OPTIMAL OPERATIONS PLANNING UNDER UNCERTAINTY BY USING PROBABILISTIC PROGRAMMING

Pu Li*, Moritz Wendt and Günter Wozny
Institute of Process and Plant Technology, Technische Universität Berlin
10623 Berlin, Germany

Abstract

Due to the lack of systematic reliability analysis, intuitive decisions can usually be made in planning process operations under uncertainty. We propose a new analysis and optimization framework to address this problem. By using the technique of probabilistic programming, the solution provides comprehensive information on profit as a function of the confidence level as well as its sensitivities to different uncertain variables. For operations under multiple uncertainties, the sources of risk that have the most significant impacts on the profitability can be identified. An optimal decision can be made, from which a suitable compensation between profit achievement and risk of constraint violation can be achieved. The approach is applied to problems of production planning, unit operations and inventory management. In particular, a novel closed framework for planning unit operations under open-loop uncertain disturbances is proposed and applied to a distillation column operation.

Keywords

Operations planning, Uncertainty, Probabilistic programming.

Introduction

Intuitive decisions can be made in planning process operations due to lack of systematic reliability analysis. One may emphasize the danger of an accident in a process and makes a conservative decision which leads to a very low profit, but in reality the probability of taking place of the accident may be extremely small (e.g. 10^{-6}). In other cases, due to profit expectations, an aggressive decision may be taken, which will probably result in constraint violations and lead to an accident, or frequent modifications of the operating point have to be made. A proper decision should be a compensation between values of profit and risk. Almost all previous studies on optimization under uncertainty for design and operation used the two-stage programming with recourse to deal with constraint violations. This method requires a model for the penalty function which however is not available in many cases.

Probabilistic (chance) programming is a suitable tool for solving such problems (Prékopa, 1995). Its unique feature is that the resulting decision ensures the predefined probability of satisfying constraints. Studies on model predictive control using chance constrained programming have been carried out for linear processes (Schwarm and Nikolaou, 1999; Petkov and Maranas, 1997, Li et al., 2002). Recently, a method to nonlinear chance constrained problems is proposed (Wendt et al., 2002).

The aim of this work is to study the insight and significance of the relation between reliability and profitability for operations under uncertainty. The basic idea is to integrate the available stochastic information into the optimization of operations planning. Based on the method of probabilistic programming, efficient solutions are achieved to problems of production planning, unit operation and inventory management.

* To whom all correspondence should be addressed.

Production Planning

Consider planning problems of profit maximization for processes shown in Fig. 1. The process produces several outflows by processing some inflows. The inflows consist of both raw materials as feedstocks and utilities. Some inflows and outflows can be taken as decision variables. The other inflows and outflows are uncertain. We may know their stochastic distributions or at least a range of values they may take based on historic data.

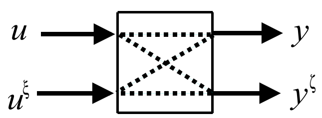

Figure 1. Processes for production planning

Operations planning for such processes is to determine the decision variables so that the profit can be maximized and meanwhile the uncertain inflow availability as well as the uncertain outflow demand be satisfied. However, due to the existing uncertainties, we may not be able to ensure a 100% success of the planned operation. For production planning linear models are always used. Thus the optimization problem can be defined as a stochastic programming problem under single chance constraints

$$\begin{aligned} \max \quad & \text{Profit} = \sum_{i=1}^{I} c_i^y y_i - \sum_{j=1}^{J} c_j^u u_j \\ s.t. \quad & P\left\{ u_k^\xi = \sum_{i=1}^{I} a_{k,i}^{(1)} y_i + \sum_{j=1}^{J} b_{k,j}^{(1)} u_j \le \xi_k \right\} \ge \alpha_k \qquad k = 1,\cdots,K \\ & P\left\{ y_l^\zeta = \sum_{i=1}^{I} a_{l,i}^{(2)} y_i + \sum_{j=1}^{J} b_{l,j}^{(2)} u_j \ge \zeta_l \right\} \ge \alpha_l \qquad l = 1,\cdots,L \\ & y_{\min} \le y \le y_{\max}, \quad u_{\min} \le u \le u_{\max} \end{aligned} \tag{1}$$

