A Continuous-Time Formulation for Simultaneous Consideration of Planning and Scheduling Decisions

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Abstract : Planning and scheduling of batch and semi-continuous plants involving problem formulation, information flow and solution methodologies are the subject of numerous publications during the last two decades. A typical planning problem includes hundreds of different products produced utilizing a variety of process unit operations over a time horizon of several weeks to few months. The mathematical formulation of such planning problem, involving detailed scheduling decisions, results in models with thousands of variables that cannot be directly addressed due to their computational complexity. In this work a new formulation is presented to address the simultaneous consideration of planning and scheduling problems based on the continuous time representation and the idea of periodic scheduling. In this paper, demand and prices are assumed fixed along the time horizon under consideration. The proposed formulation corresponds to a MINLP problem and results in the determination of the optimal cycle length and detailed schedule. Although the approach does not guarantee to provide the optimal solution to the planning problem, it results in excellent near optimal solution in reasonable computation time. Case study is presented to illustrate the efficiency of the proposed model.

Keywords : Planning, Continuous time formulation, Periodic scheduling

1 Introduction

The planning problem concerns the determination of the optimal allocation of resources within the production facility through a time horizon of few weeks up to few months. However detailed scheduling decisions should be simultaneously considered in order to guarantee feasibility. This leads to intractable problem formulation in terms of computational time. Thus one has to trade-off optimality with computational efficiency. Planning problem also involves the problem of uncertainty considerations, while for a smaller time horizon product demand and prices can be considered deterministic, this is not the case when larger time horizon is considered. Additional disturbances may also upset the production schedule as for example rush order arrival and machine break-down.

A large number of publications are devoted to modeling and solution of the planning and scheduling problem. An extended review can be found in Wu and Ierapetritou (2002). In this paper, a new model is proposed to address the planning and scheduling problem simultaneously based on the basic concept of periodic scheduling as well as continuous time representation. A motivating example is presented in section 3 to illustrate the complexity of solving simultaneously the planning and scheduling problem. The proposed mathematical formulation is presented in detail in section 4 and applied to the motivating example in section 5. The results are compared with existing approaches.

2 Approach and Concepts

The planning problem that is considered in this paper is defined as follows. Given (i) the production recipe (i.e., the processing times for each task at the suitable units, and the amount of the materials required for the production of each product), (ii) the available units and their capacity limits, (iii) the available storage capacity for each of the materials, (iv) the time horizon under consideration, and (v) the market requirements of products, the objective is to determine the optimal schedule to meet the specified criterion such as maximal profit, or minimal cost while satisfying all the production requirements. It should be noted however, that the product demands are considered at the end of time horizon and all of the above constraints are fixed within this time horizon.

The continuous time representation proposed by Ierapetritou and Floudas (1998) is used in this work that avoids the shortcoming of prepostulation unnecessary time slots or intervals. Given a number of *event points* that correspond to either the initiation of a task or the beginning of unit utilization, the starting time and duration of a task are optimally determined.

The idea of periodic scheduling is frequently utilized for the solution of planning problem described above. The optimal solution of planning problem implies that the schedule does not exhibit any periodicity (Pantelides, 1994). However, one has to balance against the computational complexity of solving non-periodic schedules for a long time horizon.

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The presented periodic scheduling approach resides on the following assumption. For the case that the time horizon is long compared with the duration of individual task, a proper time period exists, which is much smaller than the whole time horizon, within which, some maximum capacities or crucial criteria have been reached so that the periodic execution of such schedule will obtain results very close to the optimal one by solving the original problem without any periodicity assumption. Thus the size of the problem is reduced to a much smaller one that can be efficiently solved. Besides its computation efficiency the proposed operation plan is more convenient and easier to implement since it assumes repetition of the same schedule. In this approach, the variables include the length of the cyclic time period as well as the detailed schedule of this period, which are defined as unit period and unit schedule respectively. Unlike the short-term scheduling where all intermediates other than those provided initially have to be produced before the beginning of the tasks, unit schedule can start with certain amounts of intermediates as long as storage capacity constraints are not violated. These initial amounts of intermediates are the same as the ones stored at the end of unit period, so as to preserve the material balance as shown in Figure 1.

