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A precedence-based monolithic approach to lot-sizing and scheduling of multiproduct batch plants

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Abstract

Batch scheduling is a highly combinatorial problem involving two major components: the lot-sizing or batching problem (P1) defining the set of batches to be scheduled, and the “pure” short-term batch scheduling problem (P2) assigning resources to batches and sequencing batches at every resource item. Due to the large computational requirements to cope with the whole problem at once, precedence-based optimization strategies have traditionally solved sub-problems P1-P2 in a sequential manner. In contrast, this work presents an effective precedence-based approach that integrates both subproblems into a unique MILP formulation and solves the problem in a single step. A pair of examples involving the scheduling of multistage, multiproduct batch facilities carrying out linear processes have been solved. Comparison of the results found with the ones reported by other authors leads to conclude that the proposed approach shows a much better computational perfomance.

Keywords: Scheduling, MILP model, batch operation, lot-sizing.

1. Introduction

Numerous mathematical formulations and solution approaches for the short-term scheduling of chemical batch facilities have been published in the last decade. A comprehensive state-of-the-art review can be found in Méndez et al. [1]. The batch scheduling problem generally involves four major issues: the lot-sizing, the batch-resource allocation, the batch sequencing at every resource unit and the batch timing. Usually, the batching problem is concerned with the lot-sizing issue while the other three operational decisions are found by tackling the classical batch scheduling problem. With few exceptions (Lim and Karimi [2]), batch-oriented scheduling approaches generally assume that the lot-sizing problem defining the set of batches (number, sizes and due dates) based on the customer requirements has already been solved. Consequently, such techniques just determine when and where the pre-defined batches are to be produced, i.e. the "pure" batch scheduling. Although this typical sequential procedure has been widely used in practice and academia (Méndez et al. [3]; Neumann et al. [4]), the quality of the production schedule is indeed highly dependent on the lot-sizing decisions already taken. Unless both subproblems are simultaneously tackled, there is no guarantee on either the optimality of the proposed schedule or, even the feasibility of the "pure" batch scheduling problem. In contrast, some other general scheduling methodologies based on the state (STN) or resource (RTN) task network concept (Castro et al. [5]; Maravelias and Grossmann [6]) have integrated both sub-problems into a single optimization framework. Though they can handle non-linear product recipes involving batch mixing and splitting, a common drawback of such network-oriented approaches comes from the wide range of operational decisions to simultaneously consider and the large size of the related problem modeling. As a result, they seem more appropriate to find the optimal schedule of batch facilities over a rather short time horizon.

In order to overcome one of the major shortcomings of precedence-based scheduling methods, this work introduces a new MILP-based integrated approach to also handle lot-sizing decisions while seeking the optimal production schedule of multiproduct batch plants. Multiple customer orders over a weekly time horizon can be managed. The proposed MILP formulation is also capable to account for variable batch sizes and processing times, multiple processing units running in parallel and sequence-dependent changeover times without compromising the optimality of the solution. The best schedules generated through the proposed methodology for some benchmark examples were compared with the ones reported in the literature by other authors in order to highlight not only its higher computational efficiency but also the better results that were discovered.

2. Coupling batching and scheduling decisions

Taking advantage of the batch process knowledge, the proposed integrated approach is implemented in two steps. First, a systematic procedure is applied just to get a good, conservative estimation of the number of batches for each product to be processed and the latest date at which each one should be ready. A batch $b \in B_p$ of product p can be assigned to satisfy several demands with different due dates $d \in D_p$ but it must be completed before the earliest one. Next, a continuous-time MILP model aimed at finding and scheduling the optimal set of product batches over the time horizon is to be solved.

2.1. First step: Converting product requirements into a tentative set of product batches

The so-called lot-sizing or batching problem converts product requirements given in tons or kgs into an equivalent set of product batches to be produced in the plant. For each product, the preliminary procedure defines a set of batches sufficiently large to meet its requirement at every specified due date $d \in D_p$. The number of batches to be processed depends on the unit capacities as well as the operating constraints. In case equivalent batch units with a similar fixed capacity are available at every processing stage s, the batch size can be known beforehand and, consequently, the number of batches needed to meet the total requirement of product p can be easily computed. Otherwise, it should be applied Equation (1) to have a good, conservative estimation of the total number of batches nb_p based on the total pth-product requirement, i.e. $\Sigma\, r_{pd}$.

