OPTIMIZING THE SUPPLY CHAIN OF PETROCHEMICAL PRODUCTS UNDER UNCERTAIN OPERATIONAL AND ECONOMICAL CONDITIONS

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Abstract

The main objective of this work is to develop an optimization model for the supply chain of a petrochemical company operating under uncertain operational and economical conditions. First a deterministic model is developed and tested, followed by introducing uncertainties in key parameters. The proposed objective function is based on optimizing the system resources by minimizing the total production costs, and raw material procurement, as well as lost demand, backlog, transportation and storage penalization. The stochastic formulation is then developed, which is based on the two-stage problem method with finite number of realization. The optimization model has been tested on a typical petrochemical industry, manufacturing different grades of PolyEthylene, operating in a single site and using two reactors. Uncertainties have been introduced in demands, market prices, raw material costs and production yields. The main conclusion of this study is that uncertainties have a drastic effect on the planning decisions of the petrochemical supply chain. Market demands was found to be the most important parameter, which showed strong impact on the production decisions, followed by the production yields. The stochastic approach was found to be quite effective in handling uncertainties, and the resulted production plans have considerably low *expected value of perfect information* (EVPI).

Keywords

Supply Chain Optimization, Petrochemical Products, LP, MIP, Stochastic Programming.

Introduction

Petrochemical industries are global multinational organizations in which business decisions involve a number of players, which span sourcing, manufacturing and distribution. For such industry with multiple suppliers of raw material and multiple markets, it is vital to plan all activities along the entire supply chain network. This includes allocating demand quantities at all levels over the time horizon (usually 120 days and 18 month). Another imperative characteristic is that the product mix varies widely. One plant may be capable of producing various products with different grades. From this realization emerged the importance of the supply chain management and optimization of petrochemical industry.

An extremely important factor characterizing the planning and scheduling problems of process industries is the high degree of uncertainty. The stochastic nature of the problem arises from the fact that there are a number of parameters whose value cannot be controlled by the decision maker and are uncertain. Uncertainty propagates through the supply chain network from the market at supply side, quantity and quality of raw material, to production quality and yield, and from the other side to the market economics and customer demands.

Supply chain and supply chain management are relatively new terms. They crystallize concepts about integrated business planning that have been espoused by logistics experts, strategists, and operations research practitioners

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as far back as the 1950s (Shapiro, 2001). Since that early age, most researchers have been addressing a single component of the overall production-distribution system. To day, a number of researchers are actively addressing the integration of such single components into the overall supply chain. A small number of publications have been cited during late 1990's. However, in the last two years increasing research interests in this area have certainly boosted the number of publications.

Published books addressing supply chain management include Handfield and Nicholos (1999), Simchi-Levi et al. (2000) and Shapiro (2001). An extensive literature review of supply chain models was presented by Vidal and Goetschalckx (1997), while Beamon (1998) provided a research agenda for future research in this area.

Supply chain concepts and models have been applied to a number of process industries. Schenk (1998) and Dempster et al. (2000) reported their experience in applying and modeling supply chain in oil companies, and Kafoglis (1999) addressed the application of supply chain approach in refinery operations. Gibson (1998) outlined ICI's experience in applying supply chain management to their operations. Application of supply chains in petrochemical industry has been addressed by Bodington and Shobrys (1996) and Zhou et al. (2000). Other applications include the pharmaceutical industries (Papageorgious et al., 2001), paper industry (Philpott and Everett, 2001) and automotive sector (Escudero et al., 1999). Uncertainty in durations of manufacturing processes and projects was addressed by Gerchak (2000). Applequist et al. (2000) used the risk premium construct to introduce a measure of risk for investments in a supply chain. Moreover, fuzzy logic was used to handle uncertainty by Petrovic et al. (1999) and Sohn and Choi (2001).

