# A MULTI-WEEK SCHEDULING APPROACH FOR THE STEEL-MAKING PROCESS 

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#### Abstract

This work further elaborates on the steel plant MILP production scheduling method presented in Harjunkoski and Grossmann (2001). The proposed improvements include a more flexible model for the casting step, as well as combining orders from consecutive weeks to improve one of the main targets, which is increasing the number of heats per casting sequence. The paper presents a strategy to obtain optimal casting sequences by using an MILP approach.


## Keywords

Planning and scheduling, meltshop planning, casting sequencing

## Introduction

Steel making is a very energy-demanding and time-critical process where the scheduling decisions are still often performed manually, even though the highly complex and constrained environment makes this especially difficult. The final stage, continuous casting, is often considered as the most complex task in the production planning and several studies focus only on this part of the process (Lally et al. 1987). An overview of commonly used expert system approaches is given in Dorn and Doppler (1996). The steel-making process considered in this paper (Fig. 1) comprises two electric arc furnaces combining the melt steel with scrap, an argon oxygen decarburation unit for removing the carbon, a ladle metallurgy facility where the metal chemistry and temperature are adjusted and a continuous caster to form the end product, steel slabs. The casting is normally processed continuously for a group, containing up to $8-10$ heats, after which the equipment needs regular maintenance. With the term heat
we refer to a certain amount of melt steel, a batch, that is transported through the process in a ladle. A heat can be equivalent to a customer order but large orders may have to be split into several heats. There are several strict rules for sequencing and grouping the heats at the casting stage, which makes it hard to produce longer sequences that would reduce the number of necessary production stops. The approach in Harjunkoski and Grossmann (2001) allows only casting in decreasing width order. In this paper we propose a model improvement that makes it possible to do the casting in both directions. This will affect both the preprocessing step and the mathematical model presented in the earlier work. Apart from adding flexibility these modifications also contribute to diminishing the number of casting sequences needed. The resulting model is both more realistic and leads to optimal caster schedules.


Fig 1. Example process
Another major improvement in the proposed model is achieved by combining two consecutive weeks and merging the orders in a near optimal fashion for improving the casting efficiency. Considering only one week at a time typically results in some short "unwanted" casting sequences with only 1-4 products (or heats). Through a "rolling horizon" strategy we will show a clear improvement where the number of casting sequences can approximately be reduced by $10 \%$.

## Casting order

The strategy for the casting order was already discussed in Harjunkoski and Grossmann (2001). Here we will use the same type of concept but allow casting to be done both in decreasing and increasing width order. The main idea lies in first presorting the products into possible casting orders and then using mixed integer linear programming (MILP) to find the optimal casting sequence.

In the following, we will also use the word "group" to denote a casting sequence and define it as active if it contains one or more products. The result is not only the optimal grouping, but also the correct internal casting order for each group. The MILP formulation assumes that the orders are presorted exactly as in Harjunkoski and Grossmann (2001), by sorting the products subsequently according to their speed, due date, width, grade and thickness. All products of the same grade and thickness form a product family and the casting sequences are created among these. An upper bound for the number of groups needed for a product family can be obtained by starting from the top of the preordered list and selecting products into a group until there is either a compatibility conflict or the group has achieved the maximum number of products (for instance 8). As before, a compatibility matrix, $P_{i i}$, is built to represent the most complex or nonlinear rules. The elements have the value one, if product $i^{\prime}$ can be cast after product $i$, else zero. This matrix together with the constraints embedded in the MILP formulations provides the full information for a successful sequencing.

