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Derivative-Free Optimization of Offshore Production Platforms Sharing a Subsea Gas Network ⋆

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Abstract: Several systems in the real-world arise from the integration of subsystems, bringing about the need for coordination to share limited resources and drive economic gains. Large offshore oilfields fall in this category, such as the Santos Basin, which consists of multiple reservoirs that are operated by platforms interconnected by a subsea gas network. The high content of CO2 imposes constraints on the produced gas delivered to an onshore terminal. To cope with complexity, previous works proposed modeling the production platforms in terms of boundary conditions, thereby decoupling the platform local problems from the optimization of integrated operations. This work follows the same strategy, however proposes the use of derivative-free optimization to find boundary conditions that optimize the overall production. Such an approach adds flexibility from the direct use of simulators for the gas network, while allowing the platform problems to be solved with the method of choice, possibly using mixed-integer nonlinear or linear optimization. The derivative-free approach was shown to be effective in the optimization of the integrated production of the Santos Basin.

Keywords: Derivative-free optimization, production optimization, multi-reservoir oilfields, CO2 constraints, surrogates.

1. INTRODUCTION

The Santos Basin is an oil field that spreads over a large area in the deep waters of the Atlantic, about 300 km off the coast of Brazil. Several Floating Production Storage Offloading (FPSO) platforms have been deployed to produce oil and gas from this multi-reservoir oilfield (Fraga et al., 2015). The oil is transported by special vessels, known as shuttle tankers, to onshore terminals and refineries, whereas the gas is transferred by a network of subsea pipelines to an onshore terminal for further processing. Because of the high content of contaminants in the produced gas, the platforms are equipped with special units that sequestrate most, but not all of the CO_2 which is reinjected in the reservoirs.

To meet constraints on the maximum concentration of CO_2 at the onshore terminal, the gas exports from the Pre-Salt platforms must be coordinated and mixed with gas devoid of contaminants. The CO_2 free gas is produced by FPSO 4 and FIX platform, with the latter being a fixed platform that produces from gas reservoirs and which serves as the hub for the gas network. Figure 1 shows a schematic of the production system in the Santos Basin.

Some works have appeared in the literature that deal with the production optimization from multiple platforms that share a gathering network, not unlike the Santos Basin. These efforts can be roughly divided in *long-term optimization*, which extends over years of operations and concerns reservoir models, and *short-term optimization*, which focuses on daily or weekly optimization and considers the production infrastructure.

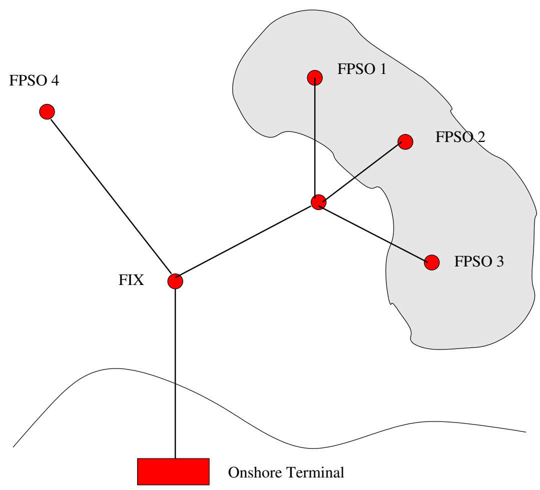

Fig. 1. Simplified representation of the production system in the Santos Basin.

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In the long-term, Huseby and Haavardsson (2009) optimized the production rates of multiple reservoirs in terms of the remaining recoverable oil volumes. More recently, Klokk et al. (2010) developed a simplified network model for long-term optimization of the value chain of such oilfields, taking into consideration CO_2 injection sites and shared facilities.

In the short-term, Selot et al. (2008) presented strategies to optimize gas transportation systems in order to honor contractual agreements. Rømo et al. (2009) developed models to optimize the network transport of natural gas on the Norwegian Continental Shelf. Camponogara et al. (2017) proposed a methodology for production optimization of multi-reservoir systems that was applied to the Santos Basin. This methodology relies on surrogates for platforms and piecewise-linear approximation of fluid flow, yielding a mathematical formulation to which standard algorithms can be applied.

