

## **Robust implementation of optimal strategies accounting for controller performance and uncertainty**

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### **Abstract**

In this work, two different approaches to avoiding infeasibility are discussed. They basically imply a direct or an indirect back-off from the nominal optimal solution. First, in order to push the nominal optimal set-points of the regulatory control layer inside the feasible region, a direct adjustment is used which is based on real-time feasibility correction. Second, feasible operation can be obtained by explicit inclusion of closed-loop deviations and model uncertainty in the optimization problem formulation. This leads to an indirect adjustment of the optimal decisions solving a chance constrained optimization problem. The efficiency and robustness of the novel approach will be demonstrated for two different scenarios on a high-pressure column embedded in a coupled two-pressure column system.

### **Keywords**

Integrated Process Control and Optimization, Chance Constraints, EKF, NMPC

### **1. Introduction**

The generation and implementation of optimal control strategies can be achieved through model-based advanced process control and optimization schemes. In an optimization upper layer decisions about the optimal process state with respect to various objectives are made. The results are then sent in

form of set-points to the regulatory control layer where the strategies are implemented to keep the system state at the optimal operating point. In practical applications, however, these optimal decisions lay often at process boundaries, e.g. product specifications, safety restrictions or physical limitations. Overstepping the constraints makes the operation infeasible, which not only means a loss of quality but also a safety risk. Main bottleneck for the implementation of nominal optimal decisions is the presence of uncertainty in the form of model mismatch and disturbances [1]. Accordingly the challenge of plant operation optimization lies in implementing optimal decisions, while guaranteeing feasible operation in the presence of uncertainty. In this work, the uncertainties,  $\Xi$ , are model parameters, which result from the lack of accurate models for industrial processes, dynamic random variables such as varying operating conditions (e.g. feed concentration) and finally the implementation error as a result of disturbances, which cause deviations around the set-points in the regulatory control layer.

## 2. Problem Statement

A high-pressure column embedded in a coupled two-pressure column system for the separation of an azeotropic mixture is considered. The operating point is defined by the distillate and bottom product specifications, as well as the maximum pressure of the considered high-pressure column. Figure 1 shows the individual high-pressure column and the control loops corresponding to the regulatory control layer.

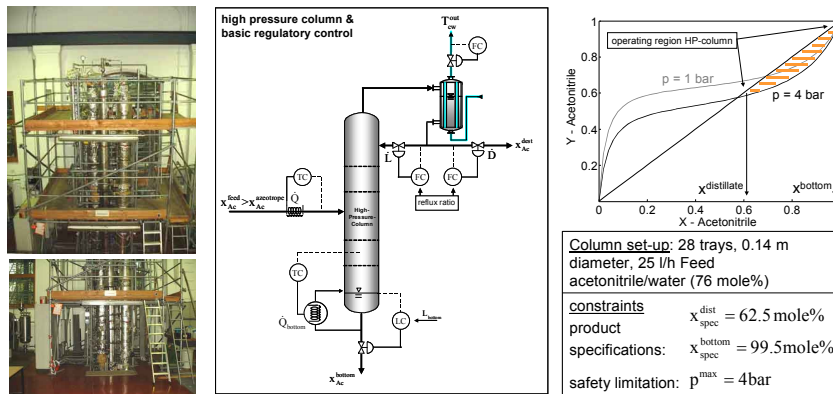


Figure 1: left: column system; middle: regulatory control layer of the high pressure column; top right: VLE diagram acetonitrile/water; down right: parameters and operational conditions.

Operating the illustrated high-pressure column above the azeotropic point results in pure acetonitrile as bottom product. The complete separation task is carried out by means of pressure swing distillation. The system pressure is

controlled taking influence on the energy transfer in the condenser unit. Here, the total condenser is flooded such that variations in the condenser level change the effective surface area, whereas the heat transfer between mixture and cooling medium is influenced by variations in the cooling water flow rate. Thus, the decision variables for the steady state operating point are: reboiler duty, reflux ratio, cooling water flow-rate, and the condenser level. However, nominal optimal conditions for the energy minimal operation are limited by the product specifications and the maximal system pressure [7]. This means that the energy-optimal operation is defined by means of covering the minimal requirements for product quality and driving the plant at its upper safety limit. Furthermore, for the optimal operation problem the condenser outflow temperature is also maximized.

