17th European Symposium on Computer Aided Process Engineering – ESCAPE17
V. Plesu and P.S. Agachi (Editors)



# Logistics Optimization Using Hybrid Meta-heuristic Approach under Very Realistic Conditions

Yoshiaki Shimizu, Takeshi Wada, and Yoshihiro Yamazaki

*Dept. of Production Systems Engineering, Toyohashi University of Technology, shimizu@pse.tut.ac.jp*

## Abstract

In this study, we have concerned with a hierarchical logistic network design problem from two aspects that have been scarcely considered previously though they are very common in real-world. That is, to cope with a multi-commodity problem and a problem with stair-wised discount cost of transportation effectively as well as practically, we have developed the extended hybrid tabu search method for each problem. Validity of the methods is verified through comparison with the commercial software for the multi-commodity problem while the algorithm is implemented as software amenable for supporting a daily logistic planning for the cost discount problem.

**Keywords:** Logistics, Large-scale Combinatorial Optimization, Multi-commodity Product, Stair-wised Discount Cost, Hybrid Tabu Search

## 1. Introduction

Logistic optimization has been weighed increasingly as a key issue to improve the efficiency of business process under global competition [1]. In the field of OR, many studies since the earlier facility location problems [2] have been taken place for more than three decades. However, most of these studies concern with the problems formulated simply and major emphasizes have been paid to developing a new algorithm, and to evaluating its validity through benchmarking. To cope with the complex and complicated real-world situations, we still need more earnest efforts. This is because it causes such a dramatic increase in problem size that makes it impossible to solve the resulting problem by any currently available software.

With this point of view, this study concerns with a multi-commodity product problem and a problem with stair-wised discount regarding transportation cost and presents the extended hybrid meta-heuristic methods developed for each problem.

## 2. Problem Statement

Taking a hierarchical logistic network composed of members such like plants, distribution centers (DC), and customers as shown in Fig.1, we formulate a typical hierarchical logistic problem as a mixed-integer programming problem. Its objective function is a total cost composed of production cost, every transportation cost, holding cost and the fixed-charge cost for opening DC. Notably, compared with the conventional formulation, in this study, we have imposed realistic conditions. Since thus formulated problem (Reference model) belongs to a NP-hard class, its solution becomes extremely difficult according to the increase of problem size.

To cope with various aspects under such circumstance, we have proposed a method termed hybrid tabu search (HTS) [3-5]. It is a two-level method whose upper level problem decides the location of DC in terms of the sophisticated tabu search [6] and the lower derives the routes among those members. At the lower level, the pegged DC location problem refers to a linear program (LP) that is possible to transform into the minimum cost flow problem (MCF). Hence, we can apply the graph algorithm known as CS2 [7] to solve the resulting problem extremely fast. These procedures will be repeated until a certain convergence criterion has been satisfied. Figure 2 sketch the procedure of HTS.

In what follows, the extended procedures will be described for two cases that should be taken into account for real-world applications.

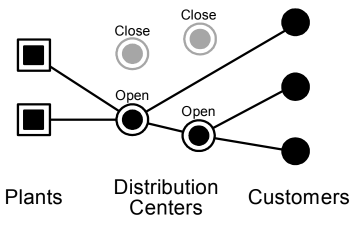


Fig.1 Logistic system concerned

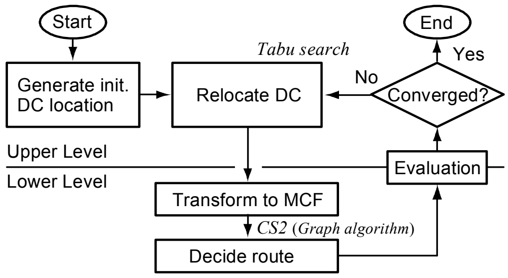


Fig.2 Scheme of hybrid tabu search (HTS)

## 3. Formulation under very realistic conditions

### *3.1. Multi-Commodity Problem*

We impose the same mild assumptions as the reference model except for the capacitated condition on DC whose upper bound is limited regarding the total sum amount of each kind. Eventually, the major extensions refer to expansion of the decision variables (from $f_{ij}$ to $f_{ij}^{p}, \forall p \in \mathrm{P}$,) and the modification of the capacitated condition on DC from Eq.(1) to Eq.(2).

$$\sum_{i \in I} f_{ij}^{1} + \sum_{j' \in J} f_{j'j}^{2} \le U_j x_j \quad \forall j \in \mathrm{J} \qquad \text{(for single-commodity)} \qquad (1)$$

$$\sum_{p \in P}\sum_{i \in I} f_{ij}^{1p} + \sum_{p \in P}\sum_{j' \in J} f_{j'j}^{2p} \le U_j x_j \quad \forall j \in \mathrm{J} \qquad \text{(for multi-commodity)} \qquad (2)$$

where $f_{ij}^{\#}$ and $f_{ij}^{\#p}$ denote respectively the shipping amounts from $i$ to $j$ for single-commodity and multi-commodity case regarding the item referred by superscript #. Moreover, index sets I, J and P correspond to plants, possible DCs and kinds of product, respectively. On the other hand, the right hand side of both equations gives the upper bound of the holding capacity at DC. It could be $U_j$ if $j$-th DC would be open ($x_j$=1), otherwise 0 ($x_j$=0).