Aiming to contribute to the state of the art, this paper proposes the application of derivative-free algorithms to coordinate the production from multiple offshore platforms, which are coupled by a subsea gas pipeline network and jointly limited by processing constraints at onshore facilities. As in (Camponogara et al., 2017), platforms are represented by surrogates but instead the subsea gas network is modeled directly in simulation software. The derivative-free approach brings about flexibility and reduces the effort of model synthesis and maintenance, but does not ensure a globally optimal solution.

The paper is organized in the following way. Section 2 gives a formal statement for the problem of optimizing production in offshore platforms that share a subsea gas network. Section 3 presents a brief review of derivative-free methods and discusses their application to the problem of concern. Computational results from the application to the production system of the Santos Basin appear in Section 4. Some conclusions are drawn in Section 5.

2. PROBLEM FORMULATION

Consider the set $\mathcal{I}$ of offshore platforms responsible for production from the reservoirs. Such platforms are complex systems that require high investments in capital for deployment and operation. They often have dozens of wells for hydrocarbon production and injection of water or gas, separators, compressors and CO_2 sequestration units, among several other topside processes. The daily production optimization of platforms has attracted the interest of scientists and practitioners, which led to the development of different models, algorithms, and systems. Arguably, the most prominent approaches are based on Mixed-Integer Nonlinear Programming (MINLP) and Mixed-Integer Linear Programming (MILP). MINLP typically uses existing nonlinear models or relies on the synthesis of models from data, obtained from simulation and field measurements (Grimstad et al., 2016). MILP formulations, on the other hand, are often put together from piecewise-linear interpolation of existing data or approximation of existing models (Silva and Camponogara, 2014).

Here, the production platforms are represented by surrogates, whose domain variables are the gas exportation rate and CO_2 concentration, and whose simulated functions are the optimal oil production rate and gas export pressure. The domain variables act as boundary conditions that decouple the platform optimization problems from the coordination of their integrated production and the handling of system constraints. Unlike in other disciplines, here a boundary condition is not a given value but rather a variable which is manipulated by an algorithm. This way, the integrated optimization problem seeks boundary conditions for the platforms that maximize the total oil production, while ensuring that the simulated variables satisfy system constraints, particularly the gas processing capacity and the maximum CO_2 concentration at the onshore terminal.

The boundary conditions of a platform i are represented by a set $\mathcal{X}_i$, such that each feasible boundary condition $x_i = (q_{\mathrm{gas},i}, r_{\mathrm{CO2},i})$ induces an optimal oil production $f_i(x_i)$. The CO_2 content in the gas phase is the ratio of CO_2 to gas flow, i.e. $r_{\mathrm{CO2}} = q_{\mathrm{CO2}}/q_{\mathrm{gas}}$. The pressure p_i for gas export is a complex function that depends jointly on the boundary conditions of all platforms, and the network itself. The compressor at a platform i can operate with a pressure p_i provided that it lies within the range $[p_i^{\min}, p_i^{\max}]$.

The terminal operates at a given inlet pressure p_{t}, maximum gas rate $q_{\mathrm{gas,t}}^{\max}$, and maximum CO_2 content $r_{\mathrm{CO2,t}}^{\max}$.

Let $x = (x_i : i \in \mathcal{I})$ be a vector with boundary conditions of all platforms, and $p = (p_i : i \in \mathcal{I})$ be the vector with the pressures for exportation. Then, conceptually, the problem of maximizing production of offshore platforms sharing a subsea gas network can be cast as follows:

$$S: \max_{x} \quad f(x) = \sum_{i \in \mathcal{I}} f_i(x_i) \tag{1a}$$

$$\text{s.t.}: \quad H(p, x, x_{\mathrm{t}}) = 0 \tag{1b}$$

$$p^{\min} \leq p \leq p^{\max} \tag{1c}$$

$$q_{\mathrm{gas,t}} \leq q_{\mathrm{gas,t}}^{\max} \tag{1d}$$

$$r_{\mathrm{CO2,t}} \leq r_{\mathrm{CO2,t}}^{\max} \tag{1e}$$

$$x \in \mathcal{X} = \mathcal{X}_1 \times \cdots \times \mathcal{X}_n \tag{1f}$$

where:

- H is a function that represents the subsea network. The pressures p that will be established at the platforms are a function of boundary conditions, namely the gas flows and CO_2 concentrations given by x.
- $p^{\min}$ and $p^{\max}$ give the pressure bounds.
- $x_{\mathrm{t}} = (q_{\mathrm{gas,t}}, r_{\mathrm{CO2,t}})$ has the total gas flow received by the onshore terminal and its CO_2 concentration.

