FINANCIAL RISK MANAGEMENT IN PLANNING UNDER UNCERTAINTY

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Abstract

This paper discusses several theoretical developments related to financial risk management in the framework of two-stage stochastic programming for planning under uncertainty. The well-known capacity planning problem is used to illustrate the concepts. A probabilistic definition of financial risk is used in the framework of two-stage stochastic programming and its relation to downside risk is analyzed. Thus, new two-stage stochastic programming models that manage financial risk are presented.

Keywords

Risk Management, Planning under Uncertainty, Two-stage Stochastic Programming.

Introduction

Planning under uncertainty is a common class of problems found in process systems engineering. Some examples are capacity expansion, scheduling, supply chain management, resource allocation, etc. The first studies on planning under uncertainty can be accredited to Dantzig (1955) and Beale (1955), who proposed the two-stage stochastic models with recourse. The industrial importance of planning under uncertainty has been discussed by Murphy et al. (1982), Eppen et al. (1989), Sahinidis et al. (1989), Berman and Ganz (1994), Lui and Sahinidis (1996) and Ahmed and Sahinidis (2000). Several approaches were proposed to formulate and solve this kind of problems (Charnes and Cooper, 1959; Bellman and Zadeh, 1970; Zimmermann, 1987, Ierapetritou and Pistikopoulos, 1994). A major limitation of all these approaches is that they do not consider the variability of the solutions explicitly. This shortcoming was first discussed by Eppen et al (1989), who proposed the use of *downside risk* to measure the cost variability. More recently, Ahmed and Sahinidis (1998) proposed to use the robust optimization framework, which had been introduced by Mulvey et al. (1995). They use the *upper partial mean* (*UPM*) as a measure of the variability of the recourse cost. However, the *UPM* may unnecessarily penalize favorable scenarios, resulting in non-optimal solutions that provide misleading information about the variability (**Error! Reference source not found.**).

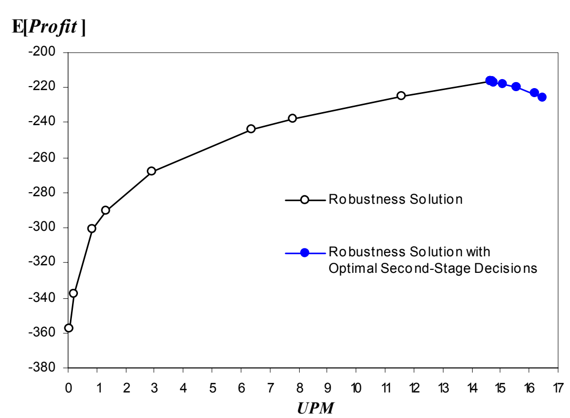

Figure 1. Expected Profit vs. Upper Partial Mean

Consequently, solutions that are considered "robust" may exhibit high levels of financial risk due to the non-optimality of the second-stage decisions. Other approaches to financial risk management were proposed by

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Ierapetritou and Pistikopoulos (1994); Cheng et al (2001); and Applequist et al. (2000).

Financial Risk

The financial risk associated with a planning project under uncertainty is defined as the probability of not meeting a certain target profit (maximization) or cost (minimization) level referred to as Ω. That is, the risk associated with a design *x* and a target profit Ω is therefore expressed by the following probability:

$$Risk(x,\Omega)=P(Profit(x)<\Omega) \quad (1)$$

where *Profit*(*x*) is the actual profit, i.e., the profit resulting after the uncertainty has been unveiled and a scenario realized. Since profit can be related to a summation over a set of independent scenarios, we have

$$Risk(x,\Omega)=\sum_{s\in S} p_s\, z_s(x,\Omega) \quad (2)$$

where $z_s(x,\Omega)$ is a new binary variable that takes the value of 1, when $Profit_s(x)<\Omega$, and zero otherwise. Equation (2) constitutes a formal definition of financial risk for two-stage stochastic problems. When profit has a continuous probability distribution, financial risk –defined as the probability of not meeting a target profit Ω– can be expressed as:

$$Risk(x,\Omega)=\int_{-\infty}^{\Omega} f(x,\xi)\, d\xi \quad (3)$$

where $f(x,\xi)$ is the profit probability distribution function. The interpretation of this equation is given in Figure 2.

