Optimization of a hydraulic fracturing process to enhance the productivity of a fractured well in ultra-low permeability environment

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Abstract

In this work, we initially focus on modeling of a hydraulic fracturing process to describe fracture growth, proppant transport and proppant settling. The developed model involves the coupling of multiple nonlinear dynamic equations that show the spatiotemporal evolution of the important physical variables in the hydraulic fracturing processes. To solve these equations by capturing the detailed process dynamics of the system with a time-dependent spatial domain, a fixed-mesh strategy is employed by adopting the size of integration time steps. Then, we identify a linear time invariant state-space model by applying the MOESP algorithm to regress a linear model of a hydraulic fracturing process. In this regard, a series of step inputs are used to generate the input (flow rate and proppant concentration) and output (average width and propped length of fracture) data using the dynamic model. In ultra low permeability formations, horizontal wells are typically drilled with multiple hydraulic fractures for enhanced recovery of oil and gas. We find the optimal number of fractures (equivalently, optimal fracture length) and well aspect ratio that maximize the productivity of a fractured well for a given amount of proppant, well drainage area and the total length of all the fractures. To achieve the optimal fracture length, a pumping schedule is generated using the developed empirical model by solving an optimization problem that directly takes into account the practical constraints such as the total amount of proppant to be injected for each fracture and the required minimum average width at the end of pumping. The generated pumping schedule is applied to the dynamic model, and a series of results demonstrate that the propped length is close to the optimal fracture length while satisfying the practical constraints to enhance the productivity of a fractured well.

Keywords

hydraulic fracturing, optimal pumping schedule, optimal fracture geometry computation, dynamic modeling.

Introduction

The shale gas refers to natural gas trapped in rock of very low porosity and permeability. Even though shale formations contain many naturally formed fractures, production would be so slow that extracting gas from such a well would be considered to be economically infeasible. Two stimulation technologies (hydraulic fracturing and horizontal drilling) are widely used to render shale gas recovery economically attractive. Typically, several horizontal wells are drilled. Within each well, tens to hundreds of fractures are generated through the operation of a hydraulic fracturing treatment.

When designing a hydraulic fracturing treatment, optimization techniques have traditionally been employed to determine the number of fracture stages, the distance between adjacent stages, and the total amount of a fracturing fluid to be introduced and its injection rate for well completion. Specifically, for a given amount of proppant, a unified fracture design that provides the optimal fracture geometry has been addressed by Econo-

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mides et al. (2002) for conventional (high-permeability) oil and gas reservoirs, and the approach has been recently extended to unconventional (low-permeability) resources by Bhattacharya et al. (2012). However, in the unconventional reservoir, because of the uncertainty in the measurement of basic optimization parameters such as the reservoir permeability, traditional design approaches do not perform very well in designing a fracturing treatment. Motivated by this consideration, Ibragimov et al. (2005) proposed a way to calculate the dimensionless productivity index by simply solving an eigenvalue problem, and the approach is employed in this work to find the optimal number of fractures as well as the desired fracture length and well aspect ratio for a given amount of proppant, well drainage area and the total length of all the fractures to maximize the total productivity of a fractured well.

Once we identify the desired fracture geometry and the number of fractures, we have to develop a technique to generate an optimal pumping schedule to achieve this target. One of the most common approaches for generating pumping schedules is developed by Nolte (1986). Assuming no proppant settling, he provided a pumping schedule in the pre-defined form as a power-law, requiring the final fluid efficiency, pad time and total time a priori. The generated pumping schedule can be applied to a variety of fracture geometries, however it notably underestimates the pad size, which may lead to premature tip screen out. Therefore, we propose an optimization framework to design a pumping schedule that will achieve the desired fracture geometry at the end of pumping.

The remainder of the paper is structured as follows: First, we present the dynamic model of a hydraulic fracturing process. Then, a simulator is developed to describe the spatio-temporal evolution of fracture geometries, suspended proppant concentration and proppant bank formation across the fracture, by effectively handling the computational requirement attributed to coupling of multiple equations over time-dependent spatial domain. Then, the optimal fracture geometry is obtained based on the method proposed by Ibragimov et al. (2005). Lastly, we present a methodology to design a pumping schedule and show that the generated pumping schedule performs very well producing a fracture with the desired fracture geometry.

