Norm based approaches for Integrated Design of Wastewater treatment plants

M.Francisco, P.Vega

University of Salamanca, Dept. Informática y Automatica, Escuela Técnica Superior de Ingeniería Industrial. C/ Fernando Ballesteros s/n, 37700 Bejar (Salamanca), Spain

Abstract

In this work the Integrated Design (ID) of the activated sludge process of a wastewater treatment plant (WWTP) has been performed, including a linear multivariable predictive controller with constraints. In the Integrated Design procedure, the process parameters are obtained simultaneously with the parameters of the control system by solving a multiobjective constrained non-linear optimization problem, taking into account investment, operating costs and a set of performance indexes based on the weighted sum of the H_∞, l1 and H_2 norms of different closed loop transfer matrices of the system, subject to a set of constraints (process, controllability, physical constraints).

Keywords: Integrated Design, Activated sludge process, H_∞, l1 and H_2 norms, Sensitivity transfer function, Control sensitivity function

1. Introduction

The traditional mode of designing processes has been the use of heuristic knowledge concentrated on determining the economically optimal process configuration among many possible alternatives. After the configuration is selected, the process parameters and a steady state working point are evaluated in order to satisfy operating requirements and reduce investment costs. In this procedure, operability and input-output controllability is not considered, obtaining plants very difficult to control. Once the process has been designed, the following step is the selection of the controller structure and tuning. The design and control of processes are tasks performed sequentially, and examination of controllability occurs only after the optimal process configuration and parameters are known.

Integrated Design methodology allows for the evaluation of the plant parameters and control system at the same time, making the designed system more controllable [4],[9].

At design stage, controllability indicators are evaluated together with economic considerations, in order to give an optimum plant. This problem is stated mathematically as a multiobjective nonlinear programming problem with differential and algebraic constraints (NLP/DAE). Many works apply Integrated Design techniques, particularly to chemical process design, such as distillation systems or reactors, stressing the interactions of design and control [8],[13]. These works also tackle process structure selection by solving a synthesis problem. A comprehensive review of advances in the area is given by [14].

Some good examples of Integrated Design applied to the activated sludge process are given in [5], where PI controllers and the plant were obtained, including linear matrix inequality (LMI) constraints to state stability conditions and some desired closed-loop behaviour, and in [17], that presents a study of Integrated Design with PI controllers applied to different plant structures. Despite of the complicated dynamics of the process under design, works adding advanced controllers to the Integrated Design procedure have not been reported in the literature and it could be a good way to improve control performance. In this work, model predictive control (MPC) has been selected as advanced control method because of the existence of several successful applications in activated sludge control ([16], [17]) and the easiness to deal with constraints.

One important issue in Integrated Design is the tuning of controller parameters. Usually the tuning of these parameters has been performed using expert knowledge and a trial and error procedure. However, some works deal with automatic tuning of MPC. Reference [2] proposed an off-line procedure for tuning the algorithm parameters of a nonlinear predictive controller specifying time-domain performance criteria. Results are good, but the tuning of integer parameters such as horizons is performed using a non intelligent grid search. For linear MPC, [1] has developed an on-line tuning strategy based on the linear approximation between the closed-loop predicted output and the MPC tuning parameters, but without considering output constraints on the on-line optimization step. Recently, [6] and [7] have developed a new method taking into account input and output constraints, and it has been applied to linear plants and the activated sludge process, but only considering ISE norm as performance index for tuning the controller.

In this work, the Integrated Design problem is stated mathematically as a constrained non-linear multi-objective optimization problem, in which economic and control objectives are considered together with some constraints. The solution of the ID problem is obtained following a constrained numerical cost optimization procedure that uses dynamic models and real data records of disturbances together with a set of predefined constraints to evaluate the plant dimensions, the optimal operation points and the control system parameters.