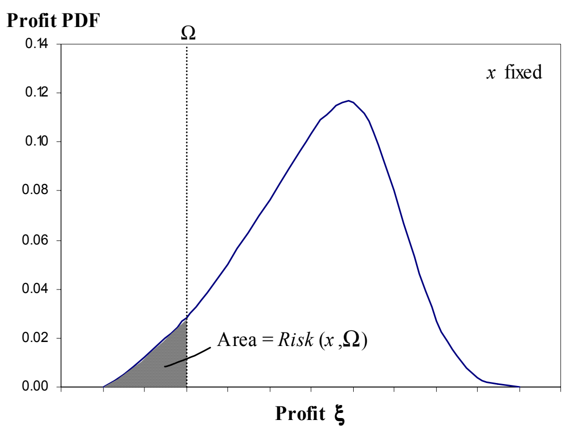

Figure 2. Probabilistic definition of risk

Alternatively, in the discrete scenario case, financial risk is given by the cumulative frequency obtained from the profit histogram. Figure 3 depicts the cumulative probability curves for both the discrete and continuous cases.

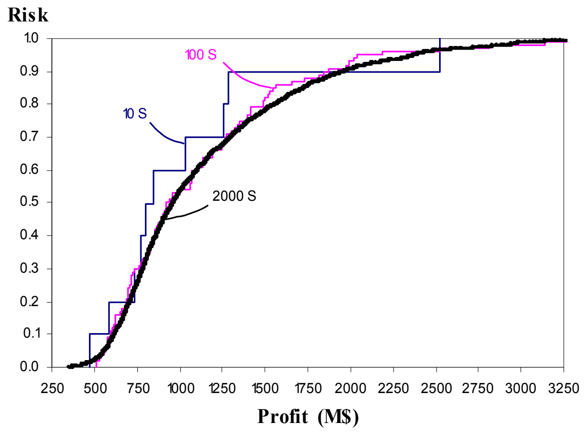

Figure 3. Cumulative risk curves

In this article, it is assumed that continuous probability density functions of uncertain parameters give rise to a set of infinite number of scenarios, which in turn give rise to a continuous Profit PDF. Thus, the technique of scenario sampling is considered to converge to a continuous Profit PDF and consequently to a smooth risk curve.

For a given design *x*, the cumulative risk curve shows the level of incurred financial risk at each profit. Handling the shape and position of the curve are the main interests of the decision maker. A risk-averse investor may want to have low risk for some conservative profit aspiration level, while a risk-taker decision maker would prefer to see lower risk at higher profit aspiration level, even if the risk at lower profit values increases. Figure 4 illustrates a hypothetical example with these two types of risk curves.

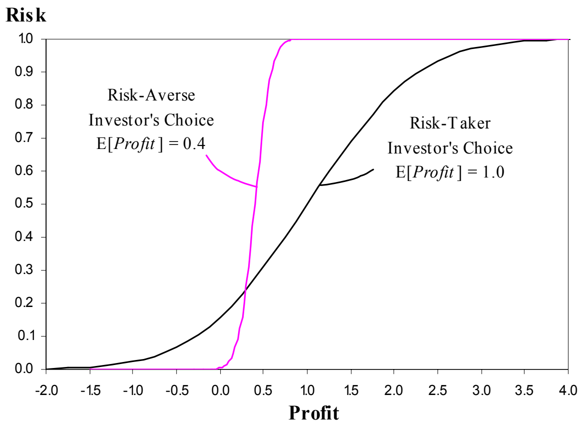

Figure 4. Different kinds of financial risk curves

Barbaro and Bagajewicz (2002a) proved that no feasible design or plan can have a risk curve that lies entirely beneath the risk curve of the optimal solution that maximizes the expected profit. Consequently, both risk curves either cross at some point(s) or the latter lies entirely above the former. This behavior is depicted in Figure 5.