$$nb_p^{\#} = \max_{s \in S_p} \left\{ \sum_{d \in D_p} \frac{r_{pd}}{\min\limits_{j \in J_{ps}} \left[B_j^{\max} \right]} \right\} \qquad \forall p \qquad\qquad nb_p = \left\lceil nb_p^{\#} \right\rceil \tag{1}$$

Once the number of batches nb_p has been estimated, the subset of batches nb_{pd} allocated to every specified due date d for product p should be determined so as to meet condition (2) for any $d \in D_p$.

$$\sum_{d' \le d} nb_{pd'}\, B_p^* \ge \sum_{d' \le d} r_{pd} \qquad \forall d \in D_p \qquad\qquad where: \sum_{d \in D_p} nb_{pd} = nb_p \;\;,\;\; B_p^* = \frac{\sum_{d \in D_p} r_{pd}}{nb_p^{\#}} \tag{2}$$

The set of batches B_p with $|B_p| = nb_p$ is then incorporated in the scheduling model as a known datum. Assuming that $D_p = \{d_1, d_2, d_3\}$, then $nb_{p,d1}$ batches in B_p will have a due date equal to d_1, $nb_{p,d2}$ batches will have to be completed before d_2 and $nb_{p,d3}$ will feature a due date d_3. Note that the condition $nb_{p,d1} + nb_{p,d2} + nb_{p,d3} = nb_p$ must be complied. It is worth noting that the number of batches included in the final schedule will depend on the sizing decisions taken in the second and final step. Consequently, the main purpose of Eqns. (1)-(2) is to just postulate a sufficient number of batches for each product requirement at every due date in the integrated scheduling model. We can adopt a number of batches nb_p lower than that suggested by Eqn (1) by selecting an average batch size over the whole set of eligible units for product p, i.e. J_{ps}, $s \in S$. In this case, however, the optimality of the solution may no longer be guaranteed and the MILP-formulation should be repetitively solved with $nb_p^{new} = nb_p^{old} + 1$ until the optimal value of the objetive function remains unchanged.

2.2. Second step: Solving the MILP-based integrated scheduling formulation

Results from the first step for each product p just allow to define the cardinality of the set B_p and the due date for each element of B_p, i.e. a bound on the number of batches of product p to be completed at due date $d \in D_p$. The proposed MILP formulation can then be used to determine: (a) the optimal number and sizing of batches to be produced; (b) the optimal allocation of resources to batches over time and (c) the selected batch sequence at every equipment unit. Therefore, predefined batches are treated individually in the scheduling problem, i.e. allocation, sequencing, timing and sizing decisions are made for each individual batch. Equation (3) enforces the condition that an individual batch b can at most be allocated to a single unit j. No unit allocation is required if batch b is finally ignored. A batch $b \in B_p$ is excluded from the final schedule if the related allocation variables Y_{bj} are all equal to zero. Equation (4) defines the size of batch $b \in B_p$ which depends on the minimum/maximum permissible size in the assigned unit. Based on the sizing variables Q_b , Equation (5) forces that the accumulated demand for each product p at each due date d, i.e. $\Sigma r_{pd'}$, must be satisfied by the batches whose due dates are earlier than or equal to due date d. Equation (6) determines the batch processing time by taking into account fixed and batch size-dependent components. In turn, Equations (7) and (8) apply the general precedence concept to sequence each pair of batches allocated to the same equipment unit in each stage s. Since $X_{bb'} = 1$ if $b,b' \in B_p$ and $d_b < d_{b'}$ then such a sequencing variable is deleted from the problem formulation. Equation (9) forces to start stage s for batch b after completing all prior ones. Finally, Equations (10), (11) and (12) compute the makespan of the schedule as well as the tardiness and earliness associated to each batch task. These measurements can alternatively be selected as the problem goal to be minimized in the proposed optimization framework.