The cost functions include the investment, operating costs, and dynamical indexes based on the weighted sum of the H_∞, l1 and H_2 norms of different closed loop transfer functions matrices of the system subject to a set of constraints. The constraints are selected to ensure that the process variables, some closed loop controllability measures and several closed loop performance criteria lay within specified bounds.

The methodology for the integrated design is subdivided in several steps:

1) Initial plant information: It is where all the information necessary is defined to carry out the WWTP design. It includes wastewater and control system characterisation (plant and control type, models, plant load, …)
2) Definition of design objectives, performance and controllability criteria and constraints: It is where the preliminary goals and the corresponding measurement criteria are proposed and classified according to different categories (environmental, economic, operational, control, ...)
3) Optimization procedure
4) Validation of results: It is where the optimal plant can be simulated, evaluating the proposed criteria, and carrying out a comparison with other plants.

The paper is organized as follows. First, the activated sludge process is presented and the way to implement an MPC for this process is explained. Secondly, a method for automatic tuning of the MPC is presented and applied to the activated sludge process. Then, the Integrated Design procedure is stated and solved for the activated sludge process, showing some results and ending with conclusions.

2. Description of the activated sludge process and model predictive controller

2.1. Plant description

For applying Integrated Design methodology, a wastewater treatment plant has been selected. The plant layout is represented in Fig. 1, which is a benchmark developed for the research European program COST 624 as a framework to compare different control strategies [3]. The complete benchmark includes substrate, oxygen and nitrogen control, with two anoxic and three aerobic reactors and one secondary settler, but in this work only oxygen and substrate control has been considered.

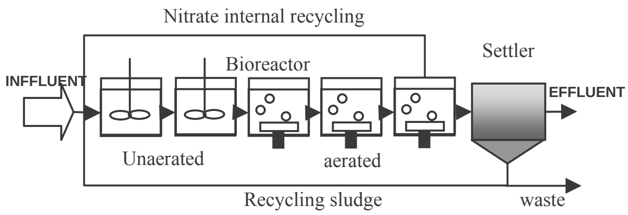

Figure 1: Benchmark plant

Our simplified plant consists of one aeration tank (reactor) and one secondary settler (Fig. 2). The basis of the process lies in maintaining a microbial population (biomass) into the bioreactor that transforms the biodegradable pollution (substrate) when dissolved oxygen is supplied through aeration turbines. Water coming out of the reactor goes to the settler, where the activated sludge is separated from the clean water and recycled to the bioreactor to keep there an appropriate level of biomass. The whole set of variables is also presented in Fig. 2. Generically, "*x*" is used for the biomass concentrations (mg/l), "*s*" for the organic substrate concentrations (mg/l), "*c*" for the oxygen concentrations (mg/l) and "*q*" for flow rates (m^3/h).

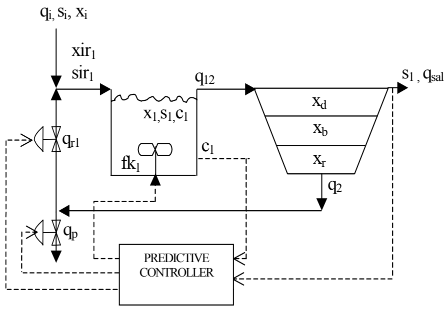

Figure 2: Selected plant for Integrated Design

A first principles model of the system is obtained by considering mass balances of oxygen, biomass and organic substrate in the whole plant, together with the equilibrium equations for the flows of water and sludge. Note that three layers of different and increasing biomass concentration are considered in the settler. This model has been linearized to use it as internal model in the MPC studied.