or a problem under a joint chance constraint

$$s.t. \quad P\left\{ \begin{aligned} u_k^\xi &= \sum_{i=1}^{I} a_{k,i}^{(1)} y_i + \sum_{j=1}^{J} b_{k,j}^{(1)} u_j \le \xi_k, \quad k = 1,\cdots,K \\ y_l^\zeta &= \sum_{i=1}^{I} a_{l,i}^{(2)} y_i + \sum_{j=1}^{J} b_{l,j}^{(2)} u_j \ge \zeta_l, \quad l = 1,\cdots,L \end{aligned} \right\} \ge \alpha \tag{2}$$

where $u \subseteq \Re^J, y \subseteq \Re^I$ are vectors of input and output decision variables, while $\xi \subseteq \Re^K, \zeta \subseteq \Re^L$ are vectors of stochastic inflows and outflows. $a^{(1)}, b^{(1)}, a^{(2)}, b^{(2)}$ are vectors with known parameters and c^u, c^y are price vectors of the input as well as output decision flows. Note that the uncertain variables do not appear in the objective function, since we have the obligation to follow their values and thus they have no impact on the profit at the solution. Based on the distribution of ξ and ζ, the constraints in (1) and (2) can be transferred into deterministic inequalities due to the inverse probability functions of the stochastic variables and the values of the defined confidence levels. They lead to an LP and an NLP problem, respectively. Let us consider a planning problem:

$$\begin{aligned} \max \quad & \text{Profit} = 0.5 y_1 + y_2 \\ s.t. \quad & y_1 + y_2 \le \xi_1, \;\; 2y_1 + y_2 \le \xi_2, \;\; y_2 \le \xi_3 \\ & y_1 \ge 0, y_2 \ge 0 \end{aligned} \tag{3}$$

where y_1, y_2 are decision outflows and ξ_1, ξ_2, ξ_3 are uncertain inflows. The decision for the maximum profit should be made while ensuring the availability of the inflows. ξ_1, ξ_2, ξ_3 are assumed having uniform, exponential and normal distribution (Fig. 2), respectively. Fig. 3 shows the profit under single and joint chance constraints. To analyze the sensitivity of the profit to the individual uncertain inflows, the profiles of single confidence values allocated by the optimizer are depicted in Fig. 4, where the joint confidence level is specified. The optimal decision will lead to a 100% confidence to ξ_2 in any case, i.e. the uncertain inflow ξ_2 will never be violated if the decision is implemented. One has to take some risk of violations of inflows ξ_1 and ξ_3. In particular, ξ_1 will be most probably violated. Thus it makes sense to use a middle buffer for the input ξ_1. It means that the result of probability analysis can be used for decision of choosing buffers.