The coordination problem S can be seen as the search for the boundary conditions x to maximize total oil production, while satisfying the flow equations given by H and the system constraints (1c)–(1f). This problem is conceptual because it cannot be directly implemented without a representation of boundary condition sets $\mathcal{X}_i$, the production functions f_i, and the flow equations H.

In (Camponogara et al., 2017), $\mathcal{X}_i$, f_i and H where modeled with piecewise-linear functions leading to a MILP approximation of S, which in turn can be optimized with standard algorithms such as branch-and-bound and branch-and-cut. Here we propose to keep the piecewise-linear representation of surrogates for platforms, but instead use a simulator directly to compute network flows, pressures, and compositions. The platform surrogates were validated against simulators. Notice that to obtain points $(x_i, f_i(x_i))$, the end user can use the model and algorithm of their choosing, which can be MINLP, MILP obtained with piecewise-linear approximation, and even derivative-free optimization applied to platform simulators.

3. DERIVATIVE-FREE OPTIMIZATION

Herein derivative-free optimization concerns the problem of minimizing a function $f(x) : \mathbb{R}^n \to \mathbb{R}$ over a domain $x \in \mathcal{X}$. The function f is available as a black-box, and its derivatives are either unavailable or unreliable (Conn et al., 2009; Rios and Sahinidis, 2013). Such problems arise, for instance when the evaluation of f is subject to noise or too costly for the application of finite difference methods. Below we present a brief review of derivative-free optimization (DFO), with some of the approaches for tackling constraints. As in derivative-based methods, the handling of constraints may rely on the iterative solution of unconstrained (or less constrained) problems, such as in penalty methods and the augmented Lagrangian (Giuliani and Camponogara, 2015). The handling of constraints is discussed in the sequence, with a particular focus on constraints defined by functions without derivatives.

3.1 Brief Introduction

Generally speaking derivative-free methods can be classified as *direct-search* and *model-based*. Direct-search methods are based on successive evaluation of f at appropriate directions. On the other hand, model-based methods construct a surrogate for f around the current iterate, which is then combined with standard algorithms to define the next iterate. The models are updated iteratively to capture the behavior of f around the incumbent solution.

Model-Based Methods These methods first sample the function around the incumbent solution to build an initial surrogate. They proceed by optimizing the surrogate, with derivative-based methods, and then evaluate the proposed solution which can be accepted or rejected. The surrogate is then updated according with this decision.

Trust-region methods rely on a surrogate that is sufficiently accurate within a neighborhood of the current iterate, the so-called trust-region. As models are used, these methods tend to be efficient in finding descent. But this comes at the expense of model maintenance, a procedure that is costly. The size and position of the trust-region are adjusted depending on whether or not the surrogate solution is accepted, and if the error between the predicted and actual function value decreases.

Direct-Search Methods Direct search methods, such as Hooke and Jeeves' (Hooke and Jeeves, 1961) and Nelder Meads' (Nelder and Mead, 1965) work based only on function values, without attempting to build any model. While in the 1960's these methods were heuristic, more recent variations have proven global convergence.

The simplex method of Nelder and Mead operates with a set of points defining a simplex at each iteration. Various operations are performed about the centroid of the simplex in order to produce an improved solution, with which the simplex is revised.

Methods related to Nelder and Meads' are the Generalized Pattern Search (Torczon, 1997), the Generating Set Search (Kolda et al., 2003), and the Mesh Adaptive Direct Search (MADS) (Audet and Dennis Jr, 2006). At each iteration, these methods sample the objective in a finite number of points around the current approximate solution. If the current iterate is not already optimal, one of the search directions is guaranteed to be a descent direction. So descent may be found for a sufficiently small step length.

