

# **A population balance approach for continuous fluidized bed dryers**

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## **Abstract**

Continuous fluidized bed drying is widely used to remove moisture or solvents from granular materials. It is known that different residence times of the wet particles may lead to a distribution of product properties, e.g. different moistures. The prediction of such moisture distributions in fluidized bed dryers is from particular interest in industrial practice. In the present study a simplified analytical model is introduced, to calculate moisture distributions in a continuous fluidized bed dryer. Furthermore, the results of the analytical calculations are compared with results from experiments under variation of particle flow rate and gas inlet temperature.

**Keywords: fluidized bed, particle, drying, modeling, population balance,**

## **Introduction**

Drying in fluidized beds is of great practical significance because of good performance, low investment and maintenance costs and robustness of the respective equipment. Many different types of materials, from chemicals to foodstuffs and from plastics to fertilizers, are dried this way, usually with large throughputs in continuous processes. The moisture content represents an important property of the dried material. This characteristic is decisive for durability, transport, use and subsequent treatment of the product and is, thus, an important quality criterion. Traditional modeling approaches for continuous dryers consider the disperse solids as a phase with average properties such as particle size, moisture content and enthalpy. Naturally, such a model can only describe uniform moisture content for the solids in the dryer. However, Kettner et al. [1] demonstrated that uniform properties do not occur in practice. They have measured the moisture content of single particles in a continuous fluidized dryer using a combination of NMR and coulometric techniques. They proved that the moisture of dried solids can be widely distributed; different residence times cause such broad distributions. By tracer experiments Burgschweiger and Tsotsas [2] demonstrated that a small fluidized bed dryer, as investigated in this study, can be treated as a well mixed system where spatial distributions of the solid phase can be neglected. For such an application a model based on the concept of population balances will be provided, which is capable to predict the interaction between process parameters, material properties and moisture distributions. The model will be simplified in such a way that an analytical solution is feasible.

## **Population balance model for a continuous fluidized bed dryer**

In this study a simplified population balance approach will be applied to calculate the moisture distribution in the solid phase of a continuous fluidized bed dryer. The objective is to provide an analytical solution for the distributed properties of the solid phase. In contrast to previous studies (see Kettner et al. [1], Burgschweiger and Tsotsas [2]) the presented model does not involve energy balances of the solid phase. In a first step we consider a fluidized bed dryer as shown in Figure 1. The particles are conveyed into the dryer at a constant flow rate. The discharge of particles is conducted by an internal weir pipe. Furthermore, it is assumed that the dryer is sufficiently small, so that the spatial distribution of solids can be neglected. For such a system the number density of particles can be described by the residence time distribution of a single Continuous Stirred Tank Reactor (CSTR).

$$n(\tau) = B_0 \exp\left(-\frac{\tau}{\bar{\tau}}\right) \quad (1)$$

$B_0$  is the flow rate of particles and  $\bar{\tau}$  indicates the mean residence time. It is assumed that all solids have the same initial moisture content  $X_0$ . Consequently, particles with the same residence time  $\tau$  will have the same moisture content  $X(\tau)$ . In the framework of this study we will use the normalization concept of van Meel [3] to determine the actual drying rate of the particle. After van Meel the normalized drying rate  $\dot{v}(\eta)$  is defined as the ratio of the actual drying rate  $\dot{m}$  (external and internal resistance) to the so called first drying period rate  $\dot{m}_1$  (only external resistance), where  $\eta$  is the normalized moisture content.

$$\eta = \frac{X - X_{eq}}{X_{cr} - X_{eq}} \quad (2)$$

$X_{eq}$  denotes the equilibrium moisture content and  $X_{cr}$  is the critical moisture content that indicates the end of the first drying period. In this way, the function  $\dot{v}(\eta)$  is unity for  $\eta \geq 1$  and is supposed to describe intraparticle inhibition at  $\eta < 1$ . To generalize the presented model we define the normalized drying rate using a quite general function (see also Figure 3)

$$\dot{v}(\eta) = \frac{p \cdot \eta}{1 + \eta(p-1)} \quad (3)$$

The parameter  $p$  will change the curvature of the drying curve. For  $p > 1$  the normalized drying curve is hyperbolic; for  $p < 1$  the curve is parabolic and for  $p = 1$  it is linear.

