HIERARCHICAL APPROACH FOR PRODUCTION PLANNING AND SCHEDULING UNDER UNCERTAINTY

Dan Wu and Marianthi Ierapetritou*
Rutgers University
Piscataway, NJ 08854

Abstract

In this paper, production planning and scheduling problems are addressed through a hierarchical framework. The planning problem aggregates orders in the planning period and considers uncertainty utilizing a multi-stage stochastic programming formulation where three stages are considered with increasing level of uncertainty. The planning model includes material balances and time horizon constraints involving a sequence factor to reflect the recipe complexity in the planning model. The production for the current stage is then provided to the scheduling problem, which is solved using an existing continuous-time formulation. In order to reduce the computational complexity of the overall approach, a Lagrangean decomposition method is utilized and the general framework is implemented based on rolling horizon strategy.

Keywords

Production planning, Scheduling, Multi-stage Optimization, Lagrangean decomposition

Introduction

Production planning determines the optimal allocation of resources within the production facility over a time horizon of a few weeks up to few months. However detailed scheduling decisions for the current period should be simultaneously considered in order to guarantee feasibility of the production objective, which leads to computationally intractable mathematical models. Uncertainty should also be taken into account since product demands and prices are fluctuating over the planning horizon.

A most commonly used approach to address this problem in literature is to follow a hierarchical solution methodology with two levels of decision-making, the planning level and the scheduling level in order to generate smaller and thus more tractable sub-problems (Papageorgiou and Pantelides, 1993). At the planning level an aggregated model is used to determine the optimal production requirements for each sub-period whereas the scheduling problem is solved to realize these production targets following a detailed production schedule. Feasibility is enforced through an iterative solution procedure of planning and scheduling problems. Although only the simultaneous consideration of planning and scheduling can result in the optimal production schedule, it is expected that the hierarchical approach can generate near-optimum solutions with reasonable computational time. Most of the existing approaches however are limited due to overly simplified planning level problem, the lack of uncertainty and task sequence feasibility consideration.

This paper proposes a general hierarchical framework for planning and scheduling problems utilizing a multi-stage planning model to account for uncertainty as presented in the next section. Lagrangean decomposition is utilized to reduce the size of the planning problem by decomposing it into several sub-problems of the same structure. Following the basic ideas of the existing short-term scheduling formulation proposed by Ierapetritou and Floudas (1998) a modified model is proposed to determine the production schedule for the current period. In the last section, an overall solution framework is presented and illustrated with an example problem.

Planning model

The overall decision process is based on the idea of rolling horizon strategy. The planning time horizon is decomposed into three periods with various durations considering the significance of variability and forecasting capability. The first period with the smallest duration is denoted as 'current' period where operation parameters are considered deterministic. The second period with

* To whom all correspondence should be addressed

larger duration is subject to small variability of demands and prices, and the final period with largest duration has higher level of fluctuations regarding demands and prices. Uncertainty is modeled using the ideas of multi-stage programming (Dantzig, 1955), where each planning period corresponds to different stage. Uncertainty is expressed by incorporating a number of scenarios at different stages with more scenarios towards the last period in order to represent the increasing level of uncertainty in future parameter realizations. Each scenario is associated with a weight representing the probability of such realization. Moreover, in order to reduce the model size it is assumed that at each stage each unit will process a certain number of batches at a full capacity and a single batch at flexible size.

Due to space limitations, a brief description of the problem formulation follows. The time horizon constraints are the most important ones in the planning model since they represent the requirement that all tasks should be implemented by the end of the specific time period as shown in equation (1).

$$\sum_{l}\sum_{i\in I_j} proc\_time(i,j,l,q,k) \le \mu_k \times H_k, \quad \forall j\in J, q\in Q, k\in K \tag{1}$$

where J, Q and K are the sets of units, scenarios and stages, respectively; H_k is the time duration of stage k; *proc_time(i,j,l,q,k)* is the processing time of task i in unit j for batch l, scenario q and stage k; and μ_k ($0\le\mu_k\le 1$) is a sequence factor, which represents the impact of the task sequence requirements to the planning level problem and is included to avoid the incorporation of the detailed sequence constraints at the planning problem.

Other constraints include capacity constraints that enforce the requirement of lower and upper bounds on batch sizes, duration constraints that define the processing time of batch sizes, mass balances and demand constraints in order to determine the necessary batches to satisfy the aggregated orders. These constraints are similar to those in the short-term scheduling formulation of Ierapetritou and Floudas (1998). The key variables, however, are the production levels of the first period, which are linked to the demand in the scheduling problem.