After all, the problem requires us to decide the location of DC, and delivery routes from plants to customers via DCs to satisfy demand of every product so as to minimize the total cost mentioned already. Due to the existence of binding condition on the holding capacity of DC i.e., Eq.(2), getting simply every solution of single-commodity problem together cannot be the present solution. To cope with the multiple commodities for the lower level problem in HTS, therefore, we propose to give the following ingenious procedure as depicted in Fig.3, and outlined below.

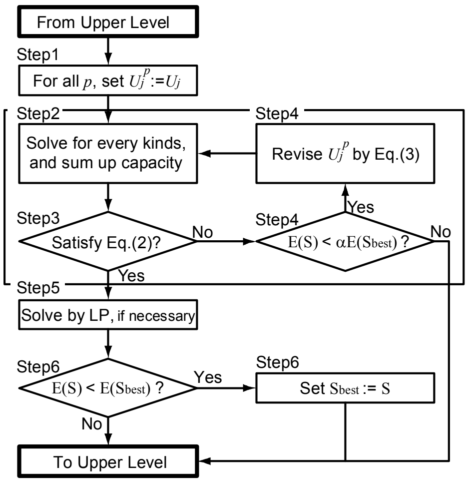


Fig.3 Lower level procedures for multi-commodity model

- Step 1: Set the initial values.
- Step 2: Solve the lower level problem per each kind and collect every solution.
- Step 3: Examine every capacity constraint of DC by summing up the above results. If no

violations are observed, go to Step 5 & 6. Otherwise, go to the next step.

Step 4: Impose/revise the forcing conditions Eq.(3) on the routes where the violations occur, and go back to Step 2 if the present search is promising. Otherwise, return to the upper level.

Step 5 & 6: Based on sophistication by LP if necessary (Step 5), the result of evaluation will be updated only if it is improved (Step 6). Then return to the upper level.

$$U_j^p := \frac{e_j^p}{\Sigma_{k\in P} e_j^k} \cdot U_j \quad \forall p \in \mathrm{P}, \quad \forall j \in \mathrm{J}_{\mathrm{vio}} \tag{3}$$

where $e_j^p$ denotes the holding amount of kind $p$ at DC $j$ obtained in Step 2 and $\mathrm{J}_{\mathrm{vio}}$ an index set of the capacity overfilling DC. In real implementation, the algorithm contains more elaborate ideas not described here.

### *3.2. A Stair-wised discount cost problem*

By considering the stair-wised volume discount transportation cost as shown in Fig.4, we need to distinguish the shipping amount depending on the discount level. This induces the introduction of additional 0-1 variables. For example, relation

$$N^l y_{ij}^{1l} \le f_{ij}^{1l} \le M y_{ij}^{1l} \quad \forall l \in \mathrm{L}, \forall i \in \mathrm{I}, \forall j \in \mathrm{J} \tag{4}$$

means that shipping amount from plant $i$ to DC $j$ must be greater or equal to $N^l$ to apply the $l$-th level discount for this transportation. Here, $y_{ij}^l$ denotes the 0-1 variable that takes 1 if this is true, and otherwise 0. Moreover, $M$ is a very large number and L index set of discount level. Apparently, necessary shipping amount will be calculated by

$$f_{ij}^1 = \sum_{l\in L} f_{ij}^{1l} \quad \forall i \in \mathrm{I}, \quad \forall j \in \mathrm{J} \tag{5}$$

From so far discussion, even if the location is pegged at the upper level problem, there still remain 0-1 variables in the lower level problem. In other word, the problem refers to the mixed 0-1 programming problem. To refrain from applying the computationally intensive solution method like branch and bound method, and to keep applying a high-speed graph algorithm to solve the transformed MCF problem, we invent a sequential substitution method to handle the discount practically and effectively as follows.