Considering the mentioned assumptions, the temporal change of particle moisture content can be obtained from the water balance for a single particle.

$$\frac{dX}{dt} = \frac{dX}{d\tau} = -\frac{6}{d_p} \cdot \frac{\rho_g}{\rho_p} \cdot \beta(Y_{as} - Y) \cdot \dot{v}(\eta) \quad (4)$$

with the adiabatic saturation moisture content  $Y_{as}$  at the surface of the particle,  $Y$  the moisture content in the bulk gas and  $\beta$  the temperature dependent mass transfer coefficient. The moisture content of particles as a function of residence time in the first drying period can be obtained by integration of eq.(4). For the first drying period the coefficient  $\dot{v}$  is unity. Thus the integration yields

$$\tau = \frac{X_0 - X}{K} \quad (5)$$

The parameter  $K$  was introduced to substitute

$$K = \beta \cdot \frac{\rho_g}{\rho_p} \cdot \frac{6}{d} (Y_{as} - Y) \quad (6)$$

The critical moisture corresponds to a critical residence time of the particle in the dryer  $\tau_{cr}$ , which can be determined from eq.(5). for  $X = X_{cr}$ . This point in time represents the transition from first to second drying period. The main objective of this derivation is to provide an analytical expression for the moisture distribution of solids. Thus one needs to transform the number density distribution  $n(\tau)$  using the relationship

$$n(X) = -n(\tau) \frac{d\tau}{dX} \quad (7)$$

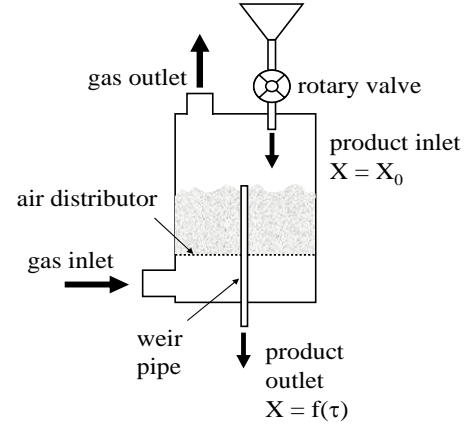


Figure 1: Scheme of a continuous fluidized bed dryer

For the first drying period one obtains

$$n_{1st}(X) = \frac{B_0}{K} \exp\left(-\frac{X_0 - X}{K \cdot \bar{\tau}}\right). \quad (8)$$

The temporal change of moisture content of the particle in the second drying period can be deduced in a similar way. First, the temporal change of moisture content as a function of residence time is determined. Therefore, eq. (4) must be integrated in the range of  $X_{cr}$  to  $X$  and  $\tau_{cr}$  to  $\tau$ . Because the normalized moisture falls below one in the second drying period, the normalized drying rate  $\dot{v}$  is not equal unity anymore. Thus the definition given in eq. (3) has to be applied. The moisture distribution for the second drying period  $n_{2nd}(X)$  can be obtained using the transformation provided in eq. (7)

$$n_{2nd}(X) = B_0 \exp\left(-\frac{\tau_{cr}}{\bar{\tau}}\right) \left(\frac{X - X_{eq}}{X_{cr} - X_{eq}}\right)^{\frac{X_{cr} - X_{eq}}{K \cdot p \cdot \bar{\tau}}} \exp\left(\frac{p-1}{K \cdot p \cdot \bar{\tau}}(X - X_{cr})\right) \left[\frac{X_{cr} - X_{eq}}{(X - X_{eq}) \cdot p \cdot K} + \frac{p-1}{p \cdot K}\right] \quad (9)$$

For some problems it is useful to determine the cumulative number distribution  $N(X)$  instead of the number density  $n(X)$ . For both cases (first and second drying period) the cumulative distribution is obtained by integration of the density distribution.