The set of equations for the nonlinear model (reactor and settler) are the following [12]:

$$
\begin{aligned}
\frac{dx}{dt} &= \mu_{\max} y \frac{sx}{(K_s+s)} - K_d \frac{x^2}{s} - K_c x + \frac{q}{V_1}(xir - x) \\
\frac{ds}{dt} &= -\mu_{\max} \frac{sx}{(K_s+s)} + f_{kd} K_d \frac{x^2}{s} + f_{kd} K_c x + \frac{q}{V_1}(sir - s) \\
\frac{dc}{dt} &= K_{la}\, fk_1 (c_s - c) - K_{01}\mu_{\max} \frac{x^2}{(K_s+s)} - \frac{q}{V_1} c \\
A \cdot l_d \frac{dx_d}{dt} &= q_{sal} x_b - q_{sal} x_d - A \cdot vs(x_d) \\
A \cdot l_b \frac{dx_b}{dt} &= q x_1 - q_{sal} x_b - q_2 x_b + A \cdot vs(x_d) - A \cdot vs(x_b) \\
A \cdot l_r \frac{dx_r}{dt} &= q_2 x_b - q_2 x_r + A \cdot vs(x_b)
\end{aligned}
\tag{1}
$$

2.2. Control problem

The control of this process aims to keep the substrate at the output (s_1) below a legal value despite the large variations of the flow rate and the substrate concentration of the incoming water (q_i and s_i). Another control objective is to keep dissolved oxygen concentration (c_1) around 2 mg/l, concentration that is necessary for the proper working of activated sludge process.

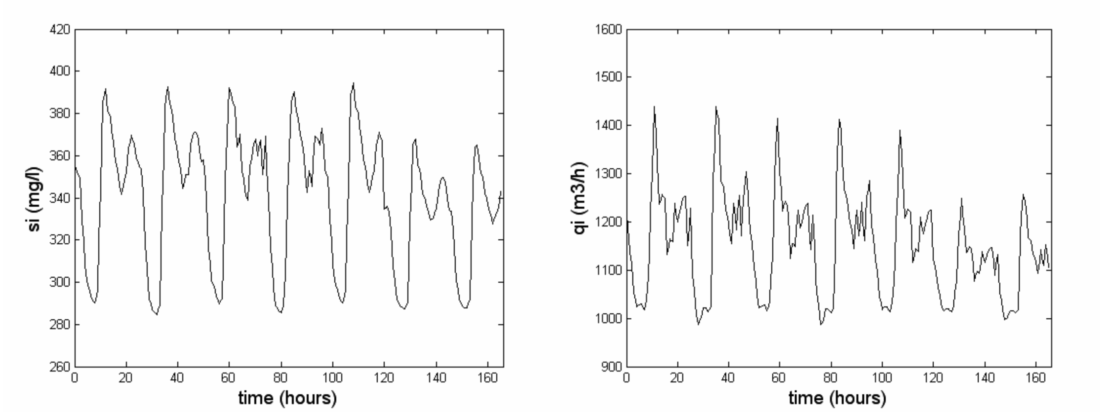

Figure 3: Substrate and flow disturbances at the influent (s_i,q_i)

One of the main problems when trying to control the plant properly is the existence of large input disturbances (q_i and s_i). The set of disturbances for designing the plant (Fig. 3) has been taken out from the benchmark plant.

The general structure of a multivariable controller applied to the activated sludge process can be seen in Fig. 4. Three manipulated variables are considered: recycling flow (qr_1), purge flow (q_p) and aeration factor (fk_1); and also three outputs: biomass (x_1), oxygen (c_1) and substrate (s_1) in the reactor. In our case, although the methodology is general, in order to simplify the problem only substrate control with the recycling flow as manipulated variable is considered.

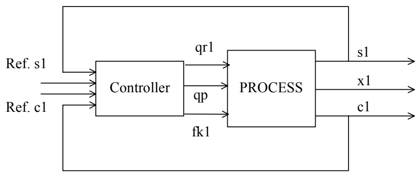

Figure 4: General controller structure

2.3. MPC applied to the process

A standard linear multivariable MPC has been considered to apply the automatic tuning procedure and the Integrated Design methodology proposed in this paper. It calculates manipulated variables by solving an on-line constrained optimization problem [10].

