

Design and Scheduling of Periodic Multipurpose Batch Plants under Uncertainty

Tânia Rute Pinto,^a Ana Paula F. D. Barbósa-Póvoa^b, Augusto Q. Novais^a

^a*Dep. de Modelação e Simulação, Instituto Nacional Engenharia de Tecnologia e Inovação, 1649-038 Lisboa, Portugal, tania.pinto@ineti.pt; augusto.novais@ineti.pt*

^b*Centro de Estudos de Gestão, Instituto Superior Técnico, Universidade Técnica de Lisboa, Lisboa 1049-101 Lisboa, Portugal, apovoa@ist.utl.pt*

Abstract

This work deals with the design of multipurpose batch plants under uncertainty. The model proposed by Pinto et al. [1] for the detailed design of batch plants is extended to address the problem of uncertainty associated with production demands. Equipment choices as well as plant topology and associated schedule are defined simultaneously under an uncertain environment. Demand uncertainty is considered as a set of scenarios and probabilities are assumed to be known a priori. A cyclic operation is assumed where time is treated using a uniform discrete time grid. Mixed storage policies and sharing of resources are considered and a MILP formulation is obtained. An illustrative example is solved to test the model applicability.

Keywords: Design, uncertainty, periodic, scheduling, optimization.

1. Introduction

In multipurpose batch plants, a wide variety of products can be produced via different processing recipes by sharing all available resources, such as equipment, raw material, intermediates and utilities. In order to ensure that, any resource in the design can be utilised as efficiently as possible, an adequate representation is necessary in order to address such type of problems without creating ambiguities in the process/plant representation. The Resource-Task

Network (RTN) is one of the possible adequate representations to describe the design of multipurpose batch plants [2]. Most of the work in the design area deals with the deterministic optimisation problem where all the parameters are considered to be known. In real plants, uncertainty is a very important issue, since many of the conditions which affect the operation of a real plant, are often subject to changes. Such is the case, amongst others, of raw material availability, prices, machine reliability and market requirements, which vary with respect to time and are often subject to unexpected deviations. The development of approaches to systematically consider uncertainty is a research subject matter of great importance. Methodologies for the design and scheduling under uncertainty with the aim of producing optimal solutions are then required. Most of the research that addresses uncertainty can be distinguished as two primary approaches, referred as the probabilistic approach and the scenario planning approach. The choice of the appropriated method is context-dependent, with no single theory being sufficient to model all kinds of uncertainty [3]. The scenario planning attempts to capture uncertainty by representing it in terms of a moderate number of discrete realisations of the stochastic quantities, constituting distinct scenarios [4]. Each complete realisation of all uncertain parameters gives rise to a scenario [5]. The objective is to find robust solutions, that perform well under all scenarios.

This work deals with this problem and studies the design and the scheduling of multipurpose batch plants under uncertainty. The model proposed by Pinto et al [1] for the detailed design of batch plants is extended to address the problem of uncertainty in production demands. Equipment choices as well as plant topology and associated schedule are defined simultaneously under an uncertain environment. A cyclic operation is assumed where time is treated using a uniform discrete time grid. Mixed storage policies and sharing of resources are considered and a MILP formulation is obtained.

2. Problem Definition

The problem can be defined as follows:

Given:

- The process/ plant description (in RTN terms) ;
- Resources availability, characteristics and costs;
- Time horizon of planning;
- Mode of operation;
- Demand over the time horizon (production range);
- Cost data;
- Probability density function.

Determine:

The optimal plant configuration (i.e. number and type of equipment units and their connections as well as their sizes under all scenarios).

The optimal process schedule (i.e. timing of all tasks, storage policies, batch sizes, amounts transferred, allocation of tasks and consumption of resources); So as to optimize an index of economic performance of the plant, measured in terms of capital expenditure, operating costs and revenues under all scenarios. A scenario planning approach is adopted for handling the uncertainty in products demands.

The plant operation is in a cyclic mode where a cycle time T is used that models the shortest interval of time at which a cycle is repeated. Each cycle represents a sequence of operations involving the production of all desired products.

3. Modelling Framework

The production is related to the planning horizon H . Since a constant cycle time is assumed, the following constraints are defined:

Excess resource balance - expresses the excess amount (unused) of resources along the cycle time for each scenario.