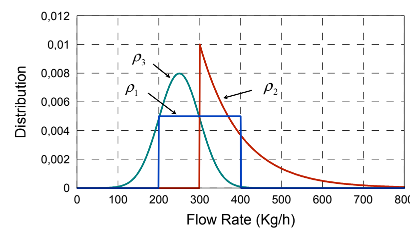

Figure 2. Density function of uncertain variables

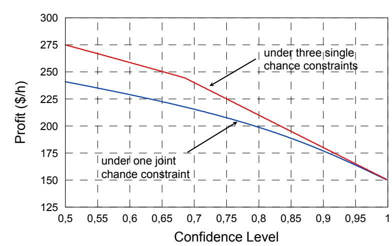

Figure 3. Profit under chance constraints

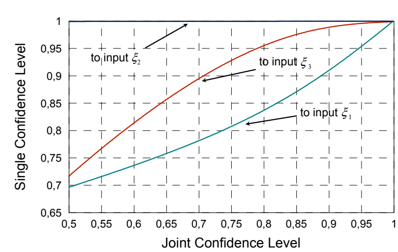

Figure 4. Single and joint confidence levels

Unit Operations Planning

At the stage of unit operations, in addition to follow the production plan, product specifications have to be emphasized. Compositions are usually not on-line measurable and thus not directly controlled (temperatures are often used as references). Moreover, the pressure and load restrictions have to be satisfied. Usually these variables are monitored but not controlled (open-loop), either. There exist some uncertain variables such as feed composition, recycle flows or atmospheric conditions which are additional disturbances to unit operations. The task of unit operations planning is to decide the setpoints for control loops, so that the purity specifications and safety restrictions are satisfied and the operation costs minimized. The optimal unit operation problem under single chance constraints can be formulated as

$$\begin{aligned} \min \quad & \text{Cost} = \mathrm{E}\left(\sum_{j=1}^{J} c_j^u u_j\right) \\ s.t. \quad & g(x, y, y^C, u, \xi) = 0 \\ & P\left\{y_l^C \le y_l^{CLIM}\right\} \ge \alpha_l \qquad l = 1, \cdots, L \\ & y_{\min} \le y \le y_{\max}, \quad u_{\min} \le u \le u_{\max} \end{aligned} \tag{4}$$

or under a joint chance constraint

$$P\left\{y_l^C \le y_l^{CLIM}, \quad l = 1, \cdots, L\right\} \ge \alpha \tag{5}$$

where $u \subseteq \Re^J, y \subseteq \Re^J$ are manipulated and controlled variables, $x \subseteq \Re^M, y^C \subseteq \Re^L$ the unmeasured state and constrained outputs, and y_l^{CLIM} the value of the upper limit of the constrained output y_i. $\xi \subseteq \Re^K$ is the uncertain inputs and $g \subseteq \Re^{M+J+L}$ the process model equations. Note the decision variables are not the controls *u* rather than the controlled variables *y* (setpoints). The controls have to be changed corresponding to the realized uncertain inputs. Thus they are stochastic variables and the expected value of the costs should be defined in the objective function. Since it is difficult to calculate their expected value, the objective function is replaced by $\min \sum_{j=1}^{J} \beta_j y_j$, where β_j is a weighting factor with a sign based on the relation of u_j and y_j, because the cost minimization of the flows (controls) corresponds to the minimization or maximization of the setpoints of the controls. The controlled output variables *y* are closed-loop and thus can be positioned at their setpoints through manipulating the controls *u*. However, the outputs y^C are open-loop but have to be constrained, under the uncertain disturbances. The solution of problem (4) or (5) provide the setpoints for the closed-loops which can achieve a desired compromise between optimality and reliability. This leads to a novel operation concept, i.e. control of an open-loop process under uncertainty by using closed-loop control.

We take a distillation column with 20 bubble-cap trays to separate a methanol-water mixture as an example. The feed flow *F*, composition x_f and atmospheric pressure *P* are considered as uncertain disturbances, with their mean and standard deviation as $\mu_F = 20$ l/h, $\sigma_F = 3$ l/h, $\mu_{x_f} = 30$mol%, $\sigma_{x_f} = 3$mol% and $\mu_P = 1013$mbar, $\sigma_P = 5$mbar, respectively. The optimality requires the column to be operated right at the specified product purity (99mol%), so its energy consumption is minimized. The temperatures (T_3^S, T_{18}^S) on the sensitive trays are selected as the controlled variables. However, ensuring a constant temperature can not ensure the required purity, if the pressure changes due to swing of atmosphere and vapor/liquid load. A conservative operation with a much lower top setpoint and a much higher bottom setpoint has been used so far, which leads to a much greater reboiler and condenser duty than necessary.