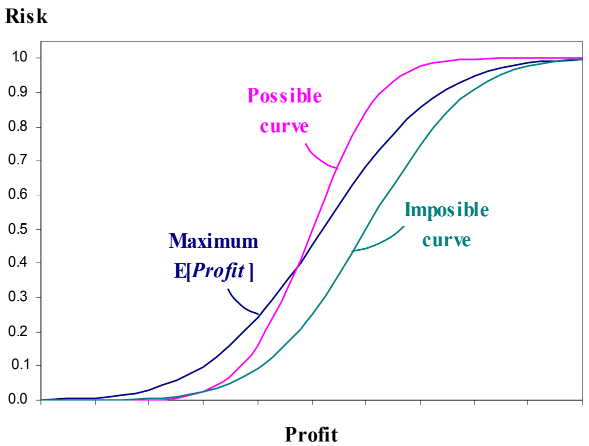

Figure 5. Possible risk curves

Using the definition of risk given by Eq. (2) a multi-objective model for risk management were developed. This model considers that the intention of the decision maker is to maximize the expected profit and at the same time minimize the financial risk at every profit level.

Model SP-FR

$$\begin{aligned}
&Max\ \ \mathrm{E}[Profit] = \sum_{s\in S} p_s q_s^T y_s - c^{\mathrm{T}} x \\
&Min\ \ Risk(\Omega_1) = \sum_{s\in S} p_s z_{s1} \\
&\qquad\vdots \\
&Min\ \ Risk(\Omega_i) = \sum_{s\in S} p_s z_{si} \\
&st \\
&Ax = b \\
&T_s x + W y_s = h_s && \forall s\in S \\
&U_s z_{si} \ge \Omega_i + c^{\mathrm{T}} x - q_s^T y_s && \forall s\in S, i\in I \\
&U_s (z_{si} - 1) \le \Omega_i + c^{\mathrm{T}} x - q_s^T y_s && \forall s\in S, i\in I \\
&x \ge 0 \qquad x\in X \\
&y_s \ge 0 && \forall s\in S \\
&z_{si} \in \{0,1\} && \forall s\in S, i\in I
\end{aligned} \tag{4}$$

Two multi-parametric representations of this model are possible. The first one includes a goal programming weight $\rho_i \ge 0$ in the objective function, which is then varied in order to obtain different solutions. The second one imposes a limit (upper bound) $\varepsilon_i \ge 0$ to financial risk in order to satisfy the decision maker's criterion (Barbaro and Bagajewicz, 2002a,b). However, the inclusion of new integer variables represents a major computational limitation for large-scale problems.

Downside Risk

To tackle the mentioned computational difficulties, the concept of *downside risk* (Eppen et al., 1989) was examined. Downside risk is defined as the expected value of the positive deviation from the target, $\delta(x,\Omega)$:

$$DRisk(x,\Omega) = \mathrm{E}[\delta(x,\Omega)] \tag{5}$$

where $\delta(x,\Omega)$ is continuous and equal to $\Omega - Profit(x)$ when $Profit_s(x) < \Omega$, and equal to zero otherwise. Thus,

$$DRisk(x,\Omega) = \sum_{s\in S} p_s \delta_s(x,\Omega) \tag{6}$$

In turn, the definition downside risk for continuous distributions is:

$$DRisk(x,\Omega) = \int_{-\infty}^{\Omega} Risk(x,\xi)\, d\xi \tag{7}$$

Therefore, downside risk is defined as the area under the cumulative risk curve from profits $\xi = -\infty$ to $\xi = \Omega$, as shown in Figure 6.