### Mean particle moisture content

For many industrial applications, the mean moisture content is of particular importance for the process. The solution of the number density distribution provides the possibility to determine the mean moisture content of the particles in the dryer. The mean value is defined as the ratio of the 0<sup>th</sup> and the 1<sup>st</sup> moment of the density distribution  $n(X)$ . The 0<sup>th</sup> moment of particles corresponds to the total number of particles. The first moment has to be determined separately for the first and second drying period as follows

$$\bar{X}_{PBE} = \frac{\int_{X_{eq}}^{X_0} X \cdot n(X) dX}{\int_{X_{eq}}^{X_0} n(X) dX} = \frac{\int_{X_{cr}}^{X_0} X \cdot n_{1st}(X) dX + \int_{X_{eq}}^{X_{cr}} X \cdot n_{2nd}(X) dX}{B_0 \cdot \bar{\tau}} \quad (10)$$

For the first drying period the 1<sup>st</sup> moment can be determined analytically,

$$\int_{X_{cr}}^{X_0} X \cdot n_{1st}(X) dX = B_0 \bar{\tau} \left[ (X_0 - K \cdot \bar{\tau}) - \exp\left(-\frac{X_0 - X_{cr}}{K \cdot \bar{\tau}}\right) (X_{cr} - K \cdot \bar{\tau}) \right] \quad (11)$$

For the second drying period the moment has to be determined by numerical integration.

### Moisture content of the gas phase

One important value of the Parameter  $K$ , introduced in eq. (9) is the gas sided moisture content  $Y$ . In a good approximation, that the gas stream can be described as a plug-flow. Furthermore the moisture content is assumed to remain constant throughout the dryer, thus a mean moisture content  $\bar{Y}$  can be applied. In order to calculate the water content in the gas phase, the mass balance for the evaporating water has to be satisfied.

$$\dot{M}_p \cdot (X_0 - \bar{X}_{PBE}) = \dot{M}_{gas} \cdot (\bar{Y} - Y_{in}) \quad (12)$$

The time derivation of the water mass can be calculated out of the spatial change and the differential amount of evaporating water. Under steady state conditions the time derivation equals zero.

$$\frac{\partial}{\partial t} dM_w = 0 = -\frac{\partial \dot{M}_w}{\partial z} + d\dot{M}_v \quad (13)$$

By assuming a mean normalized moisture  $\bar{\eta}$  and applying the transformations

$$\frac{\partial \dot{M}_w}{\partial z} = \frac{\dot{M}_G}{\dot{M}_G} \cdot \frac{\partial \dot{M}_w}{\partial z} = \frac{dY}{dz} \cdot \dot{M}_G \quad (14)$$

$$dM_v = \frac{A_p}{H} \cdot \rho_G \cdot \beta \cdot (Y_{as} - Y(z)) \cdot \dot{v}(\bar{\eta}) \quad (15)$$

the spatial change of the gas moisture throughout the dryer can be described.

$$\frac{dY}{dz} = \frac{A_p \rho_G \beta}{M_G} \cdot \dot{v}(\bar{\eta}) \cdot (Y_{as} - Y(z)) \cdot \frac{1}{H} \quad (16)$$

The constant values in front of the moisture difference can be summarized by the NTU-coefficient

$$NTU = \frac{A_p \rho_G \beta}{M_G} \cdot \dot{v}(\bar{\eta}) \quad (17)$$

The integration of eq. (16) leads to the gas moisture as a function of space coordinate  $z$ .

$$Y(z) = \left( 1 - \exp\left(-NTU \cdot \frac{z}{H}\right) \right) (Y_{as} - Y_{in}) + Y_{in} \quad (18)$$

The mean gas moisture is yielded by calculating the functional mean value of eq. (18).

$$\bar{Y} = Y_{as} + \frac{Y_{as} - Y_{in}}{NTU} (\exp(-NTU) - 1) \quad (19)$$

In the model, the mean moisture content of the particles is calculated by using an estimate for  $\bar{Y}$  in eq.(4). Subsequently a bisectional method is applied to iterate the values for  $\bar{Y}$  and  $\bar{X}$  until the mass balance of evaporating water (eq. (12)) is satisfied within a certain accuracy

## Parametric studies

In this section the influence of the particle flow rate and the gas temperature on the resulting moisture distributions will be examined. The results of the model calculations will be compared with the results yielded by experiments under equal conditions.