Because of the fact that both increasing and decreasing casting widths will be allowed, we cannot take full advantage of the preordering step. Instead, the mathematical model needs to be able to manage this added flexibility. In order to clarify the dependencies let us assume four major cases, where $w_{i}$ is the width of a
product, $t_{i}$ is a grade type number ordered such that type 1 should always be cast before type 2, type 2 before type 3 etc. By using the compatibility matrix we get four basic cases:

$$
\begin{array}{lll}
\mathrm{C} 1: P_{i i^{\prime}}=1, & w_{i} \leq w_{i^{\prime}}, & t_{i} \leq t_{i^{\prime}} \\
\mathrm{C} 2: P_{i i^{\prime}}=1, & w_{i^{\prime}} \leq w_{i}, & t_{i} \leq t_{i^{\prime}} \\
\mathrm{C} 3: P_{i^{\prime} i}=1, & w_{i^{\prime}} \leq w_{i}, & t_{i^{\prime}} \leq t_{i} \\
\mathrm{C} 4: P_{i^{\prime} i}=1, & w_{i} \leq w_{i^{\prime}}, & t_{i^{\prime}} \leq t_{i}
\end{array}
$$

In C1, a suitable (satisfying the compatibility matrix requirement) next product, $i^{\prime}$, should be wider than the previous one, $i$, and have a larger grade type. C 2 is similar but here the following product must be narrower. Cases C 3 and C 4 define the same for a preceding product, which also must meet either of the width criterias and be of a smaller grade type to be valid for casting. In the following mathematical formulation we will refer to these cases for readability.

Let $z_{g}$ be a binary variable that is one if a sequence $g$ is used, else zero. The objective is to minimize the total number of casting sequences (groups) needed.

$$
\begin{equation*}
\min \sum_{g \in G} z_{g} \tag{1}
\end{equation*}
$$

The assignment of products $i$ into groups $g$ is handled by the binary variables $x_{i g}$. Each product must be assigned to exactly one group and there is an upper bound, $M_{\max }$, for the number of products per group. Also, the variables for unused groups should be forced to zero.

$$
\begin{align*}
& \sum_{g \in G} x_{i g}=1 \quad \forall i \in I  \tag{2}\\
& \sum_{i \in I} x_{i g} \leq M_{\max } \cdot z_{g} \quad \forall g \in G \tag{3}
\end{align*}
$$

Let $\alpha_{g}$ be a binary variable that is one if the sequence $g$ is increasing with respect to width and $q_{i g}$ be a variable to relax some of the constraints for the last product in a sequence. For an increasing width sequence $(\alpha=1)$ all products, except the last one must be followed by at least another suitable product. Eq. (5) defines the same condition for decreasing width order $(\alpha=0)$.

$$
\begin{gather*}
x_{i g}-q_{i g} \leq \sum_{i^{\prime} \in I \mid C 1} x_{i^{\prime} g}+M_{\max }\left(1-\alpha_{g}\right)  \tag{4}\\
\forall i \in I, \forall g \in G \\
x_{i g}-q_{i g} \leq \sum_{i^{\prime} \in I \mid C 2} x_{i^{\prime} g}+M_{\max } \cdot \alpha_{g}  \tag{5}\\
\forall i \in I, \forall g \in G
\end{gather*}
$$

Similarly to Eqs. (4)-(5) the two following constraints state that all products in a sequence, except the first one, must be preceded by at least one suitable product. The constraint is relaxed for the first product by the variable $r_{i g}$.
$x_{i g}-r_{i g} \leq \sum_{i^{\prime} \in I \mid C 3} x_{i^{\prime} g}+M_{\max }\left(1-\alpha_{g}\right)$
$\forall i \in I, \forall g \in G$
$x_{i g}-r_{i g} \leq \sum_{i^{\prime} \in I \mid C 4} x_{i^{\prime} g}+M_{\max } \cdot \alpha_{g}$
$\forall i \in I, \forall g \in G$

