# PINCH ANALYSIS FOR PRODUCTION PLANNING IN SUPPLY CHAINS

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*Abstract*

Global competition has made it imperative for the process industries to manage their supply chains optimally. The complexity of the supply chain processes coupled with large computational times often makes effective supply chain management (SCM) difficult. This paper introduces a novel approach for aggregate planning in supply chains. The approach derives inspiration from pinch analysis, which has been extensively used in heat and mass exchanger network synthesis. By representing demand and supply data as composites, it gives planners greater insight into the SCM process and thus facilitates re-planning and quick decision-making. Two case studies are solved, one involving a single product and another involving multiple products on a single processor. For the first case study, optimal production plans are obtained and matched with the results obtained by solving equivalent optimization problems in GAMS®. For the second case study, an algorithm is proposed to determine the sequence of production of the multiple products. The initial guess obtained by following the algorithm reduces the computational time to one-sixth of the time otherwise taken by the solver. It may be concluded that plans obtained by pinch analysis provide either the best aggregate plans or excellent starting points to reduce the computational time for solutions by mixed integer programming formulations.

*Keywords*

Pinch analysis, production planning, mathematical programming, supply chain management.

## Introduction

Supply chain for a business consists of all the stages involved directly or indirectly in fulfilling demand from a customer. Production planning, scheduling and distribution are some of the operations performed in a supply chain. In most cases, a planning model is developed and an equivalent optimization problem is solved using standard optimization algorithms (McDonald and Karimi, 1997). One of the difficulties faced with discrete mathematical programming is combinatorial complexity, which increases dramatically with the size of the problem.

The aim of this work is to introduce the approach of pinch analysis in aggregate planning and optimization of supply chains. Aggregate planning (Chopra and Meindl, 2001) aims at maximizing profit over a specified time horizon while satisfying demand.

Pinch analysis has been extensively used in chemical engineering for the optimization of various resources such as energy and water (Linnhoff and Hindmarsh, 1983; Shenoy, 1995; and Wang and Smith, 1994). Pinch is defined as the most constrained point in the process. The proposed approach determines an aggregate plan taking the pinch into consideration through graphical representation. At the pinch, the material flows in a supply chain are balanced and problem decomposition is possible. The method helps in setting targets, i.e., predicting optimal performances based on fundamental principles prior to actual scheduling of processes.

## Representation: Time vs. Material Quantity

The power of pinch analysis lies in the physical insight it provides into the supply chain process. Material flows, material holdup and time form the three important indicators of a supply chain. Pinch analysis elegantly handles these parameters by plotting demand and production composites on a time vs. material quantity plot (Singhvi, 2002). During aggregate planning (required to service demand in a time interval $\Delta t = t_k - t_{k-1}$), some of the decision variables are:

$P_k$ = Cumulative in-house production (number of units) at time $t_k$

$C_k$ = Cumulative number of units outsourced (subcontracted) at time $t_k$

$D_k$ = Cumulative demand (number of units) at time $t_k$ as per demand forecast

$I_k$ = Inventory at time $t_k$

$p_k$ = Production rate (i.e., in-house production during the period $t_{k-1} \le t \le t_k$ in time $\Delta t$)

$c_k$ = Outsourced amount during the period $t_{k-1} \le t \le t_k$
$d_k$ = Demand rate (i.e., demand during the period $t_{k-1} \le t \le t_k$ in time $\Delta t$)

A simple balance of the flow of materials at time $t_k$ in a particular stage of the supply chain with $I_o$ as the initial inventory can be written as

$$I_o + P_k + C_k = D_k + I_k \quad (1)$$

In an analogous manner, a material balance over a time interval $\Delta t$ yields

$$I_{k-1} + p_k\,\Delta t + c_k = d_k\,\Delta t + I_k \quad (2)$$

It must be noted that both inventory and stockout cannot occur in the same time period; therefore, stockout can be simply viewed as negative inventory. Figure 1 shows how Eq. (2) can be elegantly captured through typical composite curves used in pinch analysis. Some of the salient features of the composite curves are listed below.

- The demand composite curve $D(t)$ is simply a plot of the cumulative demand as a function of time, and needs to be matched by a supply composite curve $P(t)$. The demand has to be met by supply of products, some by in-house production and rest by outsourcing. This is based on the fundamental principle of material balance.