Deterministic Mathematical Model

Objective Function

The proposed objective function is based on optimizing the system resources usage by minimizing the total production costs, and raw material procurement, as well as lost demand, backlog, transportation and storage penalization. The objective function for the deterministic model is defined as:

$$Z = min\left\{\sum_{j\in P}\sum_{t\in T} PC_j PV_{j,t} + \sum_{i\in IR}\sum_{t\in T} SC_i RV_{i,t} + \sum_{j\in P}\sum_{d\in DS}\sum_{t\in T}\lambda_{j,d} LV_{j,d,t} + \sum_{j\in P}\sum_{d\in DS}\sum_{t\in T}\beta_{j,d} B_{j,d,t} + \sum_{j\in P}\sum_{d\in DS}\sum_{t\in T} TC_d YV_{j,d,t} + \sum_{j\in P}\sum_{t\in T}\sigma_j SV_{j,t} - \sum_{j\in P}\sum_{d\in DS}\sum_{t\in T} SP_{j,d} YV_{j,d,t}\right\} \quad (1)$$

The first term accounts for the production cost. The second term accounts for the raw material cost. The third and fourth terms represent penalties for lost demand and backlog, with penalties $\lambda_{j,d}$ and $\beta_{j,d}$, are lost demand and backlog penalties, respectively, of product j for demand source d. $LV_{j,d,t}$ and $B_{j,d,t}$ lost demand and backlog volumes, respectively. The fifth term accounts for transportation cost, the sixth term for storage cost and selling non-shipped products at lower prices, and the last term takes into account dissimilarities in selling prices of products for different demand sources.

Constraints

Product balance equation:

$$SV_{j,t-1} + \varphi_{j,t} PV_{j,t} = \sum_{d\in DS_j} YV_{j,d,t} + SV_{j,t} \qquad \forall j\in P,\ t\in T \quad (2)$$

Demand Balance Equation:

$$B_{j,d,t-1} + D_{j,d,t} = YV_{j,d,t} + LV_{j,d,t} + B_{j,d,t} \qquad \forall j\in P,\ t\in T,\ d\in DS \quad (3)$$

$LV_{j,d,t}$ is the lost demand expressed as:

$$LV_{j,d,t} = \delta_d\left(B_{j,d,t-1} + D_{j,d,t} - YV_{j,d,t}\right) \geq 0 \qquad \forall j\in P,\ t\in T,\ d\in DS \quad (4)$$

Raw Material Balance Equation:

$$RV_{i,t} = \sum_{j\in P} a_{i,j} PV_{j,t} \qquad \forall i\in IR,\ t\in T \quad (5)$$

where $a_{i,j}$ is the net amount of raw material that is needed per unit of product. To account for losses in raw material, this amount is defined as:

$$a_{i,j} = \frac{\alpha_{i,j}}{\left(1 - 0.001\gamma_{i,j}\right)} \quad (6)$$

Maximum production volume:

$$0 \leq PV_{j,t} \leq MP_{j,t} \qquad \forall j\in P,\ t\in T \quad (7)$$

$$0 \leq \sum_{k\in P_g} PV_{k,t} \leq MG_{g,t} \qquad \forall g\in PG,\ t\in T \quad (8)$$

$$0 \leq \sum_{j\in P} PV_{j,t} \leq MU_{l,t} \qquad \forall l\in U,\ t\in T \quad (9)$$

Maximum product stock volume:

$$0 \leq \sum_{j\in P} SV_{j,t} \leq MS_t \qquad \forall t\in T \quad (10)$$

Maximum product backlog volume:

$$\left.\begin{array}{l} 0 \leq B_{j,d,t} \leq MB_{j,d,t} \\ 0 \leq YV_{j,d,t} \end{array}\right\} \qquad \forall j\in P,\ d\in DS,\ t\in T \quad (11)$$

Maximum and minimum raw material stock volume:

$$mR_i \leq RV_{i,t} \leq MR_i \qquad \forall i\in IR,\ t\in T \quad (12)$$

Reactor Scheduling

In order to produce different grades of a given product, each reactor follows a specified production wheel. A general production wheel is shown in Figure 1. It consists of *n* possible production paths. Each path is for the production of a number of products, the first of which is known as the primary path. Products are termed $EP_{i,j}$, where *i* is the path number and *j* is the product number in the i^{th} path. Production wheels introduce mainly two constraints. First, at least one of the products in the primary path should be produced, hence the primary path should is selected:

$$\pi_{l,1} = 1 \qquad \forall l \in U \tag{13}$$

The second constraint prevents switching production from a parallel path to another before starting the cycle again from the primary path:

$$\sum_{k=2}^{n} \pi_{l,k} \leq 1 \qquad \forall l \in U \tag{14}$$

The following constraint is to force production of the products in the wheel of production unit *l*.