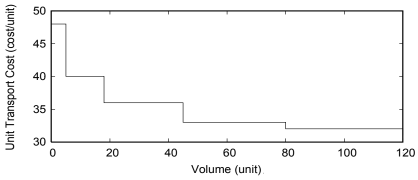


Fig.4 Stair-wised discount cost

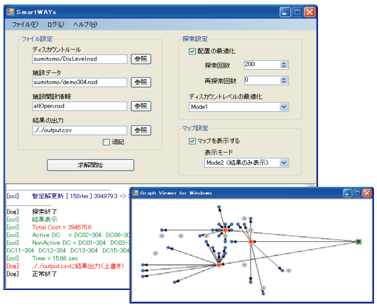

Fig.5 Screen shots of GUI of the

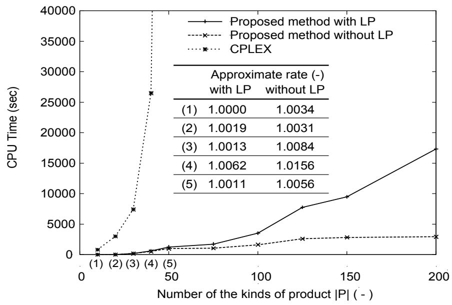


Fig.6 Computation load vs. problem size (|P|)

Step 1: Set the initial discount at level 1 (No discount) for all transportations.
Step 2: Solve the problem under the set-up discount level.
Step 3: Examine the consistency between the set-up level and the level post-determined from the result of Step 2. If there are no conflicts at all, return to the upper level. Otherwise, go to the next step.
Step 4: Replace the set-up level with the post-determined one for the inconsistent routes. Then go back to Step 2.

Also, in real implementation, the algorithm contains more elaborate ideas not described here. Moreover, we developed software amenable for supporting a daily decision making on logistic planning, and confirm its effectiveness through a real-life application for a major chemical company in Japan. Figure 5 shows a few screen shots of GUI of the software.

### *3.3. Numerical experiments*

To validate the performance of the proposed methods, we compared the results with those obtained from commercial software (CPLEX9.0). Figure 6 shows a trend of computational load along with the number of kinds of product. Three line graphs represent the CPU times to find the optimal solution using CPLEX, time to attain at the converged solution using proposed method with LP, and that without LP (Refer to Step 5 of the algorithm in Sec.3.1).

Moreover, numbers in the figure are the approximated rates of the solutions from the proposed method with LP and without LP to that of CPLEX in terms of the objective value. We can see very high approximated rates are attained by the proposed method with very short time for the smaller problems, say up to 50 (limit of the present comparison). Moreover, the proposed method can solve much larger problems with approximately linear increase in computation load. Relying on such observations, we can assert the advantage of the proposed method.

Table 1 Results of benchmark problems

| Prob.ID | No. of 0-1 vars. | CPU time [s] |
|---|---|---|
| p3-11-26 | 3867 | 22.4 |
| p3-11-31 | 4347 | 34.4 |
| p3-11-35 | 4731 | 39.5 |
| p3-11-41 | 5307 | 72.4 |

The introduction of stair-wised discount cost makes increase both numbers of 0-1 variables and constraints simultaneously many times of the number of stairs. For the problems in the chemical company, the proposed method can converge at the desirable solution within the practical computation time as shown in Table 1. It should be noticed this is achieved through a slow speed primal-dual algorithm of free software to solve the MCF instead of CS2 due to the license. By virtue of GUI shown already, the developed software is being used helpfully to support a decision making on the logistic design tasks at the company.

## 4. Conclusions

In this study, concerning with the two aspects that are very common in real-world logistics, i.e., a multi-commodity problem and a problem with stair-wised discount transportation cost, we have developed the extended methods of HTS for each problem. Novel iterative procedures are invented respectively while applying the graph algorithm to keep the high speed solution for the lower level problem. Validity of the methods is verified through comparison with the commercial software and evolution for the daily logistic planning in a real-life application.

**Acknowledgement** This study was partly supported in the 21st Century COE Program "Intelligent Human Sensing," from the Japanese Ministry of Education, Culture, Sports, Science and Technology.

## References

1. I.A. Karim, R. Srinivasan and P.L. Han, Chemical Engineering Progress, 98 (2002) 32.
2. J.F. Campbell, Studies in Locational Analysis, 6 (1994) 31.
3. Y. Shimizu and T. Wada, Transaction of the Institute of systems, control and information engineers, 17, (2004) 241. (in Japanese)
4. T. Wada and Y. Shimizu, Transaction of the Institute of systems, control and information engineers, 19, (2006) 69. (in Japanese)
5. Y. Shimizu, S. Matsuda and T. Wada, Transaction of the Institute of systems, control and information engineers, 19, (2006) (in Japanese)
6. C. Reeves (ed.), Tabu search, Modern Heuristic Techniques for Combinational Problems, Blackwell Scientific Publishing, 1993.
7. A.V. Goldberg, J. Algorithms, 22 (1997) 1.