Resources provision constraints - accounts for the maximal amount of resource that is available at any one time during the cycle time for each scenario.

Resources existence constraints - define the allocations of tasks to available resources during the cycle time for each scenario. At any one time, each equipment resource is idle or processing a single task and a task cannot be pre-empted once started.

Excess resource capacity constraints - ensure that the amount of excess resource is never negative and never exceeds the maximum storage capacity for each scenario.

Capacity and batch size constraints - ensure that the amount of material being processed must always be within the maximum and minimum equipment capacity for each scenario. This takes account of the availability of equipment in discrete and/or continuous size ranges.

Production requirements constraints - allow production to float within given upper and lower bounds for each scenario, ensuring the possibility of optimising the production levels, as part of the design calculations, taking into account the trade-off between the cost of equipment and the added value of production.

The objective function considers the annualised capital cost of the equipment and the costs and revenues arising from the plant operation taken over all of the scenarios.

3.1. Example

To illustrate the applicability of the mathematical formulation, a multipurpose batch plant that must be designed at maximum profit is considered. The plant produces five products (S5, S6, S9, S10 and S11) from three raw materials (S1, S2, and S7). The products S5 and S6 are not only final products but also intermediate. In terms of equipment suitability, the reactors R1 and R2 may

carry out two processing tasks, while each storage vessel and the reactors R3, R4, R5 and R6 are dedicated to a single state/task only. Task T1 may process S1 during 2 hours in R1 or R2; task T2 may processes S2 during 2 hours in R1 or R2; task T3 may processes during 4 hours in R3; T4 processes during 2 hours in R4; Task T5 may processes S6 during 1 hour to produce the final product 0.3 of S11 and 0.7 of S8 in R5 and finally Task T6 processes during 1 hour S8 in reactor R6 to produce the final products S9 and S10.

The product demands are such that the production is split according three scenarios: a) expected case, b) optimistic case and the c) pessimistic case. All products are produced in all scenarios and present a production range between an upper and lower bound. The demands (in tons) for the expected case are:

[0; 5100] of S5, [0; 4980] of S9 and S10, [0; 8100] of S6 and [0; 4290] of S11; for the pessimistic case are: [0; 2550] of S5, [0; 2490] of S9 and S10, [0; 4050] of S6, [0; 2145] of S11 and finally for the optimistic case are: [0; 7650] of S5, [0; 7470] of S9 and S10, [0; 12150] of S6, [0; 6435] of S11. A single campaign with periodic mode of operation was assumed over a time horizon of 720 h, with a cycle of 24 h. The reactors R1 and R2 have the maximum capacity of 150 [m.u./m²], with the fix:var costs of 20:0.5 and 55:0.5 [10³ c.u.]. R3 and R4 have the same max. capacity of 200 [m.u./m²], with the fix:var costs of 30:1 and 30:0.5 [10³ c.u.]. Finally, R5 and R6 have the max. capacity of 150 [m.u./m²], and the fix:var costs of 30:0.5 [10³ c.u.]. The connections capacity varies from 0 to 200 [m.u./m²] with fix:var. cost of 0.1:0.01 [10³c.u.]. The results presented in Fig. 1, 2 characterise the optimal plant scheduling for the expected, optimistic and pessimistic scenario, the optimal design plant for all three scenarios considered simultaneously are presented in table 1, finally the statistics for the scenario-based design problem are showed in table 2.

3.2. Results & discussion

The General Algebraic Modelling System (GAMS 22.1) was used coupled with the CPLEX 10.0. The problem was solved with a 0.1 % margin of optimality on a Pentium(R) 4, 3.00 GHz.

For each scenario there is an associated probability value of 50%, 40% and 10%, for the expected, optimistic and pessimistic respectively.

The aim is to design the equipment plant, the scheduling and all the storage policies that can handle all three scenarios while maximise the performance of the plant. The resulting optimal design plant is presented in table 1. The resulting scheduling for each scenario is presented in figure 1 till 3. The multipurpose characteristic of the resources is visible in the processing equipment R1 that perform T1 and T2 in all the three scenarios. The problem proposed used 7 670 variables from which 1 764 are binary variables and took 34.7 cpu time (s) to reach the optimal solution.