$$\min_{\Delta u} V(k) = \sum_{i=Hw}^{Hp} W_y \cdot (\hat{y}(k+i\,|\,k) - r(k+i\,|\,k))^2 + \sum_{i=0}^{Hc-1} W_u \cdot (\Delta\hat{u}(k+i\,|\,k))^2 \qquad (2)$$

subject to constraints on predicted outputs, inputs, and changes in manipulated variables:

$$20 < s_1 < 150 \;;\; 400 < x_1 < 3000 \;;\; 0 < qr_1 < 3500 \;;\; 0 < \Delta qr_1 < 1000 \qquad (3)$$

where k denotes the current sampling point, $\hat{y}(k+i|k)$ is the predicted output at time $k+i$, depending of measurements up to time k, $r(k+i|k)$ is the reference trajectory, $\Delta\hat{u}$ are the changes in the manipulated variables, H_p is the upper prediction horizon, H_w is the lower prediction horizon, H_c is the control horizon, W_u is a vector representing the weights of the change of manipulated variables and W_y is a vector representing the weights of the errors of set-points tracking.

The MPC prediction model is a linear discrete state space model of the plant obtained by linearizing the model equations. The reference trajectories *r(k)* approach the set-point trajectories *s(k)* exponentially from the current output values, with T_{ref} as the 'time constant' of the exponentials and T the sampling period, as in the following equation:

$$r(k+i \mid k) = s(k+i) - e^{-iT/T_{ref}}(s(k) - y(k)) \qquad (4)$$

When the MPC controller is linear and unconstrained, it can be represented with a transfer function K_{MPC}. The full closed loop system with measured disturbances has been represented in Fig. 5.

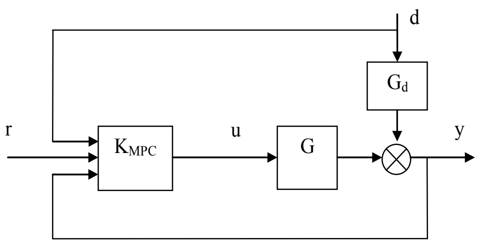

Figure 5 : Block diagram of the plant with MPC

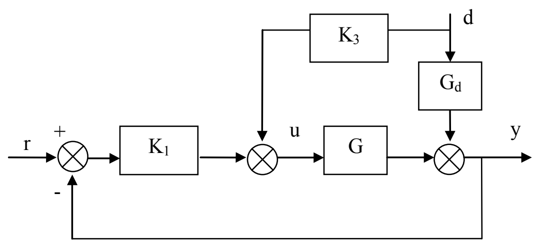

Figure 6 : Equivalent closed loop system

The controller block is multivariable, so the transfer function is

$$u = \begin{pmatrix} K_1 & K_2 & K_3 \end{pmatrix} \cdot \begin{pmatrix} r \\ y \\ d \end{pmatrix} = K_1 r + K_2 y + K_3 d \qquad (5)$$

where K_i are the transfer functions between the control signal and the different inputs (r,y,d).

On the other hand, from block diagram of Fig. 5:

$$y = Gu + G_d d \tag{6}$$

and substituting the control law (5) in equation (6) we obtain:

$$y = GK_1 r + GK_2 y + GK_3 d + G_d d$$

$$y = \frac{GK_1}{1 - GK_2} r + \frac{\left(GK_3 + G_d\right)}{1 - GK_2} d \tag{7}$$

In our MPC we have:

$$K_2 = -K_1 \tag{8}$$

and equation (7) can be expressed as:

$$y = \frac{GK_1}{1 + GK_1} r + \frac{1}{1 + GK_1} \tilde{d} \tag{9}$$

where $\tilde{d}$ are the filtered disturbances $\tilde{d} = \left(GK_3 + G_d\right) d$

From this the sensitivity (*S*) and complimentary sensitivity functions (*T*) are obtained:

$$T = \frac{y}{r} = \frac{GK_1}{1 + GK_1} \quad S = \frac{y}{\tilde{d}} = \frac{1}{1 + GK_1} \tag{10}$$

Another implication of (8) is that block diagram of figure 5 can be transformed to that in figure 6, representing a control system with feedforward compensation. So the MPC has a double effect, feedback and feedforward.