Dynamic modeling of hydraulic fracturing systems

A dynamic model of the hydraulic fracturing process is developed based on the following standard assumptions: (1) the formation layers above and below are where the fractures have sufficiently large stresses such that the vertical fracture is confined within a single horizontal rock layer; (2) the rock properties remain constant with respect to time and spatial coordinates; (3) the fracture length is much greater than its width, and thus, the fluid pressure across the vertical direction is constant; and (4) fracture propagation is described using the Perkins, Kern, and Nordgren (PKN) model which is shown in Fig. 1.

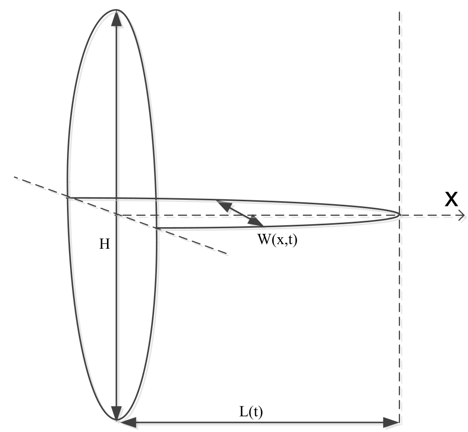

Figure 1. The PKN fracture model (Perkins and Kern, 1961; Nordgren, 1972) considered in this work.

Fluid momentum

The fluid flow rate is determined by the following equation for flow of a Newtonian fluid in an elliptical section (Nordgren, 1972; Economides and Nolte, 2000):

$$\frac{dP}{dx} = -\frac{64\mu Q}{\pi \left(H-\delta\right) W^3} \tag{1}$$

where P is the net pressure, μ is the fluid viscosity, Q is the local flow rate in the horizontal direction, H is the fracture height, δ is the bank height, and W is the fracture width.

Pressure-width relationship

For a crack under constant pressure, the fracture shape is elliptical and the maximum fracture width (i.e., the minor axis of the ellipse) with a height $(H-\delta)$ is given by (Sneddon and Elliot, 1946).

$$W = \frac{2P\left(H-\delta\right)\left(1-\nu^2\right)}{E} \tag{2}$$

where ν is the Poisson ratio of the formation and E is the Young's modulus of the formation.

Continuity equation

The continuity equation for flow of an incompressible fluid inside the fracture is given by (Nordgren, 1972),

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} + (H - \delta)\, U = 0 \tag{3}$$

where $A = \pi W\,(H - \delta)\,/4$ is the free cross-sectional area of the elliptic fracture (Nordgren, 1972), U is the fluid leak-off rate. The two boundary conditions and an initial condition are formulated as follows:

$$Q(0,t) = Q_0 \quad \text{and} \quad W(L(t),t) = 0 \tag{4a}$$

$$W(x,0) = 0 \tag{4b}$$

where Q_0 is the water/slurry injection rate at the wellbore.

Leak-off model

The fluid leak-off rate is given by the following equation (Economides and Nolte, 2000):

$$U = \frac{2C_{leak}}{\sqrt{t - \tau(x)}} \tag{5}$$

where C_{leak} is the overall leak-off coefficient, t is the elapsed time since fracturing was initiated, and $\tau(x)$ is the time at which the fracture propagation has arrived the spatial coordinate x for the first time.

Proppant transport

The transport of proppant in the vertical as well as horizontal direction is considered. The settling velocity, V_s, is computed by (Daneshy, 1978),

$$V_s = \frac{(1 - C)^2}{10^{1.82C}} \frac{(\rho_{sd} - \rho_f)\, g d^2}{18\mu} \tag{6}$$

where ρ_{sd} is the proppant particle density, ρ_f is the pure fluid density, g is the gravitational acceleration constant, d is the proppant diameter, C is the dimensionless proppant concentration and μ is the fracture fluid viscosity where its dependence on C can be modeled through the following empirical formula (Barree and Conway, 1995):

$$\mu(C) = \mu_0 \left(1 - \frac{C}{C_{\max}}\right)^{-\alpha} \tag{7}$$

where μ_0 is the pure fluid viscosity, α is the exponent in the rage of 1.2 to 1.8, $C_{\max}$ is the theoretical maximum concentration, which is determined by $C_{\max} = (1 - \phi)\, \rho_{sd}$ where ϕ is the proppant bank porosity. It is assumed that the settling velocity of proppant along the vertical direction is identical. The evolution of proppant bank is described by (Gu and Hoo, 2014),

$$\frac{d\,(\delta W)}{dt} = \frac{C V_s W}{1 - \phi} \tag{8}$$

The advection of suspended proppant in the horizontal direction can be expressed by the following equation:

$$\frac{d\,[W\,(H - \delta)\, C]}{dt} = -\frac{d\,(QC)}{dx} \tag{9}$$

Numerical simulation

A novel numerical scheme is developed for efficiently solving these equations by effectively handling the issues with time-dependent spatial domains and coupling of nonlinear equations. In order to capture the detailed process dynamics of the system that has a boundary condition associated with the time-dependent spatial domain, a fixed mesh strategy is employed by adopting the size of integration time step.

