A TWO-STAGE STOCHASTIC INTEGER PROGRAMMING APPROACH TO REAL-TIME SCHEDULING

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Abstract

This contribution deals with scheduling problems in flexible batch chemical processes with a special emphasis on their real-time character. This implies not only the need for sufficiently short response times, but also the burden of incomplete knowledge about the future.
We propose the application of two-stage stochastic integer programming techniques within a model predictive framework, which allow for the explicit modeling of recourse actions. Motivated by a real-world example process, some essential prerequisites for modeling real-time scheduling problems are discussed, and characteristic features of master scheduling models are highlighted. Numerical experiments with a problem-specific solution algorithm demonstrate the applicability of the method.

Keywords

Flexible batch processes, Real-time scheduling, Stochastic programming

Introduction

In the processing industries flexible batch plants enjoy an increasing popularity, because they allow for rapid and cost efficient adaptations of the product supply to the customers demands. Their considerable flexibility is reflected in a high combinatorial complexity of the scheduling tasks, i.e. the problem of (optimally) assigning processing steps to plant units over time. Characteristic features of scheduling problems in the chemical industries are their complex constraints due to strong couplings concerning material and time and the often significant uncertainty about the future evolution of the process.

An immense number of publications shows that mathematical programming provides appropriate and well-founded methods to formulate and solve assignment problems (an overview is given by Reklaitis (1996)). However, the aspect of uncertainty has been neglected in most models, only a few papers address this issue (e.g. Balasubramanian and Grossmann (2000), Honkomp et al. (1997), Ierapetritou et al. (1995), Petkov and Maranas (1997), Sanmarti et al. (1997)). Virtually all uncertainty conscious models are based upon a defensive strategy, which avoids extensive rescheduling activities.

In the following we present a stochastic programming approach, in which the need for recourse actions is not considered as a burden but as optimization potential. Based on a real-world example, a two-layer two-stage scheduling approach is sketched and numerical results are shown for the master level.

An Benchmark Process

The multi-product batch plant shown in Figure 1 is used to produce two types (A/B) of expandable polystyrene (EPS) in 5 grain size fractions each. The preparation stage and the polymerization stage are driven in batch mode whereas the finishing is done continuously. A polymerization batch is produced according to a certain recipe (out of 10), which determines the EPS-type and the grain size distribution. The resulting mixture of grain sizes is buffered in one out of two mixing vessels and then continuously fed into the separation stages, which must be shut down temporarily if a minimal flowrate cannot be maintained.

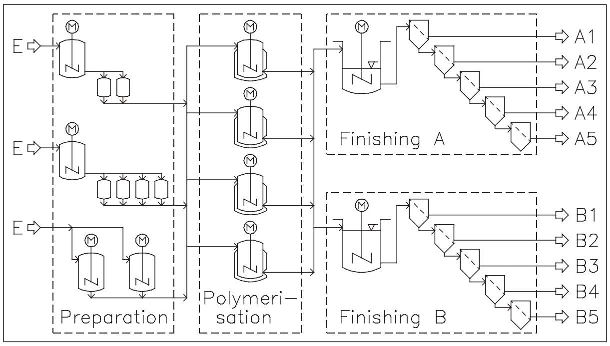

Figure 1. Flowsheet EPS-process

Scheduling decisions to be made are: 1. timing and 2. choice of the recipes of the polymerizations, 3. hold-ups of the mixing vessels, and 4. start-up- and shut-down-times of the finishing lines. They are subject to resource constraints and nonlinear equality constraints describing the mixing process. The objective is to maximize the profit calculated from revenues for satisfying customer demands in time and costs for polymerizations, start-ups/shut-downs of the finishing lines, inventory, and penalties for demand shortages.

A distinct feature of this process is its significant uncertainty. Endogenous disturbances (linked to process events) comprise polymerization times and yields; disturbances in the plant capacity and in the demand are regarded to be exogenous in nature.

Scheduling Approach

A reasonable approach to determine good decisions is a model predictive scheduling (MPS) strategy similar to model predictive control (MPC). The idea is to generate a sequence of scheduling decisions in advance, and to apply the first ones to the process. This scheme is repeated iteratively, such that making decisions and receiving information follows alternately. A widely used concept is to compute schedules for the entire horizon at each step, which ignores the potential of possible recourse decisions to compensate disturbances.

Our vision is to model the multi-stage information and decision structure explicitly by means of multi-stage stochastic mixed-integer programs (e.g. Birge and Louveaux, 1997). The basic idea is to represent possible evolutions of the process by a tree of scenarios with branches at each stage of decisions. From the optimization, a tree of schedules results, which are identical and immediately applicable for the first stage only.