3.2 Dealing with Simulated Constraints

Giuliani and Camponogara (2015) proposed the penalization of relaxable constraints using the augmented Lagrangian, and the subsequent application of a derivative-free trust-region algorithm. A comparison with a standard optimization algorithm in an application to oil production platforms indicates that the derivative-free approach can be effective. This kind of penalization is suitable for constraints defined by functions for which derivatives are not available, or too complicated to compute, such as simulated functions.

The use of the derivative-free trust-region method, which is model-based, still requires that objective and constraint functions be well defined at all points. When such functions are also the result of a black-box simulation, they may not be available at some points: in some cases the simulator may not return any meaningful result. This is sometimes called a *hidden constraint*.

The Mesh Adaptive Direct Search (MADS) methods (Audet and Dennis Jr, 2006) address such constraints through an extreme barrier approach that rejects infeasible trial points. A number of variations have been developed for generating random or deterministic orthogonal directions, as with OrthoMADS (Abramson et al., 2009). In this work we employed the NOMAD (Le Digabel, 2011) package, which implements the OrthoMADS algorithm.

4. COMPUTATIONAL RESULTS

A computational analysis is performed to assess the effectiveness of OrthoMADS in the solution of the coordination problem S, which has simulated constraints regarding the CO_2 concentration in the gas reaching the onshore terminal. The performance and robustness of the method are investigated for different initial guesses and constraint limits around the system's nominal operating conditions.

The coordination problem S seeks to orchestrate the integrated production of the Santos Basin as depicted in Figure 1. For a Pre-salt FPSO $i \in \{1, 2, 3\}$, the boundary condition $x_i = (q_{\mathrm{gas},i}, r_{\mathrm{CO2},i})$ is given by its gas export and CO_2 content. FPSO 4 and Fixed Platform have only gas export rate as boundary condition $x_i = (q_{\mathrm{gas},i})$ since they produce gas devoid of CO_2 from Post-salt reservoirs. Due to flashing and complex phenomena involved in gas transportation, the total gas $q_{\mathrm{gas,t}}$ received at the onshore terminal does not match the total gas injected by the platforms, but rather depends on the multiphase flow simulation. Consequently the constraints on total flow and CO_2 content of the gas delivered at the terminal, given respectively by (1d) and (1e), are simulated functions. The inlet pressure p_{t} at the terminal was fixed at 4.5 MPa in every simulation.

The compositional model for the gas pipeline network is available in a recent version of the commercial multiphase flow simulator Pipesim. The optimization method is implemented in Python using the OPAL interface (Audet et al., 2010) for the package NOMAD (Le Digabel, 2011), which implements the OrthoMADS algorithm. The communication between the optimization software and the simulator requires the use of the interface Openlink in Python. Because this interface is not available in recent versions of Pipesim, the simulation model was adapted to Pipesim 2009, which is compatible with this interface.

4.1 Handling Simulated Constraints

Late in the implementation of the optimization method, we discovered that OPAL cannot handle multiple nonlinear or simulated constraints. It admits multiple constraints on the decision variables that are manipulated directly, namely boundary conditions, but only one constraint on simulated variables. In view of this limitation, the constraints (1d) and (1e) of the coordination problem were combined into a single, but equivalent constraint given by:

$$\frac{\max(q_{\mathrm{gas,t}} - q_{\mathrm{gas,t}}^{\max}, 0)}{q_{\mathrm{gas,t}}^{\max}} + \frac{\max(r_{\mathrm{CO2,t}} - r_{\mathrm{CO2,t}}^{\max}, 0)}{r_{\mathrm{CO2,t}}^{\max}} \leq 0 \quad (2)$$

Some remarks on this equivalent constraint are in order:

- Notice that the left-hand side of (2) is positive if any of the original constraints is not fulfilled, deeming the constraint violated. Such a condition occurs if the total gas delivered at the terminal or its CO_2 content exceed the respective bound. The left-hand side assumes the value zero, and the constraint is satisfied, only when both original constraints are met.
- The violations in gas and CO_2 content were normalized to promote numerical stability and ensure a proper satisfaction of both constraints. Because the gas rate is given in millions of m^3/d, and CO_2 content is a fraction given in percentage (%), a small variation on total gas flow would subsume any variation in CO_2 content without the normalization.