Numerical solution procedure

The steps of the numerical algorithm are shown below:

1. At time step t_k, the fracture length $L(t_{k+1})$ is obtained by elongating the fracture tip by Δx, $L(t_{k+1}) = L(t_k) + \Delta x$.
2. The coupled equations of Eqs. (1)–(9) are solved for the fracture width $W(x, t_{k+1})$, suspended proppant concentration $C(x, t_{k+1})$, net pressure $P(x, t_{k+1})$, flow rate $Q(x, t_{k+1})$, settling velocity $V_s(x, t_{k+1})$ and proppant bank height $\delta(x, t_{k+1})$ across the fracture via a finite element method.
3. Calculate $\tau(x_{k+1})$ in Eq. (5) iteratively by repeating Steps 2 and 3.
4. The time interval Δt_{k+1} is determined.
5. Set $k \leftarrow k + 1$ and go to Step 1.

Optimal fracture geometry in ultra-low permeability environment

For the given total reservoir area per well, A_{well}, the amount of proppant to be injected, M_{prop}, and the total length of all the fractures to be created, the following parameters have to be determined to maximize the productivity of a fractured well: the aspect ratio of the drainage area, AR_{well}, the number of fractures created in one well, n_f.

The dimensionless (constant-pressure) productivity index of a single fracture, J_{Dcpfr}, is a function of the aspect ratio of the drainage area, $ar = x_e/y_e$, and the penetration ratio of the fracture, $i_x = (2x_f)/x_e$, where x_e and y_e are the width and length of drainage area for one fracture and x_f is the fracture length. According to Ibragimov et al. (2005), $J_{Dcpfr} = 2\lambda_0/\pi$ is the dimensionless productivity index where λ_0 is the first positive eigenvalue of the following eigenvalue problem:

$$\frac{\partial^2 \phi_0}{\partial x^2} + \frac{\partial^2 \phi_0}{\partial y^2} = -\lambda_0 \phi_0 \tag{10}$$

with zero Neumann boundary conditions along the rectangle cross section of drainage area everywhere except for zero Dirichlet condition along the fracture. Then, the total dimensionless productivity index of the well configuration, $J_{\mathrm{D}} = n_{\mathrm{f}} J_{\mathrm{Dcpfr}}$, can be maximized by varying n_f and aspect ratio of the well drainage area, AR_{well}.

Model identification and validation

We assume that for the hydraulic fracturing process, the nonlinear model of Eqs. (1)–(9) is not available and a dynamic model needs to be identified. First, a series of step inputs are generated and applied to the hydraulic fracturing process of of Eqs. (1)–(9). Second, a set of input/output data is obtained, and the multivariable output error state-space (MOESP) algorithm is used to regress a linear time-invariant state-space model of the hydraulic fracturing process. Additionally, a pumping schedule is generated by solving the optimization problem formulated in the following section based on the identified empirical model, and it is applied to the hydraulic fracturing process of Eqs. (1)–(9).

Design of optimal pumping schedule

In hydraulic fracturing, the same propped volume may create larger or smaller productivity. For example, in a high-permeability formation a wide and short fracture is preferred while in a low-permeability formation a narrow and long fracture is preferred Economides et al. (2002). Therefore, producing fractures with desired width and length are essential for productivity improvement.

In this work, a novel constrained optimization scheme is employed for the design of a multi-input pumping schedule with the objective of minimizing the deviation of the fracture geometry from the desired value at the end of pumping. On the basis of the empirical model developed in the preceding section, the design of the pumping schedule has the following form:

$$\min_{C_i, Q_i, \Delta t_i} \quad \left(\frac{x(t_f) - x_{\mathrm{f}}}{x_{\mathrm{f}}} \right)^2 \tag{11a}$$

$$\text{s.t.} \quad \dot{\hat{z}}(t) = A\hat{z}(t) + Bu(t) \tag{11b}$$

$$Q_{\min} \leq Q_i \leq Q_{\max} \tag{11c}$$

$$C_{\min} \leq C_i \leq C_{\max} \tag{11d}$$

$$C_{\mathrm{i}} \leq C_{\mathrm{i+1}} \tag{11e}$$

$$\sum_{i=1}^{10} 2Q_i C_i \Delta t_i = M_{\mathrm{total}}/n_{\mathrm{f}} \tag{11f}$$