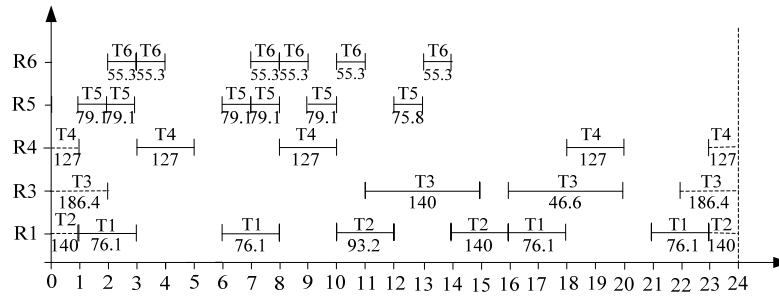


Figure 1 - Optimal plant scheduling for the expected scenario.

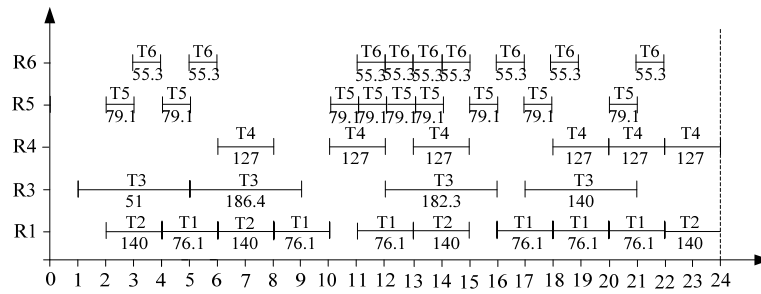


Figure 2 - Optimal plant scheduling for the optimistic scenario.

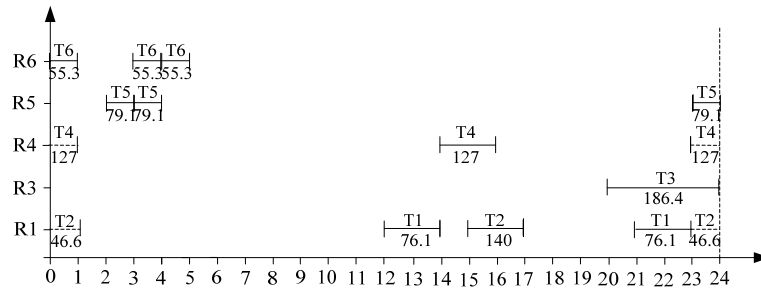


Figure 3 - Optimal plant scheduling for the pessimist scenario.

Table 1 - Optimal design plant for all three scenarios considered simultaneously. (Unit/Capacity [m.u./m2])

R1	R3	R4	R5	R6	V1,V2,V7	V4	V5	V6
139.8	186.4	126.8	79.0	55.3	unl.	266.6	305.7	405

Table 1 - Continuation

V9, V10	V11	C1, C3	C2	C4	C5, C6	C9	C13
249	213.4	76	139.8	126.8	186.4	50.7	139.8

Table 1 - Continuation

C16, C19	C17	C18	C20, C21
39.5	23.7	55.3	27.7

Table 2 - Computational statistics for the scenario-based design problem.

N° Variables	N° Binary	N° Constraints	CPU time (s)	LPs
7 670	1 764	13 880	34.703	120

4. Conclusions

This paper has addressed the design of multipurpose of batch plant operating in a cyclic mode under uncertainty in the demands. Scenarios planning approach was developed which resulted into a MILP-based formulation with high level of computational complexity. The identification of the true underlying source of uncertainty is the key so as to balance the number of representative of scenarios versus model resolution. Complementary to the use of the minimum number possible of scenarios some research should be undertaken in terms of model performance

Acknowledgements

The authors gratefully acknowledge the financial support from FCT, grant SFRH/17728/2004.

References

1. T. Pinto, A. P. Barbosa-Póvoa & A. Q. Novais, *Comp Chem. Engng.*, 29 (2005) 1293.
2. T. Pinto, A. P. Barbosa-Póvoa & A. Q. Novais, *European Symposium on Computer Aided Process Engineering*, 14 (2003) 257.
3. H.J. Zimmermann, *Eur. J. Oper. Res.*, 122 (2000) 190.
4. J.M. Mulvey, D.P. Rosenbaum, B. Shetty, *Eur. J. Oper. Res.*, 97 (1997) 1.
5. R. Sridharam, *Eur. J. Oper. Res.*, 87 (1995) 203.