We define one different sensitivity function *S'* considering disturbances without filtering. Its importance will be stressed when the tuning problem is stated. The calculation of *S'* is straightforward from (7):

$$S' = \frac{y}{d} = \frac{GK_3 + G_d}{1 + GK_1} \tag{11}$$

In the same way as *S'*, when *r*=0, sensitivity to control transfer function *M'* can be calculated from equations (5), (6) and (8):

$$M' = \frac{u}{d} = \frac{K_3 - K_1 G_d}{1 + GK_1} \tag{12}$$

3. Optimal automatic tuning of MPC

3.1. MPC tuning parameters

The main tuning parameters are those affecting the behaviour of the closed loop combination of plant and MPC. The most important are the weights W_u in the controller cost function, the lower and upper prediction horizons (H_p, H_w), the control horizon (H_c), and T_{ref} in the reference trajectories. As the MPC is multivariable, weights in the cost function are vectors, so several different values will be tuned.

3.2. Optimization problem

The automatic tuning procedure of MPC parameters is based on the resolution of a H_∞ mixed sensitivity optimization problem, as described below. The point is to find an optimal MPC controller by solving an optimization problem that considers both disturbance rejection and control effort in the same tuning function. The H_∞ mixed sensitivity problem solves that and it is stated as:

$$\min_{K_1,K_3} \left\| \begin{matrix} Wp \cdot S' \\ Wesf \cdot s \cdot M' \end{matrix} \right\|_\infty = \min_{K_1,K_3} \|N\|_\infty \tag{13}$$

s.t. $\|Wp \cdot S'\|_\infty < 1$ where $Wp, Wesf$ are suitable weights.

This problem is based on [11] with the following modifications:

- Control efforts rather than magnitudes of control are included in the objective function by considering the derivative of the transfer function *M'* (product by Laplace variable *s*)
- Controller parameters are here K_1 and K_3, directly related to the MPC tuning parameters W_u and horizons.
- One constraint over H_∞ norm of weighted *S'* has been added to assure that disturbances are properly rejected.

In order to limit control magnitudes to avoid actuator saturation, we add one of the following constraints over the l1 norm of either *M'* or *S'*.

$$\|M'\|_1 < u_{\max} \; ; \; \|S'\|_1 < y_{\max} \tag{14}$$

3.3. Multiobjective optimization approach

The optimization problems for optimal automatic tuning and Integrated Design can be stated as multiobjective problems by considering constraints as objectives f_{2i} together with constrained optimisation of $\|N\|_\infty$. Then the multiple objectives are:

$$f_{21} = \|N\|_{\infty}\,;\;\; f_{22} = \|M'\|_{1}\,;\;\; f_{23} = \|S'\|_{1} \tag{15}$$

One of the main problems when solving this optimization problem is that involves real and integer variables. In this work we propose a two iterative steps algorithm that combines a particular random search for tuning the controller horizons based on [15], and the goal attainment method, implemented in MATLAB® function *fgoalattain*, for the real variables. In this method the objectives must approach fixed goals, giving with these parameters different importance to every objective.

As for selection of weights *Wp* and *Wesf*, weight *Wp* has to be selected in such way that its inverse is smaller in magnitude that the disturbance inverse spectrums to obtain optimal controllers that reject those sets of disturbances (see a typical selection weight in Fig. 7). Weight *Wesf* is selected to determine the relevance of control efforts in the optimization.

The reason for using *S'* instead of *S* is to make the selection of weights *Wp* easier because the disturbances spectrum keeps constant. If *S* would be considered, weights *Wp* should filter $\tilde{d}$ spectrum, and as $\tilde{d}$ depends on the controller, its spectrum is variable. On the other hand, if *S'* is considered *Wp* has to filter only *d* spectrum, which is constant.