The objective function (2) maximizes the overall profit and includes the economic considerations for all three stages including product revenue, cost of materials, inventory cost, backorder cost and cost of equipment utilization which involves a fixed and a variable parts.

where *Matl_Cost(s)* is the cost of consumed material s; *revenue(s)* is the revenue from selling material s;

$$Max \sum_k weight(k)\times\Bigg(\sum_q prob(q)\times\Bigg(\sum_s revenue(s) - \sum_s Matl\_Cost(s) - \sum_s Inven\_Cost(s) - \sum_s Backorder\_Cost(s) - \sum_i\sum_j Oper\_Cost(i,j)\Bigg)\Bigg) \tag{2}$$

Inven_Cost(s) and *Backorder_Cost(s)* are the storage cost and the cost of unsatisfied orders for material s during the planning period, respectively; while *Oper_Cost(i,j)* is the operating cost calculated based on the number of batches that are performed for task i in unit j; *Prob(q)* is the probability of scenario q; and *weight(k)* is defined as a coefficient of relative importance of stage k.

It should be pointed out that the planning model involves a large number of variables and constraints due to the scenarios considered in the second and third stages, which are derived from forecasting model. Thus, in order to reduce the computational complexity of the solution of the planning problem, Lagrangean decomposition is utilized. Since the storage variables are the only set of connecting variables between the edges of the scenario tree, the problem can be decomposed to several sub-problems by duplicating these variables as shown in Figure 1. These sub-problems have the same model structure and therefore this decomposition approach can be used for large number of scenarios and stages.

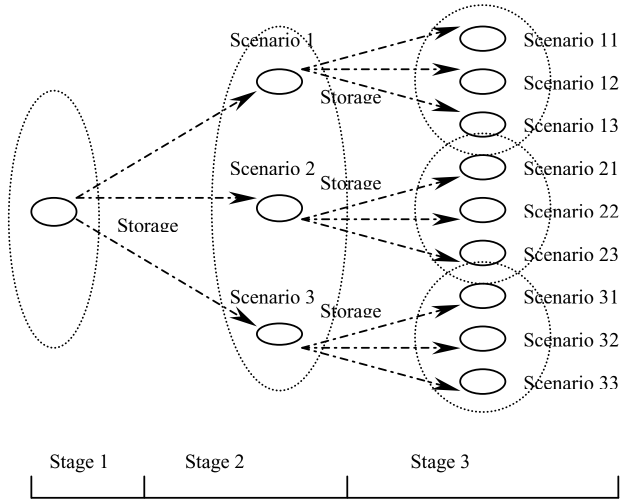

Figure 1. Lagrangean decomposition

Scheduling model

The scheduling problem provides a feasible production schedule for the current period. Since all the product orders are aggregated in the planning model, the scheduling model should determine the details of task sequence in order to satisfy all individual orders. Moreover, the production of the first period should also take into consideration the entire planning horizon that may require excessive inventory of some products in order to satisfy the demand of future periods.

Assuming that the parameters in the first period are deterministic, the scheduling problem is solved using the continuous time formulation proposed by Ierapetritou and Floudas (1998) modified to address the above two considerations. First, all the orders are required to be satisfied by their due dates as represented in constraints (3) where *S,* and *N* are the sets of materials, and event points, respectively; *production(s,n)* is the summation of all the batches that corresponds to the production of material *s* by event point *n*; while $order_{s,n}$ represents an individual order of product *s* that need to be satisfied by event point *n* (Ierapetritou et al., 1999).

$$production(s,n) \geq order_{s,n}, \quad \forall s \in S, n \in N \tag{3}$$

In order to consider the requirements imposed by the planning problem, additional production might be required at the current period due to increased demand in future time periods. The additional requirements are treated as soft constraints (4) in the scheduling model and associated with slack variables.

$$production(s,n) \geq actual\_prod_s - slack(s), \quad \forall s \in S, n \in N \tag{4}$$

where $actual\_prod_s$ is the production requirement of material *s* from the results of the planning model.