The chance constrained optimization problem has the objective function $\min(T_{18}^S - T_3^S)$ subject to a rigorous model composed of component and energy balances, vapor-liquid equilibrium and tray hydraulics for each tray and the chance constraints for the product purity. Fig. 5 shows the optimal setpoints for the two controllers by different confidence levels. The top temperature should be decreased and the bottom temperature increased if a higher confidence level is required. It is interesting to note that T_3^S is less sensitive to the disturbances than T_{18}^S is. This is because the top temperature is almost only affected by *P*, while the bottom temperature is influenced not only by *P* but also by *F* and x_f. As shown in Fig. 6, a greater reboiler duty (expected value) is needed if the required confidence level is higher. It can also be seen that the increase of the reboiler duty is higher in the region of high confidence levels (e.g. $\alpha \ge 0.97$).

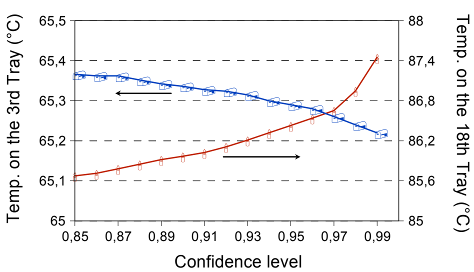

Figure 5. Optimal setpoints for the two controllers

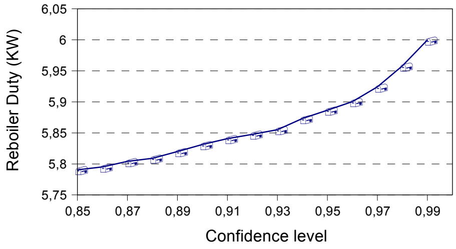

Figure 6. Expected reboiler duty profile

Inventory Management

Buffer tanks are commonly utilized to dampen variations of flows. If the size of buffers is large enough, the storage volume can be utilized for optimization. Several future time periods can be considered and allocate different storage in different time periods. The situation in industry is that buffers tend to be sized via intuition and experience. There are hidden buffers of materials and capacity as people protect themselves from uncertainty (Shobrys & White, 2000). Consider processes described by Fig. 1 and suppose there is a buffer for each uncertain inflow before feeding to the process and for each outflow before supplying to the customer. The reliability of the operation is now the guarantee of ensuring the storage inside the lower and upper limit of the buffer capacity. The optimal planning problem for multiple periods under single chance constraints is

$$\max \quad \text{Profit} = E\left\{\sum_{n=1}^{N}\left[\sum_{i=1}^{I} c_i^y(n)\, y_i(n) - \sum_{j=1}^{J} c_j^u(n)\, u_j(n)\right]\right\}$$

$$\begin{aligned}
s.t. \quad & u_k^\xi(n) = \sum_{i=1}^{I} a_{k,i}^{(1)}\, y_i(n) + \sum_{j=1}^{J} b_{k,j}^{(1)}\, u_j(n) \\
& y_l^\zeta(n) = \sum_{i=1}^{I} a_{l,i}^{(2)}\, y_i(n) + \sum_{j=1}^{J} b_{l,j}^{(2)}\, u_j(n) \\
& M_k^\xi(n) = M_k^\xi(n-1) + \Delta t\,[\xi_k(n) - u_k^\xi(n)] \\
& M_l^\zeta(n) = M_l^\zeta(n-1) + \Delta t\,[y_l^\zeta(n) - \zeta_l(n)] \\
& P\{M_{k\min}^\xi \le M_k^\xi(n) \le M_{k\max}^\xi\} \ge \alpha_k(n) \\
& P\{M_{l\min}^\zeta \le M_l^\zeta(n) \le M_{l\max}^\zeta\} \ge \alpha_l(n) \\
& M_k^\xi(0) = M_{k0}^\xi;\; M_l^\zeta(0) = M_{l0}^\zeta; \quad n = 1,\cdots,N;\; k = 1,\cdots,K;\; l = 1,\cdots,L \\
& y_{\min} \le y(n) \le y_{\max}, \quad u_{\min} \le u(n) \le u_{\max}
\end{aligned} \tag{7}$$