$$\sum_{j \in mk} PV_{EP_{j,k},t} \leq \pi_k \cdot MU_{l,t} \qquad \forall l \in U,\ t \in T,\ k = 1,\ldots,n \tag{15}$$

Stochastic Mathematical Model

The stochastic programming technique used in this study is the *two-stage stochastic linear program with fixed recourse*. This method is also known as *scenario analysis* technique because uncertainty is modeled via a set of scenarios. Uncertain parameters are assumed to have three scenarios; "*above average*", "*average*" and "*below average*". A reasonable assumption is to be neutral about the expected risk. This assumption means that the three scenarios have an equal probability of *1/3*. Hence, the objective function of the stochastic model may be represented as follows:

$$Z = min \left\{ \begin{array}{l} \sum_{j \in P} \sum_{t \in T} PC_j PV_{j,t} + \sum_{i \in IR} \sum_{t \in T} SC_i RV_{i,t} + \\ \frac{1}{3} \sum_{s=1}^{3} \left[\begin{array}{l} \sum_{j \in P} \sum_{d \in DS} \sum_{t \in T} \lambda_{j,d} LV^s_{j,d,t} + \sum_{j \in P} \sum_{d \in DS} \sum_{t \in T} \beta_{j,d} B^s_{j,d,t} + \\ \sum_{j \in P} \sum_{d \in DS} \sum_{t \in T} TC_d YV^s_{j,d,t} + \sum_{j \in P} \sum_{t \in T} \sigma SV^s_{j,t} - \\ \sum_{j \in P} \sum_{d \in DS} \sum_{t \in T} SP^s_{j,d} YV^s_{j,d,t} \end{array} \right] \end{array} \right\} \tag{16}$$

In Eq. (16), the first stage decision variables are the production ($PV_{j,t}$) and raw material ($RV_{i,t}$) volumes. The second stage decisions, $YV^s_{j,d,t}$, $LV^s_{j,d,t}$, $B^s_{j,d,t}$ and $SV^s_{j,t}$, are associated with an index, s=*1,2,3* corresponding to the three scenarios. In the above formulation, uncertainty is also assumed in the selling prices, $SP^s_{j,d}$. If $SP^2_{j,d}$ is the average selling prices, then ±20% uncertainty is introduced as $SP^1_{j,d} = 1.2 \times SP^2_{j,d}$ and $SP^3_{j,d} = 0.8 \times SP^2_{j,d}$ for above and below average scenarios respectively.

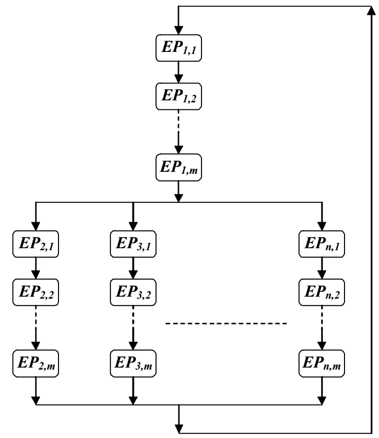

Figure 1. General representation of a production wheel

In addition to the objective function, a number of constraints should be also modified to account for the uncertainties represented by the stochastic model. In addition to the stochastic parameters mentioned above, the constraints include one more stochastic parameter, which is the market demands, $D^s_{j,d,t}$.

The proposed deterministic and stochastic models have been solved using GAMS (GAMS, 2001). The resulted optimization problem is a mixed integer linear programming (MIP) for which GAMS/XA solver was used. However, for those runs where reactor scheduling is not considered, the problem is reduced to an LP for which GAMS/BDMLP was used.

Supply Chain Network

The supply chain network that will be used in this study is shown schematically in Figure 2. This network has been developed based on the nature of business for a typical petrochemical industry.