### **3. Robust implementation of nominal optimal decisions**

As stated before, operation with a minimal amount of energy is determined by the boundary conditions of the quality specifications, the maximal allowable pressure and the appropriate variable setting, which minimizes the sub-cooling in the condenser unit. In this work, the implementation of the nominal optimal decisions is realized dividing the optimal operation problem in two decentralized sub-problems. In the 1<sup>st</sup> example, the pressure control problem and the maximization of the condenser outflow temperature are discussed. In the 2<sup>nd</sup> example, the reboiler duty and reflux ratio are manipulated in order to operate the product concentrations as close as possible at the product specifications.

#### *3.1. Pressure control and maximization of the condenser outflow temperature*

Here, a measurement-based optimization technique is applied. The constrained state variables, (here the maximal pressure and the level restrictions in the condenser) can be adjusted directly in the optimization layer [1,2,3]. A reduced order model is used for the online application which describes the liquid and vapor temperature as a function of the heat transfer during condensation and sub-cooling, as well as the dynamic behavior of the level and cooling water temperature. Although the equation system represents the nonlinear characteristics of the system, it does not predict the system behavior over the whole operating range. Thus, the vapor flow rate  $F^v$ , the vaporization heat  $q^r$  and the heat transfer coefficients  $\alpha^{[L,V]}$  are considered as unknown disturbances and parameters,  $\Theta' \subseteq \Theta$ , to be estimated by an EKF. Estimation is carried out every 24 seconds. The adapted model is then used in an NMPC-approach in order to force optimal and feasible operation (Fig. 2). Since the flow control loops are part of the superior optimization scheme, the model used for the NMPC-algorithm contains additional information about their dynamic behavior. The online optimization problem is solved using a discrete model with six

control variables over a moving horizon of 10 min. The objective is the minimization of the deviations from the pressure set-point and the minimization of the sub-cooling.

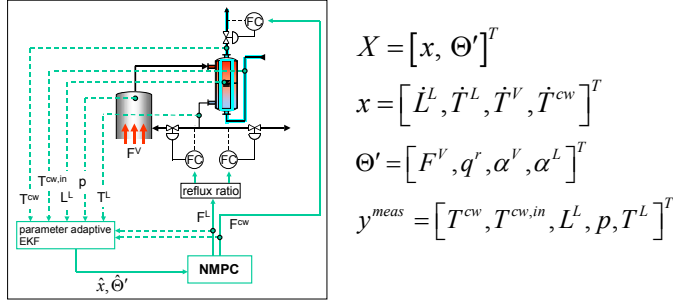


Figure 2: Control scheme, augmented state and measurement vector.

Variations in the flow rates are penalized in order to minimize set point changes for the flow control loops, which would cause unnecessary controller actions in the basic control layer. Beside the constraints on the optimization variables  $u_k$ , restrictions regarding the level in the flooded condenser  $L^L$  are also considered directly in the optimization problem. As the pressure is controlled directly, one can easily adjust its set-point in the size of the implementation error, so not to exceed critical safety limits [2]. Figure 3 shows experimental results for estimated and controlled constrained variables changing the pressure set-point.

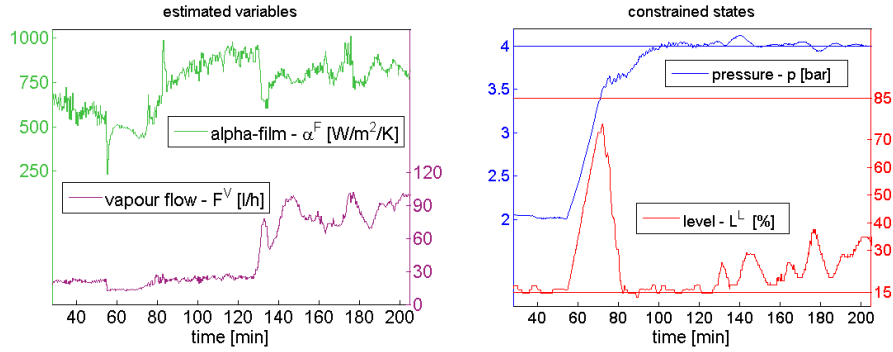


Figure 3: estimated variables and constrained state variables during operation.