$$W_{\mathrm{average}}(t_f) > 3D_{\mathrm{proppant}} \tag{11g}$$

$$\sum_{i=1}^{10} \Delta t_i = t_f \tag{11h}$$

$$\forall i = 1, \ldots, 10 \tag{11i}$$

where $\hat{z}(t) = [W_{\mathrm{average}}(t), x(t)]$ is the predicted average fracture width and propped length, t_f is the total treatment time, Δt_i is the time duration of the stage i, M_{total} is the total amount of proppant to be injected, and $u(t) = [Q_i, C_i]$ is a vector that contains the flow rate and the proppant concentration at the wellbore ($x = 0$) (i.e., decision variables) which are obtained by solving Eq. (11).

In the optimization problem of Eq. (11), the objective function of Eq. (11a) describes the deviation of the fracture length from the target value at the end of pumping. The empirical model of Eq. (11b) is used to predict the dynamic behavior of fracture propagation. The constraints of Eqs. (11c) and (11d) impose the limits on the injection rate and proppant concentration. The constraint of Eq. (11e) imposes that the proppant concentration will either increase or equal to the previous stage proppant concentration. The constraint of Eq. (11f) describes the total amount of proppant to be injected during the fracture treatment. The constraint of Eq. (11g) describes the required average fracture width at the end of pumping, which is normally greater than 3 times proppant diameter, D_{proppant}, for unconventional reservoirs. A constrained optimization algorithm will be utilized to calculate the generated pumping schedule that minimizes the cost function in the optimization problem subject to the input and state constraints.

Simulation results

In this section, we find the optimal fracture geometry and total number of fractures which will maximize the productivity of a fractured well for the given amount of proppant, well area and total length of all the fractures. Then, we identified a linear state-space model for the design of optimal pumping schedule by applying the proposed optimization method that will achieve the desired fracture geometry at the end of pumping.

Optimal fracture geometry

The values of the parameters used in our simulations are listed in Table 1. For the given well area, A_{well}, we considered three different AR_{well} values (1/6, 1/10 and 1/14) to calculate the optimal fracture geometry. The total length of all the fractures to be created is 9000 m in order to obtain the required average width at the end of fracture closure ($3D_{\text{proppant}}$).

parameter	symbol	value
leak-off coefficient	C_{leak}	6×10^{-5} m/s$^{1/2}$
maximum concentration	$C_{\max}$	0.65
young's modulus	E	1×10^{6} Pa
total reservoir area per well	A_{well}	324000 m^2
total proppant mass / well	M_{total}	4.5×10^{6} kg
vertical Fracture height	H	100 m
proppant particle density	ρ_{sd}	2648 kg/m^3
pure fluid density	ρ_f	1000 kg/m^3
Slick water viscosity	μ	0.01 Pa · s
Poisson ratio of formation	ν	0.2

Table 1. Model parameters used for example.

Table 2 shows the total dimensionless productivity index and the optimal number of fractures for different AR_{well} values. The maximum productivity will be attained for $AR_{\text{well}} = 1/14$, $n_{\text{f}} = 63$, and $x_{\text{f}} = 72$.

AR_{well}	n_{f}	x_{f}	J_{D}
1/6	40	112.5	412.63
1/10	53	84.9	422.24
1/14	63	71.4	429.13

Table 2. Total dimensionless productivity and optimal number of fractures for different well aspect ratios.

Model Identification

After obtaining the optimal fracture geometry, we generated the input/output data for a series of step inputs using the dynamic model. The inputs for the model are flow rate, Q_{i}, and proppant concentration, C_{i}, and outputs are average fracture width, W_{average}, and fracture propped length, x, as a function of time. A linear time-invariant state-space model for the propped fracture length and the average width has been identified by applying the MOESP algorithm and is shown in Fig. 2.

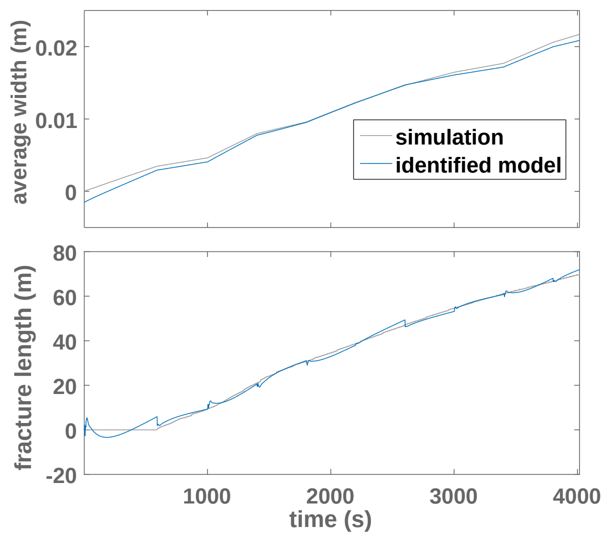

Figure 2. Comparison of results from simulation and Identified model.