The objective of the scheduling problem is to minimize the sum of the slack variables (*slack(s)*) in priority order ($priority_s$) so as to implement the production plan obtained from the planning model as shown in (5).

$$Min \quad \sum_s priority_s \times slack(s) \tag{5}$$

The resulted scheduling problem corresponds to a MILP and can be solved using available commercial solvers (e.g. CPLEX, XPRESS-MIP). However, the introduction of a large number of discrete time requirements results in an increased complexity since a number of redundant event points should be incorporated to guarantee solution feasibility and optimality. Thus, an iterative schedule is introduced here, where the discrete time requirements are first eliminated and the scheduling problem is solved. The solution is then examined for any order violation. If there is no violation the solution is optimal, otherwise a number of cuts are introduced into the scheduling problem and another schedule is generated.

Alternatively, for cases where the processing times are fixed and there are a number of discrete orders to be satisfied, discrete-time formulation can be considered for the scheduling problem.

Since this approach considers a dynamic process, there is risk that orders cannot be realized at a particular period due to inaccurate forecasting, rushed orders or unexpected events. When this situation occurs, the scheduling problem becomes infeasible. In such a case, the backorder is allowed for the current period. The amount of backorder is calculated and the planning model is reformulated to force the backorder to be produced in the earliest future period.

Due to the dynamic nature of the decision making process, the estimation of sequence factor and forecasting of scenarios may change during the planning time horizon. Therefore it is necessary to keep inspecting and updating them in the iterative procedure so that the planning results are reasonable to the scheduling problem.

Solution framework

In this section, the overall hierarchical framework is presented and its flowchart is shown in Figure 2. The planning model is obtained by using different stages where product orders are aggregated at each stage. A scenario tree is generated regarding the uncertain parameters. The resulted planning model is solved as a MILP problem or utilizing Lagrangean decomposition by duplicating the storage variables, which gives an upper bound to the original problem as well as good candidate of integer variables. Heuristics are then used by fixing the non-zero integer variables and solving the original problem to yield a lower bound (Wu and Ierapetritou, 2003). The production of the current period is then passed to the scheduling problem where all parameters are considered deterministic and the orders are disaggregated. The production schedule is determined to satisfy each individual order as well as produce the additional amounts based on the result from the planning model.

Schedule realizes the production target of planning problem when all the slack variables are zero. When this amount hasn't been fulfilled, two situations are considered. 1) The orders in the current period are not satisfied. Backorder is then allowed and the objective of scheduling problem becomes minimizing the backorder. Meanwhile, demand in the planning model is adjusted to reflect this backorder in the next period. 2) The orders are satisfied, but the additional production is not. The sequence factor is adjusted to represent precisely the production ability and the planning model is resolved. This iterative procedure continues until convergence is achieved when the planning and scheduling results agree. At the end of the first period the model is reformulated utilizing a rolling horizon strategy so that the schedule of the following period can be determined. A motivating example is presented below to illustrate the proposed methodology.

This framework has been applied to a motivating example. The State Task Network (STN) representation and the detailed data for this example can be found in Ierapetritou and Floudas (Example 2, 1998). A planning problem with time horizon of 240 hours is considered where 8-hour schedules need to be determined dynamically. The actual demand is shown in Figure 3. The planning model includes three stages corresponding to 8, 16 and 48 hours, respectively. Three demand scenarios are considered for each branch of the scenario tree corresponding to high, average and low level of demand forecasting. Therefore, there are 3 scenarios for stage 2 and total 9 scenarios for stage 3. As discussed

