{k_{xx}}{m w_0^2}, \quad w_y = \frac{k_{yy}}{m w_0^2}. \] (17)

Ignoring the superscript (*) for notational clarity, the nondimensional representation of (12) and (13) is
\[ \ddot{\dot{q}} + D \ddot{q} + K q = u - 2\Omega \dot{q} \] (18)

where
\[ K = \begin{bmatrix} w_x^2 & w_{xy} \\ w_{xy} & w_y^2 \end{bmatrix}, \quad D = \begin{bmatrix} d_{xx} & d_{xy} \\ d_{xy} & d_{yy} \end{bmatrix}. \]

Rewriting the gyroscope model (18) in state space form as
\[ X = AX + Bu \] (19)
where
\[ A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -w_x^2 & -d_{xx} & -w_{xy} & -(d_{xy} - 2\Omega) \\ 0 & 0 & 1 & 0 \\ -w_{xy} & -d_{yy} + 2\Omega & -w_y^2 & -d_{yy} \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T, \quad u = \begin{bmatrix} u_x \\ u_y \end{bmatrix}, \quad X = \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix}. \] (20)

The reference model $x_m = A_1 \sin(w_1 t), y_m = A_2 \sin(w_2 t)$ is defined as
\[ \ddot{q}_m + K_q q_m = 0 \] (21)
where $K_q = \text{diag}\{w_1^2, w_2^2\}$.

Similar to (19), the reference model can be written as
\[ \dot{X}_m = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad X_m = A_m X_m. \] (22)

where $A_m$ is a known constant matrix.

We make the following assumptions.

Assumption. There exists a constant matrix $K^*$ such that the following matching condition $A + BK^* = A_m$ can always be satisfied.

The control target for MEMS gyroscope is (i) to design an adaptive controller so that the trajectory of $X$ can track the state of reference model $X_m$; (ii) to estimate the angular velocity of MEMS gyroscope and all unknown gyroscope parameters.

### 3. ADAPTIVE CONTROL DESIGN

In this section an adaptive controller is proposed to identify the angular velocity and all unknown gyroscope parameters. The block diagram is shown in Fig. 3. In the adaptive control design, we consider (19) as the system model.
The tracking error and its derivative are
\[ e(t) = X(t) - X_m(t) \]  
(23)
\[ \dot{e}(t) = A_r e(t) + (A - A_m) X(t) + B u(t) . \]  
(24)

We define the estimation error as
\[ \tilde{e}(t) = (A_m + BK_f) v(t) \]  
(25)
where \( K_f \) is an estimate of \( K_f \), the constant matrix \( K_f \)
satisfies the condition that \((A_m + BK_f)\) is Hurwitz.

We will evaluate the proposed adaptive control on the
lumped MEMS gyroscope model by Batur et al. (2006) using
MATLAB/SIMULINK. The control objective is to design
adaptive state tracking controller so that a consistent estimate of \( \Omega \)
can be obtained.

In the simulation, we allowed \( \pm 5\% \) parameter variations
for the spring and damping coefficients with respect to their
nominal values. We further assumed \( \pm 5\% \) magnitude
changes in the coupling terms i.e. \( d_{xy} \) and \( \omega_{xy} \), again with
respect to their nominal values. The external disturbance is a
random variable signal with zero mean and unity variance.

Parameters of the MEMS gyroscope are as follows:
\[ m = 0.57e - 8 \text{ kg, } d_m = 0.429e - 6 \text{ N s/m}, \]
\[ d_{xy} = 0.0429e - 6 \text{ N s/m, } d_{yy} = 0.687e - 36 \text{ N s/m}, \]
\[ k_{ss} = 80.98 \text{ N/m, } k_{yy} = 5 \text{ N/m, } k_{xy} = 71.62 \text{ N/m,} \]
\[ w_o = 1kHz, q_o = 10^{-6} \text{ m}. \]

The unknown angular velocity is assumed \( \Omega_z = 5.0 \text{ rad/s} \)
and the initial condition on \( K \) matrix is \( K(0) = 0.95K^* \). The
desired motion trajectories are \( x_m = \sin(w t) \) and \( y_m = 1.2 \sin(w t) \)
where \( w_1 = 6.17kHz \) and \( w_2 = 5.1 kHz \). The adaptive gain of (30) is \( M = \text{diag}[20, 20] \). The \( K_f \) in
(29) is chosen as \( K_f = [-10000 -10000 1000 20000] \).

Fig. 4 depicts the tracking errors. It is observed that the
tracking errors converge to zero asymptotically. Figs. 5 and

Fig. 7 draw the adaptation of the angular velocity and controller parameters. It is shown that the estimates of angular velocity and controller parameters converge to their values. Fig.6 plots the control input using adaptive control.

The estimate of angular velocity using adaptive control has larger overshoot at the beginning but much smaller rise time. The adaptive control can deal with model uncertainties and external disturbances to some extent although the term of model uncertainties and external disturbances do not show up in the adaptive control derivation.

Simulations demonstrate that with the control laws (25), and the parameter adaptation laws (30), if the gyroscope is controlled to follow the mode-unmatched reference model, the persistent excitation condition is satisfied, i.e. \( w_1 \neq w_2 \), and all unknown gyroscope parameters, including the angular velocity converge to their true values, and tracking error is going to zero asymptotically as time go on.
5. CONCLUSIONS

This paper investigates the design of adaptive control for MEMS gyroscope. The dynamics model of the MEMS gyroscope is developed and nondimensionized. Novel adaptive controller is proposed and stability condition is established. Simulation results demonstrate that the effectiveness of the proposed adaptive control techniques in identifying the gyroscope parameters and angular velocity.

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REFERENCES