Since we cannot directly control the order of the products as in the case where only decreasing width order was allowed, two other constraints need to be formulated to eliminate mismatches: the first one eliminates the appearance of wider products with a smaller type number and narrower products with a larger type number for increasing widths. The second one does the same for decreasing width order. Note that since the products were preordered according to grade type it is sufficient to compare them by index in Eq. (9).
$\sum_{i^{\prime} \in I \mid w_{i}<w_{i}^{\prime}, t_{i^{\prime}}<t_{i}} x_{i^{\prime} g}+\sum_{i^{\prime} \in I \mid w_{i_{i}}<w_{i}, t_{i}<t_{i^{\prime}}} x_{i^{\prime} g}-M_{\max }\left(1-\alpha_{g}\right) \leq$
$M_{\text {max }}\left(1-x_{i g}\right) \quad \forall i \in I, \forall g \in G$
$\sum_{i^{\prime} \in I \mid w_{i_{i}}<w_{i}, i^{\prime}<i} x_{i^{\prime} g}+\sum_{i^{\prime} \in I \mid w_{i}<w_{i^{\prime}}, i<i^{\prime}} x_{i^{\prime} g}-M_{\max } \cdot \alpha_{g} \leq$
$M_{\text {max }}\left(1-x_{i g}\right) \quad \forall i \in I, \forall g \in G$

Equations (10) and (11) allow exactly two exceptions per group: for the first product and the last product in a sequence.

$$
\begin{array}{ll}
\sum_{i \in I} q_{i g} \leq z_{g} & \forall g \in G \\
\sum_{i \in I} r_{i g} \leq z_{g} & \forall g \in G \tag{11}
\end{array}
$$

The rest of the constraints are more or less to diminish the search space to make the model more compact. The flag for the increasing width order is forced to zero for non-existing sequences in Eq. (12). The groups are ordered by their numbers of products in Eq. (13) and additionally Eq. (14) forces the active groups to be the first ones. Here it should be noted that the number of groups available, $|G|$, is determined in the presorting step and it is common that some of these sequences turn out to be redundant after the optimization.
$\alpha_{g} \leq z_{g} \quad \forall g \in G$

$$
\begin{align*}
& \sum_{i \in I} x_{i, g+1}-\sum_{i \in I} x_{i g} \leq 0 \quad \forall g \in G  \tag{13}\\
& z_{g+1}-z_{g} \leq 0 \quad \forall g \in G \tag{14}
\end{align*}
$$

The exception variables for the first and last products are real variables ranging between zero and one.

$$
\begin{aligned}
& z_{g}, x_{i g}, \alpha_{g} \in\{0,1\} \\
& 0 \leq q_{i g}, r_{i g} \leq 1
\end{aligned}
$$

Solving the model above will automatically generate the optimal casting sequences, taking into account the width and grade type constraints, as well as, for instance the more complex chemistry constraints through the compatibility matrix.

## Multi-week approach

The method in Harjunkoski and Grossmann (2001) focuses on scheduling one week at a time. This is a natural choice, not only owing to traditional planning but also as certain products need to be delivered always on the same weekday. Being able to obtain the optimal grouping for one week is not always sufficient for practical production requirements, for instance due to unwanted short casting sequences. What is often done to deal with these is transferring some production orders from a week to the previous one, or to the following one. The main goal is to avoid rare grades to be sequenced alone by combining the order information from two consequtive weeks.

Since the main benefits from considering two weeks come from this fact, the following integration procedure is proposed:

1. Optimize casting sequences for week 1 .
2. Optimize casting sequences for week 2.
3. Select the short sequences from weeks $1 \& 2$ and solve a new casting sequencing problem with their products.

This approach should result in the same or better casting strategy. An example of the groups and their relations is shown in Fig. 2.


Figure 2. Merging two weeks
How these new groups are then scheduled into the production plan depends completely on the process itself.

If, for instance, there are special products that cannot be moved from their weekly position this should be taken into consideration. Also, the chosen due date tolerance plays a key role, since orders with a high tolerance may be shifted several times (from week 1 to week 2 and further from week 2 to week 3), as in a rolling horizon approach.