### Experimental setup

The experiments were carried out in a continuous lab scale dryer as depicted in Figure 1 with a diameter of 150 mm and a batch size of 3 liters. The instrumentation provides various measurements of temperature, pressure, pressure difference, gas flow rate, inlet and outlet gas moisture. As test material  $\gamma$ -Al<sub>2</sub>O<sub>3</sub> beads were used with a Sauterdiameter of  $d_p = 1.8$  mm and a particle density  $\rho_p = 1040$  kg/m<sup>3</sup>. For different process conditions, the moisture distributions in the product flow were determined by single particle measurements. These measurements were conducted using the nuclear-magnetic-resonance (NMR) technique and were performed in a Bruker Avance 300 MHz spectrometer with micro-imaging option.

### Comparison of measured and calculated distributions

The experimental parameters are summarized in Tab. 1 (trials 1 - 3) and Tab. 2 (trials 4 - 6) respectively. The aim of experimental trials 1 - 3 was to investigate the influence of particle flow rate on the moisture distribution. In runs 4 - 6 the influence of an increase of the gas inlet temperature was studied. The moisture distributions were estimated by calculating the cumulative number distribution out of Eqs. (8) and (9). These equations contain some parameters that are not directly accessible. The parameter  $p$  was estimated from the measured values of the drying curve provided by Kettner et al. [1] by minimizing the sum of square errors. The result of the fitting is depicted in Figure 2. A value of  $p = 0.27$  was obtained and used for all further calculations. The critical moisture content was specified by Kettner et al. [1] to  $X_{cr} = 0.27$ .

Symbol	Exp. 1	Exp. 2	Exp. 3	Unit
$M_{bed}$	0.982	0.820	0.976	[kg]
$\dot{M}_p$	1.7	1.3	1	[g/s]
$\dot{M}_g$	125	125	125	[kg/h]
$\beta$	0.0712	0.0712	0.0712	[m/s]
$\vartheta_{g,in}$	80	80	80	[°C]
$Y_{as}$	22.3	22.3	22.3	[g/kg]
$Y$	16.07	12.20	10.23	[g/kg]
$X_0$	0.67	0.67	0.65	[kg/kg]
$X_{eq}$	0.09	0.08	0.06	[kg/kg]
$K$	14.20e-4	23.04e-4	27.53e-4	[-]

Tab. 1: Experimental and simulation parameters for particle flow rate variation

A crucial model parameter is  $K$ , which contains the diameter and the density of the particle, the gas density and the adiabatic saturation moisture. Respective values can be measured or calculated. Furthermore, the gas sided mass transfer coefficient needs to be known to determine the parameter  $K$ . For this coefficient several models have been suggested in literature. In our studies we applied a Sherwood correlation recommended by Burgschweiger and Tsotsas [2], where axial dispersion in the gas is considered determining the kinetic coefficient.

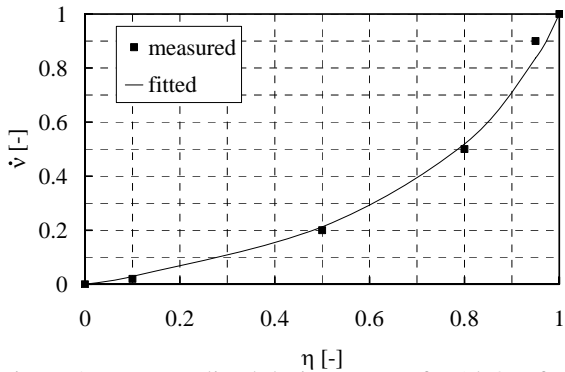


Figure 2: Normalized drying curve of  $\gamma\text{-Al}_2\text{O}_3$  after van Meel [3] as measured by Kettner et al. [1] and fitted according to Eq.(3) with  $p = 0.27$  (SSQE = 0.0055)

