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Multi-Objective Optimization of Dairy Supply Chain

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Abstract

In this paper, an important profit/market demands/milk vendors' satisfaction trade-off problem in dairy supply chain is presented as a multi-objective optimization problem. The Pareto Frontier is generated to present the front of the optimal compromise. The obtained Pareto Frontier contour plot is proposed to support the planning managers for quick plant profit estimation in case of priorities changing in dairy supply chain.

Keywords Supply chain, Multi objective optimization, Dairy complex

1. Introduction

Dairy industry is well positioned in countries world-wide. Importance of dairy products non-stop increases their market demand and presses the processing sector. Supply chain (SC) models, appears a useful tool for efficient analysis the environmental impact for dairy products [1] and product portfolio optimization of dairy complex [2]. However, in dairy supply chain three main actors take place following their own aims: 1) dairy complex – looking for maximum profit; 2) markets – pursuing customers' demands satisfaction; and 3) milk centers – aiming at selling most of collected milk. For the planning manager, it is very important to have a clear picture of the trade off margin existing between these three actors. The goal of this paper is to evaluate the existing compromise between the dairy supply chain actors by developing a deterministic multi-objective mathematical model and thus providing a decision making support.

2. Description of Case Study

The selected scenario consists of a dairy supply chain comprising two plants, two markets and two centers for milk collection. Three products could be manufactured in both dairies over a time horizon of 720 [h]: drinking milk-P_1 of 1% fat content; curds-P_2 of 1% fat contents; and butter-P_3 of 82,5% fat content.

Milk is collected from farmers, standardized to Frm=3,6% fat content and sold to plants. In dairies, it passes the separation step where the skim milk FP_1=1% fat content for P_1 and FP_2=0.233% for P_2, and cream with Fcr=30% fat content are obtained. The skimmed milk and the cream are subject to pasteurization. After its completion drinking milk is obtained, while acidification and draining must be carried out to produce curds. Average yield-YF of curds is 0.202 [kg curd/kg processed milk]. For butter manufacturing, cream ripening and butter churning have to take place. Buttermilk with Fbm=0.5% fat content is also produced. The required amount-QCR_i of cream for butter processing comes from milk-$Qcr_{i,1}$ and curds-$Qcr_{i,2}$ production. If any additional amount-$Qcr_{i,3}$ is needed, a whole milk, is skimmed to Fsm=0.05% fat content.

Capacities-$MSup$ of milk centers and milk costs-CRM are listed in Table 1. Markets demands-$MDem$ and selling costs-$Cost$ for products are given in Table 2. Distances between dairies and markets-$MDis$ and dairies and milk centers -$SDis$ and related transportation costs (TC; MC) are shown in Table 3.

Table 1. Capacities of milk centers and milk costs.

	Capacity [ton/month]	Milk cost [BGN / ton]
S_1	600	310
S_2	1800	280

Table 2. Markets demands and products selling costs.

	Market demands [ton /month]			Selling costs [BGN/ton]		
	P_1	P_2	P_3	P_1	P_2	P_3
M_1	1400	55	53	790	1430	4530
M_2	400	20	28	900	2800	6320

Table 3. Distances between dairies and markets and dairies and milk centers and respective transportation costs.

	Distance [km]				Transportation cost [BGN/ton.km]			
	M_1	M_2	S_1	S_2	M_1	M_2	S_1	S_2
Dairy 1	226	92	41	36	1	2	1	1
Dairy 2	238	89	31	61	1	2	1	1

3. Mathematical Formulation

3.1. Variables

Control variables - $(X_i)_{p,m}$ are introduced to track for the amount of product- p processed in plant- i and sold on market- m . They range in the boundaries:

$$0 \le (X_i)_{p,m} \le MDem_{m,p}\ ,\ \forall i,\ \forall p\ ,\forall m\ . \quad (1)$$

Variables- $(Y_i)_s$ account for the amount of milk bought by plant- i from the milk center- s , and vary in:

$$0 \le (Y_i)_s \le MSup_s\ ,\ \ \forall i,\quad \forall s\ . \quad (2)$$

Design variables- $QP_{i,p}$ determine the amount of each product- p that must be processed in each plant- i within the horizon *H*.