## Example

The optimization steps are illustrated and compared using a simple example problem containing only 7 products. Table 1 shows the products after presorting.

| Product | Grade | Width | Thick | Seq |
| :--- | :---: | :---: | :---: | :---: |
| P1 | 101A | 49,8 | 7,5 | 1 |
| P2 | 101A | 44,5 | 7,5 | 1 |
| P3 | 101B | 27,5 | 7,5 | 2 |
| P4 | 101C | 32,2 | 7,5 | 3 |
| P5 | 101 | 49,8 | 7,5 | 4 |
| P6 | 101 | 31,8 | 7,5 | 5 |
| P7 | 101 | 31,5 | 7,5 | 5 |

Table 1. Example orders after presorting
The orders resulted in 5 sequences separated by horizontal lines in Table 1. Given the fact that grade types should be cast in the order: $101 \mathrm{~A} \rightarrow 101 \mathrm{~B} \rightarrow 101 \mathrm{C} \rightarrow 101$ and a maximum width change of 7,0 between two consecutive products the compatibility matrix for these products is shown in Table 2. An optimal solution for the orders when allowing only decreasing casting width is shown in Table 3.

|  | P1 | P2 | P3 | P4 | P5 | P6 | P7 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| P1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| P2 | 1 | 0 | 0 | 0 | 1 | 0 | 0 |
| P3 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| P4 | 0 | 0 | 0 | 0 | 0 | 1 | 1 |
| P5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| P6 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| P7 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |

Table 2. Compatibility matrix

| Product | Grade | Width | Thick | Seq |
| :--- | :---: | :---: | :---: | :---: |
| P1 | 101 A | 49,8 | 7,5 | 1 |
| P5 | 101 | 49,8 | 7,5 | 1 |
| P4 | 101 C | 32,2 | 7,5 | 2 |
| P6 | 101 | 31,8 | 7,5 | 2 |
| P7 | 101 | 31,5 | 7,5 | 2 |
| P2 | 101 A | 44,5 | 7,5 | 3 |
| P3 | 101B | 27,5 | 7,5 | 4 |

Table 3. Optimized with decreasing width
The optimization was able to reduce the number of sequences by one. In this particular example a further sequence was eliminated by allowing a casting order with both decreasing and increasing widths.

| Product | Grade | Width | Thick | Seq |
| :--- | :---: | :---: | :---: | :---: |
| P2 | 101 A | 44,5 | 7,5 | 1 |
| P1 | 101 A | 49,8 | 7,5 | 1 |
| P5 | 101 | 49,8 | 7,5 | 1 |
| P4 | 101 C | 32,2 | 7,5 | 2 |
| P6 | 101 | 31,8 | 7,5 | 2 |
| P7 | 101 | 31,5 | 7,5 | 2 |
| P3 | 101 B | 27,5 | 7,5 | 3 |

Table 4. Optimal solution of the new approach
In this simple case the sequence 3 would be a possible candidate for being merged with other weeks.

As stated in Harjunkoski and Grossmann (2001), this methodology produces nearly optimal weekly schedules (theoretical optimality gap: $0-3 \%$ ) for the meltshop process considered. It is difficult to calculate a valid lower bound for the two-week problem but it is evident that it can on average only improve the earlier one-week schedules, as the number of casting sequences may be further reduced.

## Conclusions

The improvements presented in this paper lead to a minimum number of casting sequences needed to produce a set of products. This number has been identified as a major bottleneck in many steelmaking processes and is also a significant cost factor since the change of casting sequences requires timely and costly maintenance operations. The combination of some of the products within two consecutive weeks has therefore, apart from operational efficiency, a significant economical impact, especially if this helps in further reducing the number of changeover operations.

## References

Dorn J. and Doppler C. (1996). Expert systems in the steel industry, IEEE Expert, 11(1), pp. 18-21
Harjunkoski I. and Grossmann I.E. (2001). A decomposition approach for the scheduling of a steel plant production, Computers and Chemical Engineering, 25, pp. 1647-1660
Lally B. and Biegler L. and Henein A. (1987). A model for sequencing a continuous casting operation to minimize costs, Iron \& Steelmaker, 10, pp. 53-70