4.2 Initial Guesses

The robustness of the method is assessed by evaluating the solutions to which the method converges when starting from different initial guesses, one induced by the system's nominal operating condition and four additional points in the neighborhood. Table 1 presents five platform boundary conditions for the gas pipeline network, i.e. gas export rates and CO_2 concentrations (in percentage), which are the initial guesses provided to the optimizer.

The nominal case is denoted by BASE. $\mathrm{PRE_{gas}}$ is a variation of the nominal case in which the total gas coming from the Pre-Salt is increased by 15%, while the Post-Salt gas is reduced by the same ratio. The initial guess $\mathrm{POS_{gas}}$ represents a boundary condition in which the gas coming from the Pos-Salt is increased by 15%, and the gas with high content of contaminants is decreased in the same proportion. The same analogy holds for $\mathrm{PRE_{CO2}}$ and $\mathrm{POS_{CO2}}$, i.e., these guesses represent boundary conditions in which the CO_2 concentration is increased and decreased by 15% with respect to the nominal case, respectively. However, since the CO_2 concentration is constant for the Pos-Salt platforms, only a variation in CO_2 content of the Pre-Salt platforms is considered. The table presents the gas export ratio and the CO_2 concentrations (r_{CO2}) of each platform in all cases. It should be mentioned that the values of physical quantities are given in the international system units (SI). Gas flow is measured in million standard cubic meters per day (MSm^3/d), CO_2 concentration in percent (%), and time in seconds.

Table 1. Initial boundary conditions.

Guess	B. Cond.	P1	P2	P3	P4	FIX
BASE	$q_{\mathrm{gas},i}$	0.471	0.834	0.793	0.884	2.097
	$r_{\mathrm{CO2},i}$	5.02	4.84	4.00	0.25	0.27
$\mathrm{PRE_{gas}}$	$q_{\mathrm{gas},i}$	0.541	0.959	0.912	0.751	1.783
	$r_{\mathrm{CO2},i}$	5.02	4.84	4.00	0.25	0.27
$\mathrm{POS_{gas}}$	$q_{\mathrm{gas},i}$	0.400	0.709	0.674	1.016	2.412
	$r_{\mathrm{CO2},i}$	5.02	4.84	4.00	0.25	0.27
$\mathrm{PRE_{CO2}}$	$q_{\mathrm{gas},i}$	0.471	0.834	0.793	0.884	2.097
	$r_{\mathrm{CO2},i}$	5.77	5.57	4.6	0.25	0.27
$\mathrm{POS_{CO2}}$	$q_{\mathrm{gas},i}$	0.471	0.834	0.793	0.884	2.097
	$r_{\mathrm{CO2},i}$	4.27	4.11	3.40	0.25	0.27

4.3 Analysis of Results

Table 2 shows the computational results obtained with OrthoMADS, which was applied to solve the coordination problem S by iteratively simulating the subsea gas network. Three different upper bounds were imposed to the total flow and CO_2 concentration of the gas arriving at the onshore terminal:

- The first case (C1) considers both constraints relaxed, meaning that the problem solution is equivalent to the unconstrained case.
- The second one (C2) limits the total gas reaching the onshore terminal with respect to the first case.
- The last case (C3) constrains the maximum CO_2 content allowed in the mixture that reaches the terminal, while keeping the upper bound on the total gas relaxed.

The results shown in Table 2 lead us to draw some remarks:

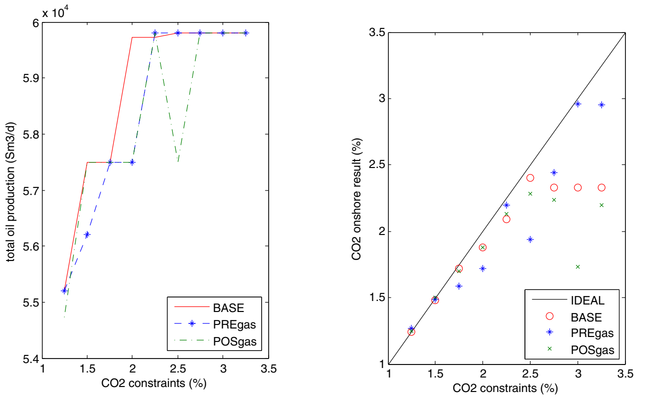
(1) The initial guess ($\mathbf{x}_0$) can influence the solution to which the derivative-free algorithm converges, which suggests that multiple runs should be performed to increase robustness.
(2) The running time of the algorithm tends to increase as the constraints become more stringent, either the bound on total gas or CO_2 content. This behavior is consistent with the nature of derivative-free algorithms, which were originally designed for unconstrained optimization. Constraints are handled by some form of penalization that can be difficult to optimize, particularly when the constraint is a simulated function such as the CO_2 content.