3.2. Supply chain model

Mass balance equations of the subsystems dairies – markets and dairies – milk centers. We assume that the supply chain is a constant over the horizon-*H* and accept that no stocks and milk accumulations are permitted in the plants.

$$QP_{i,p} = \sum_{m=1}^{M} (X_i)_{p,m}\ ,\quad \forall p\ ,\ \forall i \quad \text{and} \quad QRM_i = \sum_{s=1}^{S} (Y_i)_s\ ,\qquad \forall i\ . \quad (3)$$

Dairies mass balance equations aim to meet the amount of raw milk needed for products manufacturing:

$$QRM_i = QRM_{i,1} + QRM_{i,2} + QRM_{i,3} \qquad \forall i\ , \quad (4)$$

The milk required for P_1 manufacturing and the amount of cream processed is:

$$QRM_{i,1} = QP_{i,1} \frac{Fcr - FP_1}{Fcr - Frm} \quad \text{and} \quad QCR_{i,1} = QP_{i,1} \left(\frac{Fcr - FP_1}{Fcr - Frm} - 1 \right) \quad \forall i\ . \quad (5)$$

The whole milk for curds processing and the obtained cream are:

$$QRM_{i,2} = \frac{QP_{i,2}}{YF} \cdot \frac{Fcr - FP_2}{Fcr - Frm} \ \text{and}\ QCR_{i,2} = \frac{QP_{i,2}}{YF} \cdot \left(\frac{Fcr - FP_2}{Fcr - Frm} - 1 \right) \quad \forall i\ . \quad (6)$$

The amount of cream needed for butter manufacturing is:

$$QCR_i = QP_{i,3} \frac{FP_3 - Fbm}{Fcr - Fbm},\ \ \text{where}\ \ QCR_i = QCR_{i,1} + QCR_{i,2} + QCR_{i,3}\ . \quad (7)$$

Apart from P_1 and P_2 processing, cream could be also produced from whole milk skimming, if any additional amount is needed:

$$Q_{CR_{i,3}} = \begin{cases} QCR_i - \left(Q_{CR_{i,1}} + Q_{CR_{i,2}}\right), & \text{if } QCR_i > Q_{CR_{i,1}} + Q_{CR_{i,2}}; \\ 0 \text{ otherwise} \end{cases}, \ \forall i \qquad (8)$$

Following (8) the amount of raw milk for cream manufacturing is:

$$Q_{RM_{i,3}} = Q_{CR_{i,3}} \cdot \frac{Fcr - Fsm}{Frm - Fsm}, \quad \forall i\ . \qquad (9)$$

Accounting that the separation step is shared by P_1 and P_2 and is used for P_3 only if some additional amount of milk must be skimmed, a decomposition of butter manufacturing is applied. We accept that butter production starts with cream pasteurization and introduce a new product-P_4 passing only the separation step which quantity is determined according to (8).

3.3. Constraints

Product portfolio feasibility constraints aim to establish the feasible working frame for each dairy using a new time based approach proposed in [2]. It employs the time resource distribution over the processing nodes and products. Processing nodes joint the units belonging to a given type-*n* and have summarized volumes-U_n (Table 4). Additionally, we assume that manufacturing of each product passes through all nodes. If some product does not use units of a given type fictitious processing tasks are introduced to connect them. Size factors -$SF_{p,n}$ [m^3/ton] and processing times-$T_{p,n}$ [h], are given for the tasks of all products. For the fictitious ones they are set equal to 0.

Table 4. Plants' data.

	Separators [m^3/hour]	Pasteu-rizers [m^3]	Curds Vats [m^3]	Drain-ers [m^3]	Ripening vessels [m^3]	Chur-ners [m^3]
Dairy 1	0,6	0,8	0,95	0,3	0,4	0,6
Dairy 2	2	0,95	1,05	0,34	0,5	1

Taking into account that each processing node is shared by all products over the horizon-*H* and all processing nodes are involved in each product, the following sets of constraints must be satisfied:

$$\sum_{p=1}^{P} SF_{p,n} QP_{i,p} \frac{T_{p,n}}{U_{i,n}} \le H, \quad \forall n,\ \forall i \ \text{ and } \sum_{n=1}^{N} SF_{p,n} QP_{i,p} \frac{T_{p,n}}{U_{i,n}} \le H, \quad \forall p,\ \forall i \quad (10)$$

Market constraints and Milk distribution centers constraints:

$$\sum_{i=1}^{I} \left(X_i\right)_{p,m} \le MDem_{m,p}, \ \forall m,\ \forall p \quad \text{and} \quad \sum_{i=1}^{I} \left(Y_i\right)_{p,s} \le MSup_{s,p}, \quad \forall s\ . \qquad (11)$$