### 3.2. Robust set-points for indirect concentration control

In this section, the focus lies on the compliance with the bottom and distillate product specification. The non-measurable bottom product concentration is indirectly controlled using a temperature control on a sensitive tray, while the

distillate concentration is controlled manipulating the reflux ratio. The challenge is to define robust set-points for these decision variables, which guarantee the compliance with the product specifications during steady state operation. This can be achieved by explicit consideration of uncertainties in the model equations [4,5,6]. In this work, controller quality metrics (here the closed loop variance in the regulatory control layer), as well as parametric model uncertainties are considered. Since the constrained state variables are affected by the considered uncertainties, it is obvious that it is not possible to hold its limitations for sure. Hence, they are reformulated as chance constraints. Consequently, a probability level is defined to represent the reliability of being feasible. This leads to the formulation of single chance constraints:  $\Pr\{y(u, \xi) \leq y^{\text{spec}}\} \geq \alpha$ , where  $\Pr$  represents the probability measure and  $\alpha$  the probability level defined by the process operation requirements [7,8]. In this case study, uncertainties in the feed conditions and the controller performance metrics of the temperature and pressure control loops are considered as stochastic parameters in the steady-state model. Finally, a nonlinear chance-constrained optimization problem is formulated (P1), where the objective function is reformulated as additional chance constraint [8]. Controller deviations and model uncertainties are considered as Gaussian stochastic parameters [5,6]. Here, the constrained product concentrations have to be fulfilled with a probability level of 95 %.

$$\begin{aligned} & \min \beta && \text{(P1)} \\ \text{s.t.} & \text{ model equations,} \\ & \text{indirect adjusted constrained output variables:} \\ & \Pr\{x_{Ac}^B \geq x_{Ac, \text{spec}}^B\} \geq \alpha_1 \quad \Pr\{x_{Ac}^D \leq x_{Ac, \text{spec}}^D\} \geq \alpha_2 \\ & \text{originally replaced objective function as chance constraint:} \\ & \Pr\{\dot{Q}_B \leq \beta\} \geq \alpha_3, \\ & \text{Uncertainties and probability levels:} \\ & \bar{\xi} = [\sigma(P_{\text{top}}); \sigma(T_{\text{strip}}^{\text{sens}}); \sigma(x_{\text{Feed}})]; \sigma(P_{\text{top}}) = 0.04 \text{ bar}; \sigma(T_{\text{strip}}^{\text{sens}}) = 0.65 \text{ K}; \sigma(x_{\text{Feed}}) = 4 \text{ mole\%} \\ & \alpha = [95\%; 95\%; 90\%] \end{aligned}$$

As a result, feasible operation is obtained forcing an indirect back-off from the nominal optimal solution. The results are robust set-points, which can be implemented by the regulatory control layer. In order to operate as close as possible at the optimum, the indirect back-off is minimized while still satisfying all constraints. In the presence of changing plant conditions, a cyclic adjustment of the set-points can guarantee feasible and optimal operation. By application of the Monte Carlo sampling method the distribution of all variables for the considered uncertainties are calculated. Deviations around the robust operation point in the temperature profile over the column and the product concentrations are shown in Figure 4. Temperature control is realized on the 5th tray of the stripping section. In comparison with a conventional operating point with

product concentrations beyond the product specifications, the robust operation strategy is close to the nominal optimum satisfying safety criteria and guaranteeing the specifications with a probability of 95%.

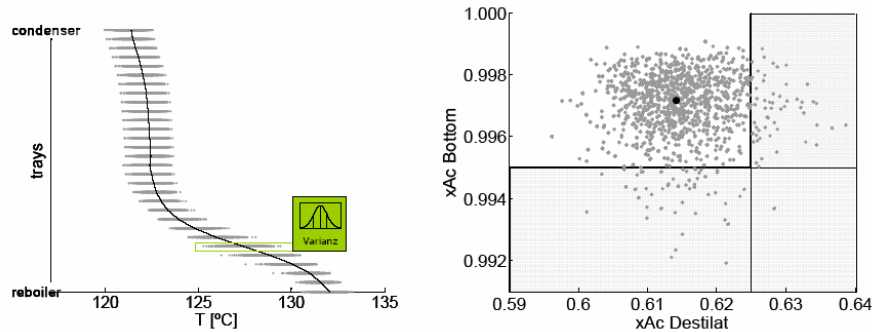


Figure 4: left: robust temperature profile and deviations, controlled temperature on the 5th tray; right: distribution of the product concentrations around the robust optimization result.

#### 4. Concluding remarks

Two different methods and its application for the implementation of optimal decisions in the presence of uncertainties are presented. Optimal and feasible operation is realized forcing a direct or indirect back-off from the nominal optimal operating point. In the 1<sup>st</sup> example, to comply with physical and safety related constraints, a measurement-based approach is used, where the constraint states are adjusted directly. In the 2<sup>nd</sup> example, the product specifications are satisfied with a desired confidence level (probability) during steady state operation. This is achieved by a cyclic adjustment of the optimal set-points solving a reduced stochastic optimization problem. The solutions are to some extent conservative but represent the best strategy available with a minimal back-off from optimal operation. The robustness of the approaches is demonstrated for different experimental scenarios on the high-pressure distillation column.

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