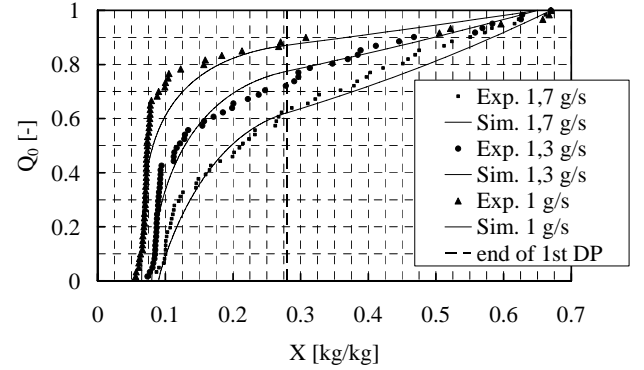


Figure 3: Comparison of experimental and calculated moisture distributions for different particle flow rates (remaining parameters according to Table 1)

The first comparison in Figure 3 demonstrates the influence of the mass flow rate of particles on the moisture distribution. An increase of particle flow rate results in a decrease of residence time. Consequently, wider moisture distributions are measured. With the given set of parameters the experimental moisture distributions can be reproduced in a satisfying manner by the model. The equilibrium moisture content increases slightly with increasing particle flow rate. An explanation of this behavior is that the test material is strongly hygroscopic and the equilibrium state depends on the gas temperature and gas humidity. For higher mass flow rates (higher moisture loads) the gas temperature in the bed will decrease and the gas moisture content increases respectively (see also Tab. 1). Thus the

relative humidity will increase and the equilibrium state of the solids moves towards higher moisture contents.

The experimental trials 4 to 6 were conducted under variation of gas inlet temperature. An increase of the gas inlet temperature will lead to higher adiabatic saturation moisture. Thus higher drying rates can be achieved, resulting in more slender moisture distributions. The comparison between experimental data and calculated values is presented in Figure 4. The measured distributions can be well reproduced applying the model equations.

Symbol	Exp. 4	Exp. 5	Exp. 6	Unit
$M_{bed}$	0.934	0.887	0.911	[kg]
$\dot{M}_p$	1.45	1.45	1.45	[g/s]
$\dot{M}_g$	125	125	125	[kg/h]
$\beta$	0.0712	0.0712	0.0712	[m/s]
$\vartheta_{g,in}$	55	65	80	[°C]
$Y_{as}$	14.87	17.73	22.06	[g/kg]
$Y$	11.73	10.37	10.74	[g/kg]
$X_0$	0.63	0.63	0.57	[kg/kg]
$X_{cq}$	0.098	0.08	0.064	[kg/kg]
$K$	7.65e-4	17.54e-4	26.04e-4	[-]

Tab. 2: Experimental and simulation parameters for temperature variation

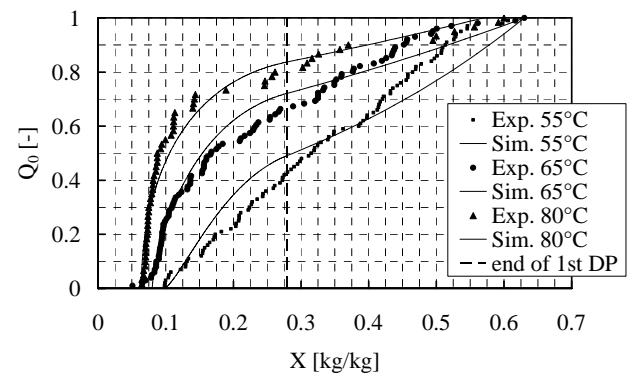


Figure 4: Comparison of experimental and calculated moisture distribution for different gas inlet moisture contents

## Conclusion

A simple possible solution for particle moisture distribution in a continuous, well mixed fluidized bed dryer has been presented. This model based on population balances enables an easy way to investigate and discuss the influence of parameters like single-particle drying curve and particle flow rate (residence time) on the distribution of residual moisture in the product.

The mean gas phase moisture content has been estimated iterative out of the mean moisture content of the particles by a bisectional method. The resulting agreement between calculated and measured particle moisture distributions is acceptable. Similarly good agreements can be obtained by numerical analysis including gas phase balances throughout the dryer, as it will be shown separately. However, respective solutions are more complicated and, thus, less instructive than the simplifications presented in this paper. Since population balances are used, the method can be expanded to capture particle properties other than the moisture content. This opens new opportunities for simultaneous improvement of product quality and process efficiency.

## References

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