Undoubtedly, a monolithic multi-stage model of a real-world process is usually intractable by standard mathematical programming algorithms within reasonable computing time. Instead, we approximate this problem by applying the following key ideas:

1. Decomposition of the problem into a master and a detailed scheduling problem (MS/DS) according to Figure 2.
2. Approximation of both problems by two-stage stochastic integer programs (2-SSIPs).
3. Formulation of compact and efficiently solvable models; linearization of the nonlinearities.
4. Application of the decomposition algorithm from Carøe and Schultz (1999).

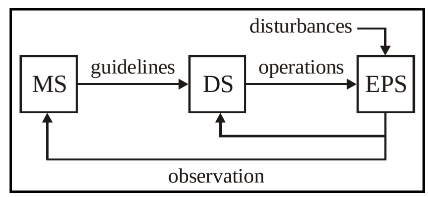

Figure 2. Cascaded feedback structure

Modeling Prerequisites

Mathematical Framework

For a linearized 2-SSIP with a finite number of scenarios a deterministic equivalent can be stated as an MILP:

$$\begin{array}{ll} \max\limits_{x, y_1, \ldots, y_\Omega} & \sum\limits_{\omega=1}^{\Omega} \pi_\omega \left(c^T x_\omega + q_\omega^T y_\omega \right) \\ \text{s.t.} & T_\omega x_\omega + W_\omega y_\omega = h_\omega,\ x_1 = \ldots = x_\Omega, \\ & x_\omega \in X,\ y_\omega \in Y,\ \omega = 1, \ldots, \Omega \end{array} \qquad (1)$$

The 1st and 2nd stage variable-vectors x and y belong to polyhedral sets X and Y with integer requirements. The parameter Ω denotes the number of scenarios ω with corresponding probabilities π. The constraints are formulated by means of the matrices T und W and the right hand side-vector h of suitable dimensions. The classical objective is to maximize the expectation value over all scenarios computed as a weighted sum of x and y subject to the weighting-vectors c and q.

Auto-Recourse

For the problem at hand models for both scheduling problems were formulated on finite moving horizons of reasonable length (DS: 4-8 days, MS: 2-4 weeks). By shifting the horizon, some of the former *recourse decisions y* become *here and now decisions x*. This *auto-recourse*, i.e. the property, that the same model is used throughout, gives rise to a uniform model structure over the entire horizon. Starting from a deterministic base model, a 2-SSIP with the classical or a more advanced optimality criterion can be formulated in a modular fashion.

Scenario Definition

According to the lengths of the horizons, the MS model reflects long-term uncertainties, i.e. demand and capacity, and the DS model reflects short-term uncertainties (time and yield). For the EPS-process *exogenous* and *endogenous* corresponds to *long-term* and *short-term*, respectively.

According to Eq. (1) the uncertainties are modeled by scenarios and their corresponding probabilities are fixed. Therefore, the base models have to be stated such that disturbances affect only parameters (not indices) and the probabilities do not depend on the decisions. Since the probability of an exogenous event only depends on the considered period length and for an endogenous event it depends on the number of process events, the probability space should be discretized wrt. time in the MS case and wrt. events in the DS case.

Time Representation

According to the demanded scenario definition, appropriate representations of time are an event-driven grid with a fixed number of variable points of time for the DS problem, and a multi-period representation with fixed time intervals for the MS problem. Schulz (2001) realized an event-driven approach for the EPS process under the assumption of certain data.

The MS period lengths have to be chosen such that the probability of a disturbance is significant. A horizon of 10 periods of 2 days is reasonable. The first 3 intervals are defined as the 1st stage, since the results serve as a guideline for the DS level and should not have a tree structure.

Master Scheduling Models

In line with the modular concept, the base models of the process and the costs and the optimality criterion under uncertainty are only loosely coupled and interchangeable. We restrict this exposition to the key ideas and refer to Engell et al. (2001) and Sand and Engell (2002) for more details.

Process Models

Scheduling decisions to be made on the master level are 1. the rough timing of start ups/shut downs of the finishing lines, 2. the rough timing of polymerizations and 3. the assignment of recipes. Given I fixed time periods i, the degrees of freedom are represented by the variables $z_{ip} \in \{0,1\}$ and $N_{ir_p} \in IN$, which represent the operation mode of the finishing line p in i and the number of polymerization starts according to recipe r_p in i, respectively. The relevant constraints are the capacity of the polymerization stage and of the finishing lines. It turned out that modeling the interaction between the periods is of major importance.

Considering the constraint for minimal throughput of the finishing lines, the formulation for *decoupled* periods reads as follows (C - mixer levels, F - feed rates):

$$\sum_{r_p=1}^{R_p} N_{ir_p} \geq C_p^{\min} + z_{ip} F_p^{\min} \quad \forall i, p \tag{2}$$

The technique to model the couplings is to constrain sums of periods (the non-linearity can exactly be linearized):

$$\begin{aligned} &\sum_{i'=i}^{i''} \sum_{r_p=1}^{R_p} N_{i'r_p} \geq z_{i''p} z_{(i''+1)p} C_p^{\min} \\ &-\begin{Bmatrix} C_p^0 \text{ if } i=1 \\ z_{(i-1)p} z_{ip} C_p^{\max} \text{ else} \end{Bmatrix} + \sum_{i'=i}^{k} z_{i''p} F_p^{\min} \quad \forall i, i'', p \mid i \leq i'' \end{aligned} \tag{3}$$

The use of constraints (2) instead of (3) leads to more shut down-procedures if a finishing line is driven at its lower capacity limit and to significantly higher costs.