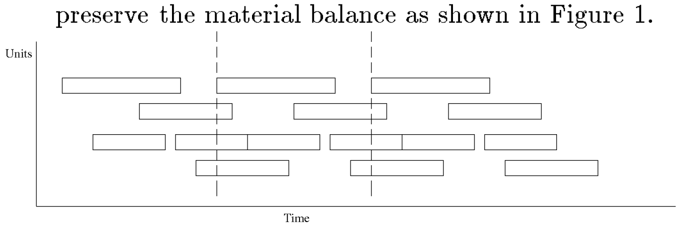

Figure 1. Periodic Schedule

It should be noticed that in the periodic scheduling, each processing unit may have an individual cycle as long as the cycle time is equal with the duration of the unit period. So as illustrated in Figure 2a, all the units do not necessarily share the same starting and ending time points. This concept can be found in Shah et al. (1993) in their discrete time representations for periodic scheduling problem as wrap-around. Schilling and Pantelides (1999) incorporated the same concept into their continuous time formulation based on the resource-task network (RTN) representation (Pantelides, 1994).

Figure 2a illustrates a unit schedule. When a larger time period has to be scheduled using the unit schedule, overlapping is allowed in order to achieve better resource utilization. In this way the equivalent unit schedule is determined as shown in Figure 2b. Note that by using this idea better schedules are determined since tasks are allowed to cross the unit schedule boundaries.

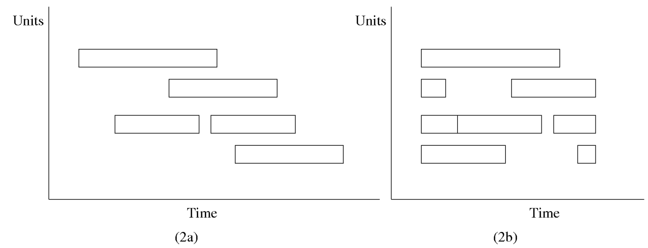

Figure 2. Unit Schedule

3 Motivating Example

The State Task Network (STN) representation and the detailed data for this example can be found in Ierapetritou and Floudas (example 2, 1998). When large time horizon is considered, the size of the model becomes intractable. For example considering a time horizon of 24 hours, the formulation of Ierapetritou and Floudas (1998) consists of 1517 constraints, 546 continuous variables and 156 binary variables using 13 event points. It takes 92367 CPU seconds to get a sub-optimal solution on Sun Ultra 60 workstation. When the same formulation were used for a time horizon of 168 hours, the solution procedure (GAMS/CPLEX) could not even generate a feasible schedule for the whole time horizon. This results point to the importance of developing a new approach for the simultaneous solution of planning and scheduling problem.

4 Mathematical Formulation

The proposed formulation is based on the framework presented by Ierapetritou and Floudas (1998) and involves the following constraints (the detailed explanation can be found in Wu and Ierapetritou (2002)).

Allocation Constraints

$$\sum_{i \in I_j} wv(i,n) = yv(j,n), \quad \forall j \in J, \quad n \in N \qquad (1)$$

Capacity Constraints

$$V_{ij}^{min} wv(i,n) \leq B(i,j,n) \leq V_{ij}^{max} wv(i,n), \quad \forall i \in I, \quad j \in J_i, \quad n \in N \qquad (2)$$