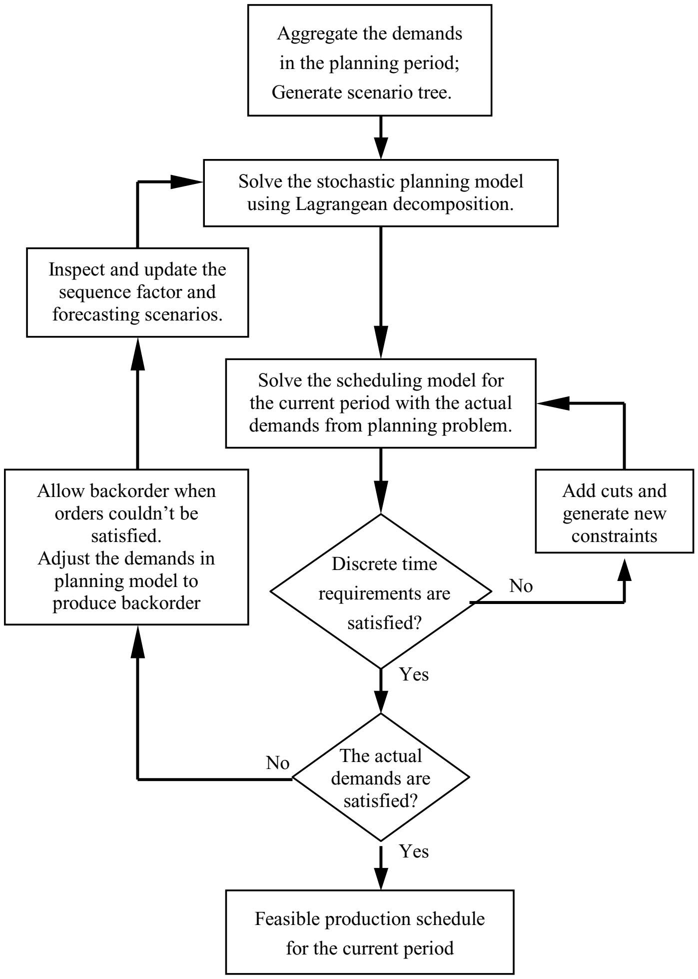

Figure 2. Flowchart of the proposed approach

above, this planning problem is decomposed into 5 sub-problems of reduced size by duplicating the storage variables between three stages as shown in Figure 1 and utilizing Lagrangean decomposition. The detailed schedule is determined when the production from schedule model matches the results of the planning model. Inventories are then updated and a new period is considered utilizing a rolling horizon approach. The process is repeated until the whole time horizon of 240 hours is covered. The results are compared to that of using short-term scheduling for the current period without considering the future time periods. Using this proposed approach, the overall profit is increased from 19886.54 to 21416.61 due to the fact that the planning model can foresee the demand peak and thus require more production when there is available capacity. Although the inventory levels increase, better control of backorders is achieved. The average backorder level is only 22.5% compared with the case where no planning periods is considered and the number of periods that has backorders is reduced from nine to seven. It should be pointed out that although this is only a motivating example, the proposed approach is general and can be applied to realistic problems with case-specific constraints.

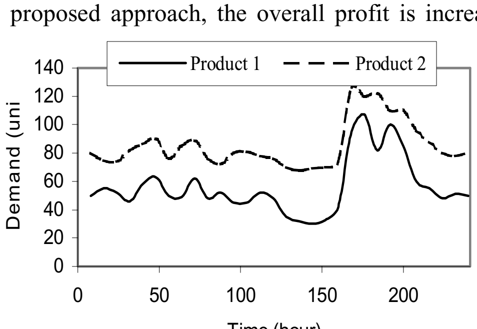

Figure 3. Demand of the motivating example

Conclusions

This paper addresses a hierarchical solution approach for solving dynamic production planning and scheduling problems. The planning model involves a multi-stage formulation while the scheduling model generates detailed schedule for the first planning period. Lagrangean decomposition is used to solve the problem efficiently. An iterative procedure between planning and scheduling level is processed to ensure the consistency of the optimality of planning and scheduling problems.

Acknowledgements

The authors would like to thank the NSF (CTS-0224745) for financial support.

Nomenclature

Indices		***Sets***	
i	*task*	*J*	*Units*
j	*unit*	*K*	*Stages*
k	*stage*	*N*	*Event points*
l	*batch*	*Q*	*Scenarios*
n	*event point*	*S*	*Materials*
q	*scenario*		
s	*material*		

Reference

Dantzig, G.B. (1955). Linear Programming Under Uncertainty. *Manage. Sci.*, 1, 197-206

Ierapetritou, M.G., and Floudas, C.A. (1998). Effective Continuous-time Formulation for Short-Term Scheduling. 1. Multipurpose Batch Processes. *Ind. Eng. Chem. Res.*, 37, 4341.

Ierapetritou, M.G., Hene, T.S. and Floudas, C.A. (1999) Effective Continuous-Time Formulation for Short-Term Scheduling. 3. Multiple Intermediate Due Dates. *Ind. Eng. Chem. Res.*, 1999; 38, 3446-3461.

Papageorgiou, L.G. and Pantelides, C.C. (1993). A hierarchical approach for campaign planning of multipurpose batch plants. *Comput. Chem. Eng.*, 17, S27-S32

Wu, D and Ierapetritou, M.G. (2003). Decomposition approaches for the efficient solution of short-term scheduling problems. Comput. Chem. Eng., 27, 1261-1276