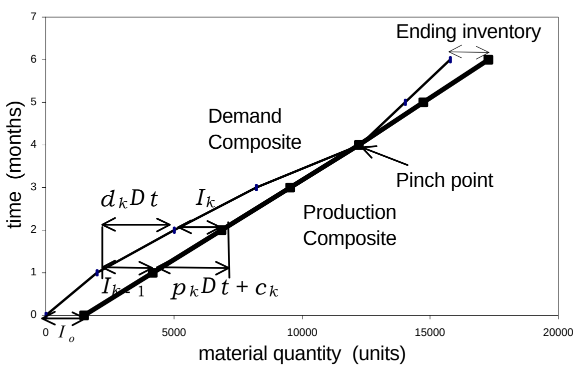


*Figure 1. Typical Composites by Pinch Analysis*

- The vertical difference between the demand and supply composites is the lead time. Here, it is the time interval between producing an order and servicing the demand. Lead time can, in general, include the time consumed in various activities like processing and transportation. There is a lower limit $T$ to the lead time. The point at which $P(t\text{-}T) = D(t)$ is the pinch. The two composites are separated by the minimum lead time at the pinch. When $T = 0$, the pinch will be the point where $P(t) = D(t)$.
- The horizontal distance between the two composites at any given time gives the total inventory in the system. This also includes the work in process (WIP). The pinch is defined as the point of minimum inventory. The area between the two composites gives the measure of inventory in the system, which when multiplied by the inventory holding cost factor provides the actual inventory costs.
- A linear composite assumes constant and continuous demand or supply in a given time period. The demand composite will be a series of step functions, if actual demand has to be met at definite time intervals, and the corresponding composite for continuous servicing of demand will then depict the limiting case and provide the lower bound.

**Planning for Single Product Scenario**

The pinch analysis approach is illustrated for the single product scenario using data of an example from Chopra and Meindl (2001). The demand for the product is seasonal and the company has the option to hire and lay-off workers, outsource some of the work, and build up inventory or backlogs. The company sells the product at \$40 per unit, but plans to give a discount of \$1 per unit in April. Table 1 shows the demand forecast.

*Table 1. Demand Data for Single Product Case Study*

| Time Period $k$ | Month | Forecasted Demand (units) |
|---|---|---|
| 1 | January | 1600 |
| 2 | February | 3000 |
| 3 | March | 3200 |
| 4 | April | 5060 |
| 5 | May | 1760 |
| 6 | June | 1760 |

The production capacity is determined mainly by the size of the workforce and not the machine capacity. At the beginning of January, there is a starting inventory of 1000 units and a workforce of 80 workers. The plant has a total of 20 working days in each month. Each employee works for 8 hours per day, and no worker can work overtime for more than 10 hours per month. Four hours of labor are needed to produce one unit. As the company desires high customer service level, the aggregate plan should meet all the demand and also result in an inventory of at least 500 units at the end of June. Table 2 gives relevant cost data.

*Table 2. Cost Data for Single Product Case Study*

| Item | Cost |
|---|---|
| Material cost | \$10/unit |
| Inventory holding cost | \$2/unit/month |
| Penalty for stockout / backlog | \$5/unit/month |
| Hiring cost | \$300/worker |
| Layoff cost | \$500/worker |
| Regular time cost | \$4/hour |
| Overtime cost | \$6/hour |
| Cost of subcontracting | \$30/unit |

*Preliminary Analysis*

The cost of producing a unit during regular time is \$26 [\$4x4 + \$10]. The cost increases to \$34 [\$6x4 + \$10], when produced with overtime. Since it is higher than the cost of subcontracting (\$30), overtime is not required. Since inventory holding cost is \$2/unit/month, inventory should not be maintained for more than two months [(\$30-\$26)/2]. The total amount of production required is 15880 units (16380 – 1000 + 500). The demand over the time periods 0 to 4 is monotonically increasing, and is followed by a decrease in demand in periods 4 to 6. As the cost of varying capacity (hiring and lay-off) is high, it is better to operate with constant capacity (level strategy). A worker can be laid-off any time, since regular time wage is $640/month whereas layoff cost is $500.

*Initial Aggregate Plan with No Stockout*

The demand composite is plotted in Figure 2 based on the cumulative demand calculated from Table 1. As lead time is not specified, it is taken as zero. This implies, that for the case of no stockout, the demand composite provides an upper bound to the production composite.