Cost Models

An essential target of profit oriented scheduling is to maximize the sales subject to demand and supply constraints. With $M_{if_p} \in IR_+$ denoting the sales of product f_p in i, $B_{if_p} \in IR_+$ the demand and $\rho_{f_p r_p} \in IR_+$ the yield of f_p according to a certain recipe r_p, (4) defines the demand and the supply constraints, respectively:

$$\begin{aligned} &\sum_{i'=1}^{i} M_{i'f_p} \leq \sum_{i'=1}^{i} B_{i'f_p} \quad \forall i, f_p, p \\ &\sum_{i'=1}^{i} M_{i'f_p} \leq \sum_{i'=1}^{i} \sum_{r_p} \rho_{f_p r_p} N_{i'r_p} \quad \forall i, f_p, p \end{aligned} \tag{4}$$

A disadvantage of this formulation is the missing distinction between timely and late sales. To control the lateness, an index d represents delay intervals; the constraints then read as follows:

$$\begin{aligned} &\sum_{d} M_{d(i+d-1)f_p} \leq B_{if_p} \quad \forall i, f_p, p \\ &\sum_{j=1}^{i} \sum_{d=1}^{j} M_{djf_p} \leq \sum_{j=1}^{i} \sum_{r_p} \rho_{f_p r_p} N_{jr_p} \quad \forall i, f_p, p \end{aligned} \tag{5}$$

Optimality Criteria

In multi-stage stochastic programs, any kind of uncertainty is reflected in the objective function. The classical optimality criterion under uncertainty is the expectation value over all scenarios. However, this approach is not suitable for risk conscious decision making since the width of the probability distribution has no effect. A reasonable solution to this problem is to extend Eq. (1) by the *minimum risk criterion* as follows:

$$\begin{aligned} \max_{\substack{x,y_1,\ldots,y_\Omega,\\ u_1,\ldots,u_\Omega}} \quad & \sum_{\omega=1}^{\Omega} \pi_\omega \left(c^T x_\omega + q_\omega^T y_\omega - \delta u_\omega \right) \\ \text{s.t.} \quad & c^T x_\omega + q_\omega^T y_\omega \geq \varepsilon - M^{big} u_\omega, \\ & T_\omega x_\omega + W_\omega y_\omega = h_\omega,\ x_1 = \ldots = x_\Omega, \\ & x_\omega \in X,\ y_\omega \in Y,\ u_\omega \in \{0,1\},\ \omega = 1,\ldots,\Omega \end{aligned} \tag{6}$$

The idea is to compute the probability that the profit falls below a threshold ε by using binary indicator variables u_ω in a big-M inequality, and to reduce the expectation value proportionally. This extension fits into the 2-SSIP framework and increases the model size only marginally.

Numerical Evaluation

For the different approaches, the base models comprise about 10^2 to 10^3 variables and constraints. Table 1 shows the number of (integer) variables and constraints for base models defined by their characteristic equations.

Table 1. Data of base models

char. Eqs.	var. (int.)	constr.	opt. gap
(2),(4)	244 (122)	194	2.4 %
(3),(4)	264 (122)	479	3.9 %
(3),(5)	634 (122)	679	2.7 %

The stochastic extension mainly consists of the introduction of scenarios and equality constraints for the 1st stage; the problem size approximately scales with the number of scenarios Ω. The 2-SSIP is solved by the decomposition algorithm from Carøe and Schultz (1999), which uses CPLEX (2000) to solve base model-like subproblems.

Mean optimality gaps for the sub-problems are given in Table 1 for 20s CPU-time on a SUN Ultra II 300. Due to its minor additional numerical cost, the more precise process model according to (3) is preferred to (2).

The stochastic extension to 1000 scenarios leads to 2-SSIPs with 10^6-10^7 variables and constraints. The classical formulation (Eq. (1)) with a cost model according to (4) can be solved with optimality gaps of less than 10 % in 8 h CPU time (SUN Ultra Enterprise 450). Preliminary experiments with a reduced model illustrate the aspect of risk: Adding the minimum risk criterion leads to solutions with 20 % less risk while the expectation value is only reduced by 2 % and the numerical performance remains unaffected.

Conclusion and Perspectives

The results demonstrate that the master scheduling problem for a real-world process can be solved by two-stage stochastic mixed-integer programming; studies on the detailed scheduling problem are under way.

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