3.4. Multi-objective function

Profit of Dairy complex. It is subject to maximization and is determined as the difference between the incomes from sold products and production, transportation and milk costs:

$$F1=\sum_{i=1}^{I}\left[\begin{array}{l}\sum_{p=1}^{P}\sum_{m=1}^{M}(X_i)_{p,m}.Cost_{m,p}-\sum_{p=1}^{P}PRC_{i,p}\sum_{m=1}^{M}(X_i)_{p,m}-\\ \left(\sum_{s=1}^{S}(Y_i)_s\,CMR_s+\sum_{s=1}^{S}MC_{i,s}.SDis_{i,s}.(Y_i)_s+\sum_{m=1}^{M}TC_{i,m}.MDis_{i,m}.\sum_{p=1}^{P}(X_i)_{p,m}\right)\end{array}\right] \quad (12)$$

Due to lack of data the production costs- PRC will not be taken into account.

Index of customers' demand satisfaction is evaluated by the ratio between products request and offer on markets and is subject to maximization:

$$F2=\sum_{m=1}^{M}\sum_{p=1}^{P}\frac{\sum_{i=1}^{I}(X_i)_{p,m}}{MDem_{m,p}}. \quad (13)$$

Index of milk venders' satisfaction is subject to maximization too:

$$F3=\sum_{s=1}^{S}\sum_{p=1}^{P}\frac{\sum_{i=1}^{I}(Y_i)_{p,s}}{MSup_{s,p}}. \quad (14)$$

Using (13)-(15) the following multi-objective criterion is proposed $\Phi = MAX(F1,F2,F3)$.The corresponding weighted multi-objective function is:

$$MAX(\alpha 1.F1+\alpha 2.F2+\alpha 3.F3), \text{ where } \alpha 1+\alpha 2+\alpha 3=1. \quad (15)$$

4. Results and Discussions

Three separate optimization problems were solved to determine the maximal values of functions F1, F2 and F3. Their values and corresponded product portfolios are listed in Table 5.

Table 5. Product portfolios.

	MaxF1=3.231.10^5 BGN; F2=3.79 F3=0.689		MaxF2=4.542; F1=2.727.10^{3}BGN F3=1.452		MaxF3=1.617; F1=1.435.10^{4}BGN F2=4.453	
	Dairy 1	Dairy 2	Dairy 1	Dairy 2	Dairy 1	Dairy 2
P_1	287.403	667.867	281.936	669.42	288.315	669.42
P_2	15.632	14.319	20.911	28.168	20.654	28.168
P_3	13.735	26.924	14.977	45.902	14.905	45.902

The results obtained are used to scale the problem. Applying the methodology proposed in [3], the Pareto-Frontier is built based on the 74 non-dominated solutions found. Its contour plot is shown on Fig. 1.

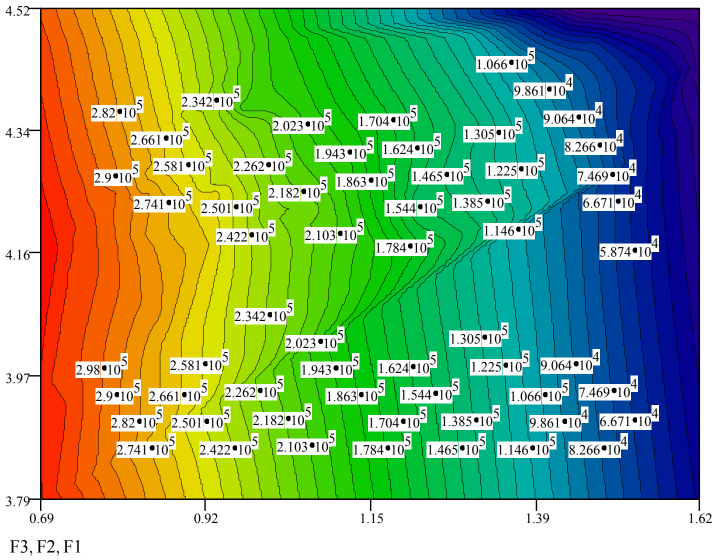

Fig. 2 Pareto-Frontier contour plot.

Analyzing the results obtained we observed that the profit is decreased rising both satisfaction indices. This is due to increased curds and butter production, which affects on plants expenditures. The obtained Pareto Frontier contour plot is proposed to support planning managers for quick plant profit estimation in case of changing priorities in dairy supply chain.

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