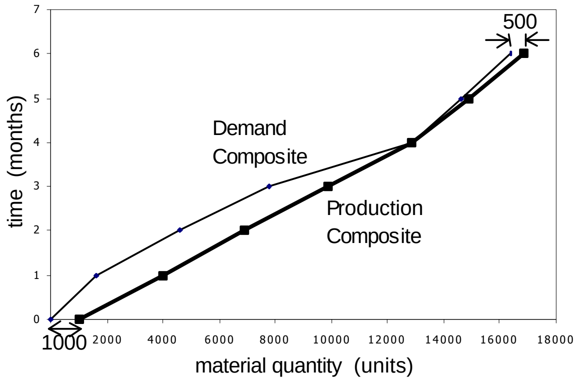


*Figure 2. Composites for Single Product Case Study*

To determine the minimum production rate with no stockout, the starting inventory (1000 units) is taken as the pivot point and a constant production line is rotated till it just touches the demand composite. The point (12860, 4) is the pinch point. This approach is similar to the rotation of the water supply line proposed by Wang and Smith (1994) for the determination of the minimum freshwater target. The reciprocal of the slope of the production composite gives the minimum production rate to be 2965 units/month [i.e., (12860-1000)/4] for the first four months. For the period after four months, the demand rate is lower and the terminal inventory is to be kept at 500 units. To meet the requirements for this period, the production composite is rotated from the pivotal point (12860, 4) till it passes through the terminal point (16880,6). The resultant production rate turns out to be 2010 units/month for the last two months. The production composite is closest to the demand composite with a constant manpower deployment resulting in minimum inventory. For the given cost data, the production plan given above is optimal and exactly matches the solution obtained by solving an equivalent linear programming (LP) formulation.

*Initial Aggregate Plan with Stockout*

Stockouts result in delayed customer delivery, but lower inventory costs. Consider the task of determining the minimum production rate with stockout. This rate equals the reciprocal of the slope of the steepest line, which begins at the starting inventory and passes through the terminal point of the demand composite with the required ending inventory. Based on this minimum production rate of 2646.67 units/month (i.e., 15880/6), Table 3 gives the plan with stockout and this matches the LP solution reported by Chopra and Meindl (2001).

*Table 3. Initial Aggregate Plan*

| Period $k$ | Cumulative Demand $D_k$ | In-house Production $p_k$ | Out-source $c_k$ | Invent-ory $I_k$ | Stock-out $S_k$ | Work-force $W_k$ | Number Laid-off |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1000 | 0 | 80 | 0 |
| 1 | 1600 | 2646.67 | 0 | 2046.7 | 0 | 66.167 | 13.833 |
| 2 | 4600 | 2646.67 | 0 | 1693.3 | 0 | 66.167 | 0 |
| 3 | 7800 | 2646.67 | 0 | 1140 | 0 | 66.167 | 0 |
| 4 | 12860 | 2646.67 | 0 | 0 | 1273.3 | 66.167 | 0 |
| 5 | 14620 | 2646.67 | 0 | 0 | 386.7 | 66.167 | 0 |
| 6 | 16380 | 2646.67 | 0 | 500 | 0 | 66.167 | 0 |
| $ | 650140 | 158800 | 0 | 10760 | 8300 | 254080 | 6916.7 |

Profit = $650140 - $438857 = $211283

*Final Aggregate Plans*

In Table 3, the number of workers is 66.167 (i.e., 2646.67/40), which may be rounded off to the nearest lower integer. Over the six-month horizon, 66 workers will produce 15840 units, which is 40 units less than the total production required. These 40 units will be subcontracted in the month when stockout occurs (April). The final plan based on the above reasoning is given in Table 4 and is the optimal solution in terms of total costs. It gives a profit of $211220 and can be validated by solving a mixed integer linear programming (MILP) formulation in GAMS®.

*Table 4. Final Aggregate Plan*

| Period $k$ | Cumulative Demand $D_k$ | In-house Production $p_k$ | Out-source $c_k$ | Invent-ory $I_k$ | Stock-out $S_k$ | Work-force $W_k$ | Number Laid-off |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1000 | 0 | 80 | 0 |
| 1 | 1600 | 2640 | 0 | 2040 | 0 | 66 | 14 |
| 2 | 4600 | 2640 | 0 | 1680 | 0 | 66 | 0 |
| 3 | 7800 | 2640 | 0 | 1120 | 0 | 66 | 0 |
| 4 | 12860 | 2640 | 40 | 0 | 1260 | 66 | 0 |
| 5 | 14620 | 2640 | 0 | 0 | 380 | 66 | 0 |
| 6 | 16380 | 2640 | 0 | 500 | 0 | 66 | 0 |
| $ | 650140 | 158400 | 1200 | 10680 | 8200 | 253440 | 7000 |

Profit = $650140 - $438920 = $211220

Alternatively, the number of workers can be rounded off to the next higher integer. Over the planning horizon, 67 workers will produce 16080 units, which is 200 units more than the total production required. So 200 units or 5 man-months may be reduced by laying-off one worker in the second month. In this plan, subcontracting is not needed. However, it gives a marginally lower profit of $211140.