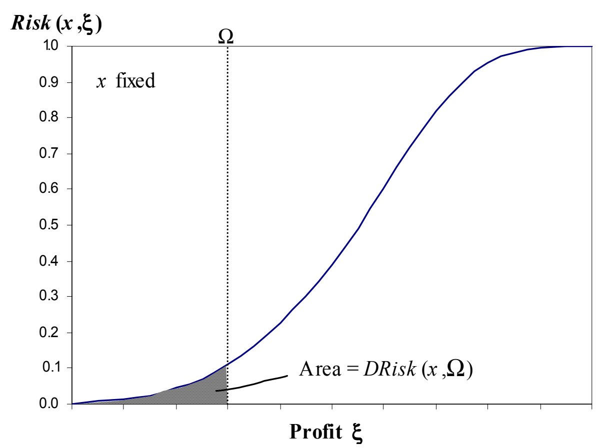

Figure 6. Different kinds of financial risk curves

Notice that downside risk is defined as an expected value (in \$), while financial risk is a probability. In addition, $DRisk(x,\Omega)$ does not require the use of binary variables in the second stage, reducing the computational burden. Using $DRisk(x,\Omega)$ as a measure of $Risk(x,\Omega)$, a new model for risk management is presented (Barbaro and Bagajewicz, 2002a,b), where risk is addressed at various levels, as above, depending on the attitude towards risk of the decision makers.

Thus, a full spectrum of solutions with different levels of risk exposure is generated, which is then presented to the decision maker, who makes the final choice. Barbaro and Bagajewicz (2002a,b) showed that this model is very effective for risk management purposes using a series of test capacity planning problems. Risk curves obtained for one of these problems are shown in the figure below.

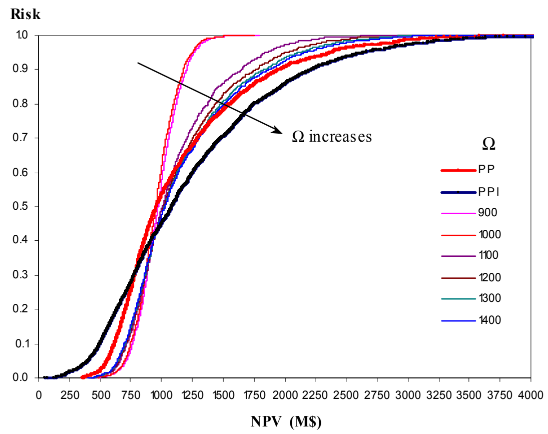

Figure 7. Spectrum of solutions

Barbaro and Bagajewicz (2002b) also studied the effect of inventory and option contracts on risk, showing that even when these risk-mitigating instruments are considered, the standard stochastic formulation with the maximization of the expected profit as single objective yields solutions that exhibit high risk exposure at low aspiration levels. It is only after risk is penalized using the models presented here that less risky solutions are obtained.

Conclusions

The cumulative risk curves were found to be very appropriate to visualize the risk behavior of different alternatives. Furthermore, the concept of downside risk was examined, and a close relationship with financial risk was discovered. It is suggested that downside risk, limited at different aspiration levels, be used to present alternatives to decision makers.

Nomenclature

A: Matrix of coefficients of first-stage constraints
b: Vector of independent terms of the first-stage constraints
c: Vector of first-stage cost coefficients
h_s: Stochastic terms of the second-stage constraints
p_s: Probability of occurrence of scenario s
q_s: Vector of recourse function's stochastic coefficients
T_s: Technology matrix of the second-stage constraints
U_s: Upper bound of the profit under scenario s
x: First-stage decision variables. $x \in X$.
y_s: Second-stage decision variables for scenario s
z_{si}: Binary variable for financial risk definition
δ_s: Profit positive deviation for scenario s
μ: Penalty weight for the expected profit in model SP-DR
ξ: Profit
Ω: Profit target

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