Storage Constraints

$$ST(s,n) \leq ST(s)^{max}, \; \forall s \in S, \; n \in N \qquad (3)$$

Material Balances

$$ST(s,n) = ST(s,n-1) - d(s,n) + \sum_{i \in I_s} \rho_{si}^{p} \sum_{j \in J_i} B(i,j,n-1) + \sum_{i \in I_s} \rho_{si}^{c} \sum_{j \in J_i} B(i,j,n), \quad \forall s \in S, \quad n \in N \qquad (4)$$

Material Balances Between Cycles

$$STIN(s) \quad = \quad ST(s,n), \ \ \forall s \in IS, \ \ n = N \quad (5)$$

Demand Constraints

$$\sum_{n \in N} d(s,n) \geq r(s), \ \ \forall s \in S \quad (6)$$

Duration Constraints

$$T^f(i,j,n) \quad = \quad T^s(i,j,n) + \alpha_{ij} wv(i,n) + \beta_{ij} B(i,j,n), \ \forall i \in I, \ j \in J_i, \ n \in N \quad (7)$$

Sequence Constraints

$$T^s(i,j,n+1) \geq T^f(i',j',n) - U(2 - wv(i',n) - yv(j',n)), \ \forall j,j' \in J, \ i \in I_j, i' \in I_{j'}, \ n \in N, n \neq N \quad (8)$$

Sequence Constraints: Completion of previous tasks

$$T^s(i,j,n+1) \geq \sum_{n' \in N, n' \leq n} \sum_{i' \in I_j} (T^f(i',j,n') - T^s(i',j,n')), \ \forall i \in I, \ j \in J_i, \ n \in N, n \neq N \quad (9)$$

Time Horizon Constraints

$$T^f(i,j,n) \leq 2H, \ \ \forall i \in I, j \in J_i, n \in N \quad (10)$$

$$T^s(i,j,n) \leq 2H, \ \ \forall i \in I, j \in J_i, n \in N \quad (11)$$

Time Length Constraints

$$\sum_{n \in N} \sum_{i \in I_j} (T^f(i,j,n) - T^s(i,j,n)) \leq H, \ \forall i \in I_j, \ \ j \in J, \ \ n \in N \quad (12)$$

Time Fitting Constraints

$$T^s(i',j',n0) \geq T^f(i,j,n) - H, \ \forall j,j' \in J, \ i \in I_j, i' \in I_{j'}, \ n = N \quad (13)$$

Objective: Maximization of Average Profit

$$\frac{\sum_s \sum_n \text{price}(s) d(s,n)}{H} \quad (14)$$

wv(i,n) and *yv(j,n)* are binary variables representing whether task i or unit j is assigned at event point n, respectively. Other continuous variables include storage *ST(s,n)*, initial input *STIN(s)*, batchsize *B(i,j,n)*, delivery amount *d(s,n)*, cycle length H, start time $T^s(i,j,n)$ and finish time $T^f(i,j,n)$. Parameter U is the maximum cycle length. Eq.(1)-(4) and Eq.(6)-(9) are the same as the ones utilized for the short-term scheduling problem. Eq. (5) represents the key feature of periodic scheduling expressing the fact that intermediates stored at the last event point of previous cycle is used as input for the current cycle. Eq.(12) states that the duration of all tasks performed in the same unit must be less than the cycle length, so that it ensures that each unit cannot have individual cycle longer than the cycle length. Eq.(13) represents the requirement that the first task in a new cycle has to start after the completion of all the required tasks in the previous cycle. The objective function for the planning problem is to maximize the profit due to product sales. The objective for periodic scheduling approach corresponds to maximize the average profit as shown in Eq.(14). The average profit is considered to express the dependence of the profit over the whole time horizon on both the production during each cycle and the cycle time. Note that the objective function involves fractional terms ($\frac{d(s,n)}{H}$), thus giving rise to a MINLP problem. Alternative objectives can also be incorporated to express different scheduling targets such as makespan minimization.

5 Examples

Due to the nonlinear term appearing in the objective function, global optimality cannot be guaranteed when using local optimization solvers. In this work, GAMS/DICOPT (Grossmann et al., 2002) is used that uses OA/ER solution procedure.