Optimal pumping schedule

After developing the empirical model, a pumping schedule is generated by solving the optimization problem to achieve the optimal fracture length by considering the respective constraints on the total amount of proppant to be injected for each fracture and the minimum required average width to be satisfied at the end of pumping. A total of 10 stages for the flow rate, Q_{i}, and the proppant concentration, C_{i} are considered in the design of a pumping schedule. The obtained pumping schedule is presented in Fig. 3. It is observed that the flow rate is nearly constant throughout the pumping schedule while the proppant concentration increases with time. Then, we applied the generated pumping schedule to the hydraulic fracturing process of Eqs. (1)–(9). It is presented in Fig. 4 that the propped length at the end of pumping is close to the optimal fracture length.

Conclusions

In this work, we developed a dynamic hydraulic fracturing model to describe fracture growth, proppant transport and proppant settling. Using the developed

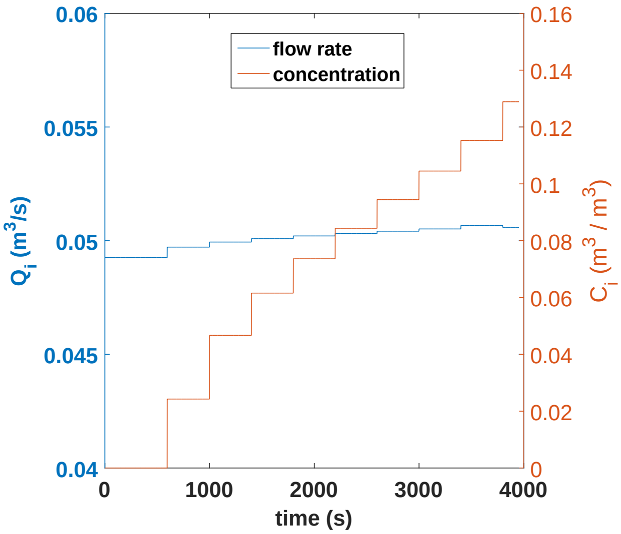

Figure 3. Pumping schedule obtained after optimization.

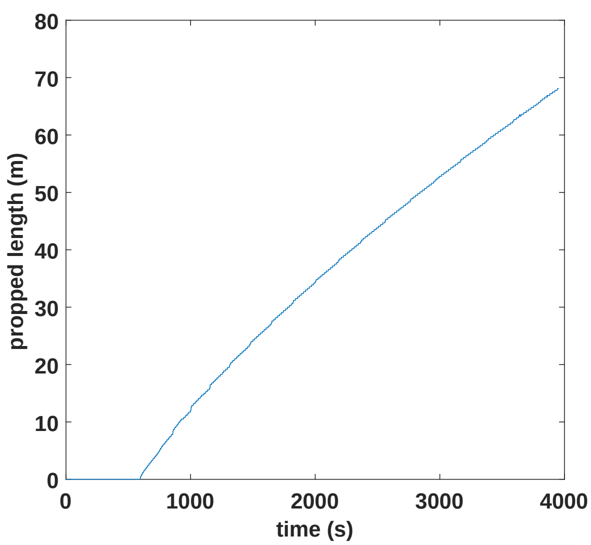

Figure 4. Propped length of fracture with time.

dynamic model a series of step inputs are used to generate the input (flow rate and proppant concentration) and output (average width and propped length of fracture) data. Then, using the generated data we identified a linear time invariant state-space model by applying the MOESP algorithm to the hydraulic fracturing process. The identified model predicted the output data with a good accuracy. After that, an offline optimization-based technique was employed to find the optimal fracture length for a given amount of proppant, well drainage area and total length of all the fractures to maximize the productivity of a fractured well. Then, a pumping schedule was generated using the identified empirical model by solving an optimization problem to achieve the optimal fracture length by considering the constraints on the total amount of proppant for each fracture and the required average width. It was demonstrated that the propped length at the end of pumping using the obtained pumping schedule was very close to the optimal fracture length maximizing the productivity of a fractures well.

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