In the proposed supply chain network, Hexene and catalysts are ordered and imported while Ethane is obtained from a local refinery. Two production plants are used to produce the needed amounts of Ethylene and Butene. Intermediate storage is provided for Hexene, Ethylene and Butene feed stocks. The production facility consists of two reactors, R1 and R2. The first reactor R1 produces nine products, A1 to A9, while R2 produces six products, B1 to B6. Production volumes are directly shipped to demand sources and excess volumes are kept in the warehouse. Demand sources represent retailers in different distribution countries. In the proposed network, eleven demand sources are considered, D1 to D11. This network can be considered as a typical network that includes the principal components. The network can be easily modified and extended to include more products and additional demand sources.

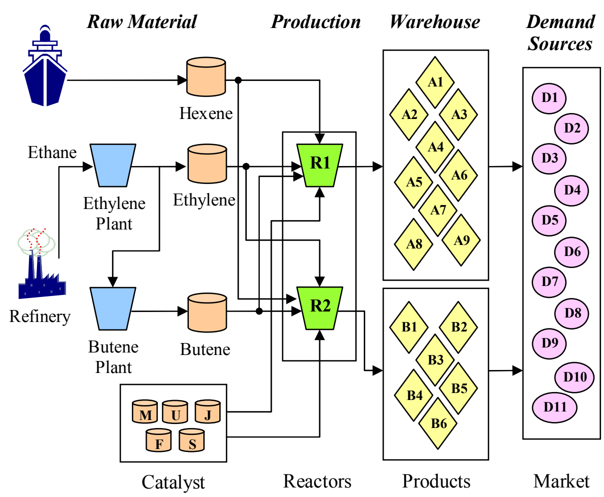

Figure 2. Supply chain network

Table 1. Case studies selected for analysis and discussion

Case Studies	Description	Cost relative to *Case-1*
Case-0	Deterministic, No Reactor Scheduling	0.91
Case-1	BASE CASE, Deterministic	1.0
Case-2	Deterministic, + 20 % Demand	0.92
Case-3	Deterministic, – 20 % Demand	1.37
Case-4	Stochastic, ± 20 % Demand	1.14
Case-5	Deterministic, – 20 % Market Prices	1.038
Case-6	Deterministic, + 20 % Market Prices	0.953
Case-7	Stochastic, ± 20 % Market Prices	0.995
Case-8	Deterministic, – 20 % Raw Mat. Costs	0.887
Case-9	Deterministic, + 20 % Raw Mat. Costs	1.117
Case-10	Deterministic, – 20 % Prod. Yields	0.73

Results and Discussion

The proposed supply chain model has been tested on a typical petrochemical industry, manufacturing different grades of PolyEthylene, in a single production site and utilizing two reactors. Eleven case studies have been selected for analysis and discussion. The case studies are listed in Table 1 and compared with respect to a base case study, *Case-1*. The case studies resulted in different values of the six cost items of the objective function. For the base case, production (34.5%) and raw material costs (38.5%) were found to contribute to more than two third of the total cost. This is followed by the difference in selling prices (13%), transportation (7.3%), storage and selling excess products (5.8%) and the least cost item is the backlog and lost demand (0.9%).

Comparing *Case-0* and *Case-1* shows that reactor scheduling introduced a number of constraints on the sequence by which the products are produced. The increase in the value of the objective function is mainly due to the storage and backlog penalties. Despite the fact that reactor scheduling converted the problem to MIP, which is more difficult to solve, it appeared to be an important element in studying the supply chain of petrochemical plants.

Effects of uncertainties in market demands have been investigated in three case studies. For the first two case studies, the deterministic model was solved for 20% increase and decrease in market demands, while the stochastic model was solved for the third one. Excess market demands (*Case-2*) resulted in ceasing production of less economical products and caused all cost items to drop below those of the base case except for the lost demand and backlog costs. On the other hand, decreased market demands (*Case-3*) subjected the supply chain to the penalties of keeping the excess production volumes in stock and/or selling them at lower prices.

Evaluating the stochastic model for ± 20% deviations in market demands (*Case-4*) resulted in determining the optimum production plan in presence of uncertainty, which is referred to as the first stage decisions. The second stage decisions provided information about the volumes shipped, lost demands, backlogs, and stocks for each of the three scenarios; above average, average, and below average demands. Stochastic market demands resulted in 14.2 % increase in the value of the objective function as compared to the base case. This increase is known as the expected value of perfect information (EVPI) and may correspond to loss of profit due to the presence of uncertainty.