$$\sum_{j \in J_{ps}} Y_{bj} \le 1 \quad , \quad \forall b \in B_p \, , s \in S_p \, , p \in P \tag{3}$$

$$\sum_{j \in J_{ps}} q_{bj}^{\min} Y_{bj} \le Q_b \le \sum_{j \in J_{ps}} q_{bj}^{\max} Y_{bj} \quad , \forall b \in B_{pd} \, , s \in S_p \, , p \in P \tag{4}$$

$$\sum_{d' \le d} \sum_{b \in B_{pd}} Q_b \ge \sum_{d' \le d} r_{pd'} \, , \forall d \in D_p \, , p \in P \tag{5}$$

$$CT_{bs} = ST_{bs} + \sum_{j \in J_{ps}} (ft_{psj} Y_{bj} + vt_{psj} Q_b) , \; \forall b \in B_p \, , s \in S_p \, , p \in P \tag{6}$$

$$CT_{bs} \le ST_{b's} - ct_{bb's} + H(1 - X_{bb's}) + H(2 - Y_{bj} - Y_{b'j})$$
$$\forall b \in B_p \, , b' \in B_{p'} \, , j \in J_{bb'} \, , s \in S_{pp'} \, , p, p' \in P \tag{7}$$

$$CT_{b's} \leq ST_{bs} - ct_{b'bs} + H\,X_{bb's} + H\,(2 - Y_{bj} - Y_{b'j}) \qquad \forall b \in B_p, b' \in B_{p'}, j \in J_{bb'}, s \in S_{pp'}, p, p' \in P \tag{8}$$

$$CT_{b,s-1} \leq ST_{bs} \qquad \forall b \in B_p, s \in S_p, p \in P \tag{9}$$

$$MK \geq CT_{bs} \quad , \quad \forall b \in B_p\,, s = s^{last}, p \in P \tag{10}$$

$$T_b \geq CT_{bs} - d_b\,, \quad \forall b \in B_p\,, s = s^{last}, p \in P \tag{11}$$

$$E_b \geq d_b - CT_{bs}\,, \quad \forall b \in B_p\,, s = s^{last}, p \in P \tag{12}$$

3. Results and discussion

In this section we illustrate the applicability and efficiency of our proposed approach by solving two challenging problems reported in the literature. For both examples, the total tardiness and the makespan were alternatively chosen as the objective function to be minimized. To carry out a fair comparison, we implemented and solved both the Lim and Karimi's model [2] and this approach in the same computer and with the same optimizer code. Unfortunately, a more extensive comparison with regards to network-oriented models presented in [5] and [6] was not possible because these general batch scheduling formulations are unable to deal with multiple due dates along the time horizon.

3.1. Example 1.

This example was first solved by Lim and Karimi [2]. It comprises three batch units, four products and fourteen product demands at four different due dates. Since this problem also involves variable batch sizes and variable processing times, an integrated approach is highly recommended. Table 1 summarizes the computational results obtained with both our formulation and the Lim and Karimi's [2] model. Results show a remarkable reduction in the computational time by a factor larger than 100 for tardiness and more than ten times when the makespan is minimized.

3.2. Example 2.

The Example 2 was initially introduced by Méndez et al. [3] and subsequently addressed by Lim and Karimi [2]. The problem involves four batch units, eight products and twenty-nine production demands at six due dates. Full problem details can be found in Méndez et al. [3]. Similar results and computational requirements were found for tardiness as the objective function. If the makespan is the problem goal to be minimized, either model was unable to find the optimal schedule in 3600 s. However, the best solution found through the proposed approach features a relative gap much lower than the one discovered by the Lim and Karimi's [2] model, i.e. 23% against 89.3%.

Table 1. Comparison of the proposed approach (1) and the Lim and Karimi's model [2]

Example	Binary vars., cont.vars, rows	OF	CPU Time[a]	Iterations	Nodes	Rel. Gap%
1-Tardiness (1)	36, 206, 436	30.51	12.95	131917	14757	0
1-Makespan(1)	36, 214, 480	106.60	4.83	67306	5962	0
1-Tardiness [2]	77, 94, 506	30.51	1844	26801597	958115	0
1-Makespan[2]	77, 94, 508	106.60	52.25	603871	15575	0
2-Tardiness (1)	141, 467, 3005	0	0.3	483	4	0
2-Makespan[1]	216, 265, 1944	41.6	3600*	12948637	3329345	23
2-Tardiness [2]	216, 265, 1938	0	0.35	455	20	0
2-Makespan(2)	141, 467, 3060	44.7	3600*	12701246	182114	89.3

*Time limit - [a] Seconds on a Pentium IV (2.8 GHz) with GAMS/CPLEX 9.0

4. Conclusions

An effective MILP precedence-based integrated optimization approach for lot-sizing and short-term scheduling of multiproduct batch plants satisfying multiple due dates along the time horizon has been developed. Besides being the first precedence-based integrated approach, numerical results found for two moderate-size benchmark problems show that is computationally efficient.

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