In practice, the constraint on the total CO_2 content is typically more challenging since it depends on the

Table 2. Sensitivity to initial guess.

	$q^{\max}_{\text{gas,t}}$	$r^{\max}_{\text{CO2,t}}$	$\mathbf{x}_0$	$q_{\text{gas,t}}$	$r_{\text{CO2,t}}$	$f(\mathbf{x})$	f. eval.	time
	5.663	2.50	BASE	5.222	2.39	59799	197	1802
	5.663	2.50	PRE_{gas}	5.622	2.06	59799	331	2995
C1	5.663	2.50	POS_{gas}	5.275	2.28	57496	320	2819
	5.663	2.50	PRE_{CO2}	5.389	2.50	59799	201	2207
	5.663	2.50	POS_{CO2}	5.080	2.29	59799	234	2499
	3.681	2.50	BASE	3.666	2.27	54113	275	2610
	3.681	2.50	PRE_{gas}	3.667	2.46	56416	264	2420
C2	3.681	2.50	POS_{gas}	3.671	2.19	56416	373	3362
	3.681	2.50	PRE_{CO2}	3.670	1.76	54113	267	3056
	3.681	2.50	POS_{CO2}	3.639	2.07	53640	251	2797
	5.663	1.25	BASE	5.140	1.24	55196	240	3010
	5.663	1.25	PRE_{gas}	5.138	1.25	55196	467	4316
C3	5.663	1.25	POS_{gas}	5.112	1.24	54723	346	3095
	5.663	1.25	PRE_{CO2}	5.111	1.25	54723	208	2227
	5.663	1.25	POS_{CO2}	5.210	1.24	55669	212	2344

compositions of all platform streams. Figure 2(a) shows the impact of the CO_2 constraint on the optimization of the coordination problem S for a range of CO_2 upper bounds and different initial guesses. Figure 2(b) correlates the optimal CO_2 content of the gas at the treatment unit with the CO_2 constraint for three initial guesses.



(a) Oil production (b) CO_2 content at the terminal

Fig. 2. Total oil production and CO_2 content at the terminal for different CO_2 constraint values.

For this analysis, the upper bound on the total gas was sufficiently large, set at $q^{\max}_{\text{gas,t}} = 5.663$ MSm^3/d, so that only the CO_2 constraint has impact in the optimization. Some conclusions can be stated from the results shown in the figure. The total oil production varied significantly with respect to the initial guess for the scenarios with the CO_2 constraint up to 2.50%. For the cases in which the CO_2 constraint is more relaxed, the total oil production converged to the same value. Notice that in most cases the objective tends to increase as the CO_2 constraint becomes more relaxed.

5. SUMMARY

The maturing of existing oil assets is forcing operators to search for new reservoirs, which are found in deep waters and inhospitable areas such as in the Arctic. The development of such assets must be carefully analyzed given the high investments, which is of particular relevance with today's price of the oil barrel. Among these assets, multi-reservoir oilfields located in the deep waters of Brazil stand out for the sheer size, complexity, and long distance from the coast. Such oilfields are operated with multiple platforms that share a subsea gas network and terminal. This gives rise to the problem of coordinating platform production to meet constraints on the CO_2 content of the gas delivered, while ensuring production maximization, which is key to make the enterprise profitable.

To that end, this work extended previous work by coordinating the overall production from multiple platforms with a derivative-free algorithm. Although this approach does not guarantee global optimality, it offers flexibility to optimize the gas-network simulation model directly, while still inducing optimal performance of the platforms from the obtained boundary conditions. Another advantage of the derivative-free approach is the ability to cope with more complex gas networks, with multiple sources and sinks that can elicit complex behavior, such as flow splitting (Silva et al., 2015). The computational experiments indicate that good, feasible solutions can be reached after an hour of computation in a modest computer and without the use of parallelization.

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