Similarly, uncertainty in market prices have been considered in three case studies. The optimum cost increased by 3.8% for the low market prices case (*Case-5*), and decreased by 4.7 when the prices are high (*Case-6*). Nevertheless, the value of the objective function for the stochastic case (*Case-7*) was very close to that of *Case-1*.

Effects of uncertainty in raw material prices are considered by *Case-8* and *Case-9*, while uncertainty in production yields is considered by Case-10. Uncertainty in the cost of raw material showed slight effect on the production plan due to the fact that all material costs are constantly changed. Thus the products' preferences in terms of production costs are not altered. It would be, however, interesting to investigate the effect of changing the cost of each raw material separately, for which the products' distribution is expected to vary inconsistently.

For *Case-10*, the 20% reduction in production yields caused the value of the cost function to drop by 27%. This is obviously due to the high increase in the costs of demand losses and backlogs.

Conclusions

The optimization model proposed in this study simulates closely the basic planning requirements of an actual supply chain of petrochemical plants. The model has been thoroughly tested by means of a number of case studies reflecting uncertainty in key parameters.

Two approaches have been applied in studying the effect of uncertainty on the supply chain. The first approach is based on introducing deviations in the deterministic model, while the scenario analysis stochastic approach is used for the second approach. The proposed stochastic model succeeded in determining the optimum production volumes that maximize the volumes of products shipped to the demand sources and minimize demand losses and backlogs, for each scenario. Both the first and second stage decisions were found quite different from those of the deterministic results. Hence, one can safely conclude that deterministic optimization models may result in unsatisfactory planning results for supply chains with uncertainty in market demands.

It is clear from the optimization results that market demands have strong effect on the planning results followed by production yields. Uncertainty in market selling prices resulted in minor effects. However, it was realized that the first production unit (R1) is more resilient to disturbances in market prices than the second unit (R2). This might be related to the fact that the production of R1 is distributed to more product types. Such observation may motivate further investigation to study the dynamics of the petrochemical supply chains.

Nomenclature

$a_{i,j}$ Net mount of raw material needed per unit of product.
$B_{j,d,t}$ Backlog volume of product j for demand source d at (the end of) time period t.
$D_{j,d,t}$ Demand of product j from demand source d at time period t.
DS Set of demand sources.
$EP_{n,m}$ Product m in n^{th} path of the production wheel.
$LV_{j,d,t}$ Lost demand of product j for demand source d at time period t.
$MB_{j,d,t}$ Max. allowed backlog for demand source d.
mR_i, MR_i Min. and max. stock for raw material i.
MS_t Max. allowed stock of products at time period t.
$MG_{g,t}$ Max. production volume for products in group g at time period t.
$MP_{j,t}$ Max. production volume for product j at time period t.
$MU_{l,t}$ Maximum production capacity of unit l for time period t.
P Set of products.
P_g Set of products that belong to group g.
PC_j Unit production cost at any time period for product j.
PG Set of product groups
$PV_{j,t}$ Volume of product j made available at time period t.
IR Set of raw materials
$RV_{i,t}$ Volume of raw material i needed at (the beginning of) time period t.
SC_i Unit cost at any time period for raw material i.
$SP_{j,d}$ Selling price of product j for demand source d.
$SV_{j,t}$ Volume of product j kept in stock at time period t.
T Set of time periods in the planning horizon.
TC_d Unit transportation cost to demand source d.
U Set of production units.
$YV_{j,d,t}$ Volume of product j shipped to demand source d at time period t.
$\alpha_{i,j}$ Amount of raw material i needed per unit of product j.
$\beta_{j,d}$ Unit penalty cost of backlog for product j and demand source d.
δ_d Expected loss demand fraction of non-served cumulated demand for demand source d.
$\gamma_{i,j}$ Fallout of raw component i in manufacturing product j.
$\varphi_{j,t}$ Yield of product j made available at time period t.
$\lambda_{j,d}$ Unit penalty cost of lost demand for product j and demand source d.
$\pi_{l,k}$ Production path k in the production wheel of unit l.
σ_j Unit storage cost of product j.

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