where Δt is the length of one time period and n is the index for period n. $M_k^\xi(n), M_l^\zeta(n)$ are the amounts of storage for feed k and product l of the period n in the corresponding buffers, respectively, while M_{k0}^ξ, M_{l0}^ζ are the initial charges of the buffers. The meanings of the other symbols are the same as in section 2. If the uncertain variables are normally distributed, the problem can be readily addressed (Prékopa, 1995), since a linear transformation of Gaussian variables also results in Gaussian variables.

We consider a planning problem as shown in Fig. 7. It has 2 processing units, a mixer and 3 buffers. There are two demand uncertainties (ζ_1, ζ_2) and one supply uncertainty ξ. The two feed flows to the units (u_1, u_2) and the outflow y are the decision variables. 12 periods are considered. In each period, the uncertain flows have a normal distribution. For instance, Fig. 8 shows 100 samples of ξ. Given the prices of the control flows and the confidence levels ($\alpha_i(n) = 0.99 - 0.01n$) to hold the buffer capacity ($100 \le M_i \le 900$), the operation policy can be gained by solving the problem. Fig. 9 shows the future holdup of tank 1 by the optimal control under 100 samples of ξ, ζ_1, ζ_2. It can be seen that there may be violations during the 4th to 8th periods (lower bound) and the 8th to 12th periods (upper bound) by the open-loop policy. These can be prevented by the method of moving horizon.

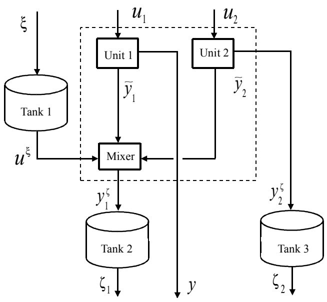

Figure 7. A plant for inventory management study

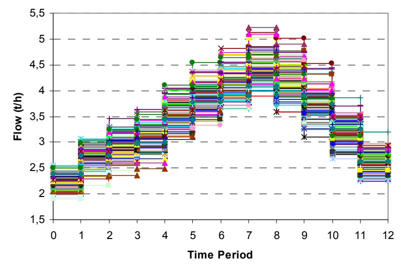

Figure 8. 100 samples of the feed flow to tank 1

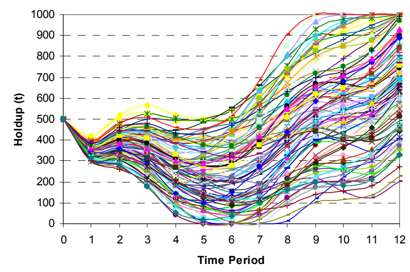

Figure 9. Holdup profiles of tank 1

References

Li, P., Wendt, M., Arellano-Garcia, H., Wozny, G. (2002) optimal operation of distillation processes under uncertain inflow streams accumulated in a feed tank, *AIChE Journal*, **48**, 1198-1211.

Petkov, S. B., Maranas, C. (1997). Multiperiod planning and schedulung of multiproduct batch plants under demand uncertainty. *Ind. Eng. Chem. Res.*, **36**, 4864-4881.

Prékopa, A. (1995). *Stochastic programming*. Kluwer, Dordrecht.

Schwarm, A. T., Nikolaou, M. (1999) Chance-constrained model predictive control, *AIChE J.,* **45**, 1743-1752.

Shobrys, D. E., White, D. C. (2000). Planning, scheduling and control systems: why can they not work together? *Comp. Chem. Eng.*, **24**, 163-173.

Wendt, M., Li, P., Wozny, G. (2002), Nonlinear chance constrained process optimization under uncertainty. *Ind. & Eng. Chem. Res.*, **41**, 3621-3629.