## Planning for Multiple Products Scenario

This section briefly deals with planning for the case of multiple products on a single processor. The demands of all products are added and a demand composite is plotted. Since the demand is to be serviced only at the end of the planning horizon, the demand composite will be a step profile. It is assumed that the changeover times are not significant compared to the planning horizon. Fixed costs for production of a product are high, so any product is produced only once. For these conditions and assumptions, the following algorithm (Singhvi, 2002) can be theoretically proven to give minimum inventory.

1. List all the products in order of increasing production rates, and produce the products in that order.
2. For products with the same production rate, produce the product with lower inventory holding cost first.
3. For products with the same production rate and the same inventory holding cost, produce the product with lower demand first.

The algorithm will now be validated with a paper and pulp industry case study. Five qualities of paper have to be manufactured on a single machine. Demand for each quality of paper is given (Table 5) and has to be met by the end of the day, which results in the demand composite having a step profile (Figure 3).

*Table 5. Data for Paper and Pulp Industry Case Study*

| Quality | Production Rate (Tons/hr) | Selling Price (Rs/ton) | Inventory Cost (Rs/ton/hr) | Changeover Time (hr) | Demand (Tons/day) | Bleached Pulp (%) |
|---|---|---|---|---|---|---|
| Newsprint | 12.5 | 22000 | 1760 | 0.1 | 150 | 22 |
| Cream wove | 12.5 | 31000 | 2480 | 0.1 | 60 | 75 |
| Super print | 15 | 34000 | 2620 | 0.1 | 45 | 75 |
| Maplitho | 15 | 32000 | 3840 | 0.1 | 30 | 75 |
| Azure | 15 | 33000 | 3960 | 0.1 | 15 | 75 |

Application of the proposed algorithm results in minimum inventory holding cost and the optimal production plan. Figure 3 shows the production composite for the case. More details are available in Singhvi (2002). As production capacity is able to meet the demand, production is not started at time zero. The discontinuity in the production composite is due to changeover times, during which no production takes place. The sequence of production of the five products is as follows: Newsprint, Cream wove, Super printing, Maplitho and Azure.

The mathematical formulation of the case study results in a MINLP problem. The initial guess obtained by following the algorithm reduces the computational time to one-sixth of the time otherwise taken by solver. Furthermore, the solution thus obtained is the global optimum, compared to the local optimum obtained when no initial guess is given. The approach needs to be further developed for a generalized scenario related to different changeover times and associated costs.

## Conclusions

Pinch analysis proposed as a graphical procedure involving production and demand composites is shown to provide not only a good qualitative understanding of the production planning problem but also optimal to sub-optimal solutions. In the case of problems following chase strategy, the approach can be shown to provide optimal solutions as it ensures minimal inventory.

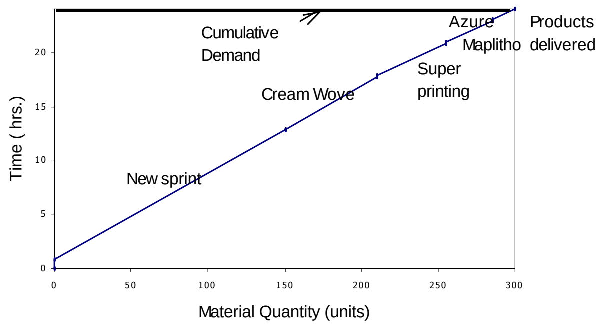


*Figure 3. Composites for Multiple Products Case Study*

Since cost factors are not explicitly incorporated in pinch analysis, it cannot always guarantee cost-optimal solutions for other cases. In the case of level strategy with no stockout, the cost influences are incorporated implicitly in the disposition of the composites. Depending on the cost data and demand profile, the solution can be either optimal or sub-optimal. This is also true for the stockout case. For multiple products, the approach can solve the product sequencing problem when all product demands are at the terminal time and product changeover effects (time and cost) can be neglected.

Pinch analysis helps in achieving targets by minimizing inventory for a given strategy. A hybrid approach, where pinch analysis provides a good starting guess, can assist considerably reduce the computational time during planning in SCM.

Pinch analysis brings in the much-required flexibility into quick re-planning. For minor changes, the effects can be observed on the composites without running mathematical formulations (Singhvi, 2002).

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