The motivating example is solved on Pentium*III* 500. In order to determine the optimal schedule and cycle length, the following strategy is considered. Instead of considering the whole cycle time range, for example 2-24 hours, several sub-ranges are considered such as 2-6 hours, 6-10 hours, up to 24 hours and the resulting problems are solved independently. The advantages are that (i) each sub-period utilizes less number of event points that speeds up the solution process, (ii) it generates a number of scheduling alternatives that can be beneficial to plant manager that has to consider additional requirements such as work shift constraints, and (iii) each sub-problem can be solved independently and thus parallelization can be easily achieved. As shown in Table 1, the optimal cycle length obtained is 23.790 with the objective value of 279.029 units. The optimal schedule is shown in Figure 3. Additional computational statistics information are shown in Table 2 for the sub-model with cycle time range of 2-6 hours.

To consider the whole planning problem the time horizon is divided in three periods, the initial period when the necessary amounts of intermediates are produced to start the periodic schedule, the main part when periodic scheduling is applied and the final period to wrap up all the intermediates. The initial and final periods are bounded by a time range and solved independently. The sum of time lengths of all three periods equals to the time horizon. Applying this approach to the motivating example for a time horizon of 168 hours, the objective value obtained is 45784.94. A more complex and larger system is currently under evaluation with this formulation.

Cycle time range	Number of event points	Objective function value	Optimal cycle time (h)	CPU time (s)
2-6 hours	4	268.289	5.094	2.86
6-10 hours	6	272.247	9.036	512.00
10-14 hours	7	273.801	12.978	5365.74
14-18 hours	9	276.447	14.407	305.88
18-21 hours	11	277.363	19.709	545.83
21-24 hours	12	279.029	23.7900	2884.41

Table 1. Solution for Motivating Example

Relative optimality creterion	0.01
Cycle time range (h)	2-6
Number of event points	4
Binary variables	48
Continuous variables	299
Constraints	530
Optimal cycle time (h)	5.094
Objective function value	268.289
CPU time (s)	2.86

Table 2. Computational Statistics for Motivating Example

This motivating example is very similar to the one used by Schilling and Pantelides (1999). Application of this solution procedure for this problem results in an optimal cycle of 36.64 with objective 28.94. The results are compared with the results of Schilling and Pantelides (1999) in Table 3. Note that the proposed formulation results in significant less number of variables and constraints. However no direct comparison of computational complexity can be made since Schilling and Pantelides employ a global optimization solution procedure.

	Proposed Approach	Formulation of Schilling and Pantelides (1999)
Cycle time range (h)	20-40	20-40
Event points or time slot	3	6
Binary variables	28	81
Continuous variables	112	437
Constraints	272	440
Optimal cycle time (h)	36.64	36.81
Objective function value	28.94	28.72
CPU time (s)	0.82	81

Table 3. Comparison of Results

6 Summary

This paper addresses the approach of solving the planning and scheduling problem simultaneously based on a continuous-time formulation to determine the optimal periodic schedules as well as the optimal cycle length for multipurpose batch plants. Compared with existing approaches, the formulation results in smaller size and thus easier to solve model. Although the proposed model corresponds to a MINLP problem, it is solved efficiently with GAMS/DICOPT solver.

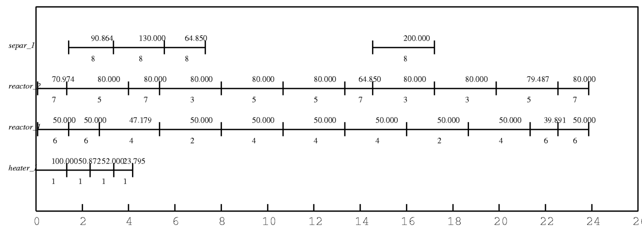

Figure 3. Optimal Schedule for Motivating Example

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