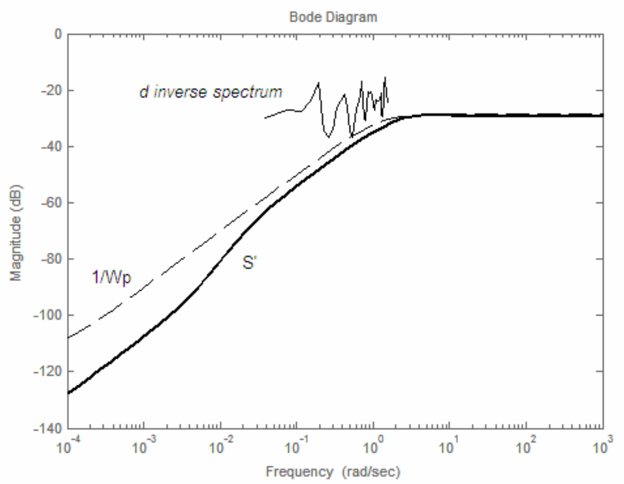

Figure 7: Typical selection of *Wp,* where *d* inverse spectrum is for wastewater example disturbances.

4. Integrated Design of Plant and MPC controller.

The Integrated Design problem consists of determining simultaneously the plant and controller parameters and a steady state working point, while the investment and operating costs are minimized. Non-linearities of the plant and the relatively high number of variables increase the complexity of the problem and make necessary the use of an iterative two steps optimization approach. In the first step the MPC is tuned using the method exposed above, and in the second step the plant is designed solving the following constrained multiobjective optimization problem. The objective functions are:

$$f_1 = w_1 \cdot V_{1n} + w_2 \cdot A_n \; ; \; f_{22} = \|M'\|_1 \; ; \; f_{24} = \frac{\|G\|_1}{\|G_d\|_1} \tag{16}$$

where V_{1n} and A_n are the normalized values for the volume of the reactor and the cross-sectional area of the settler.

The solution of this optimization problem is also solved with the goal attainment method of MATLAB, and is subject to lower and upper bounds for optimization variables x=(s_1,x_1,c_1,x_d,x_b,x_r,,fk_1,qr_1,q_p,V_1,A) and other nonlinear constraints representing process and controllability constraints. The weights w_i (i = 1, 2) are selected from CAPDET model (Computer Aided Procedure for Design and Evaluation of Wastewater Treatment Systems). Their normalized value is:

w_1=1; w_2=3.1454

The purpose of objective f_{22} is to design plants in which control magnitudes be less than one fixed value for the worst linear case. As for objective f_{24}, it is related with the l1 norm of the open loop plant transfer functions as a measure of intrinsic plant controllability. The constraints for this problem are:

- Residence time and mass load in the aeration tanks:

$$2.5 \le \frac{V_1}{q_{12}} \le 8 \; ; \; 0.001 \le \frac{q_i s_i + q_{r1} s_1}{V_1 x_1} \le 0.1 \tag{17}$$

- Limits in hydraulic capacity and sludge age in the settler and limits in the relationship between the input, recycled and purge flow rates:

$$\frac{q_{12}}{A} \le 0.7 \; ; 2 \le \frac{V_1 x_1 + A \cdot l_r x_r}{q_p x_r 24} \le 10 \tag{18}$$

$$0.03 \le \frac{q_p}{q_2} \le 0.3 \; ; 0.05 \le \frac{q_2}{q_i} \le 0.9 \tag{19}$$

- Constraints on the non-linear differential equations of the plant model to obtain a solution close to a steady state (ε close to zero).

The algorithm for solving the nonlinear optimization problem generated tackles the problem in an iterative two step approach (see Fig. 8). The first step performs the controller tuning, and the second step the plant design with the previous controller obtained. The loop is finished when a convergence criteria over costs is reached.

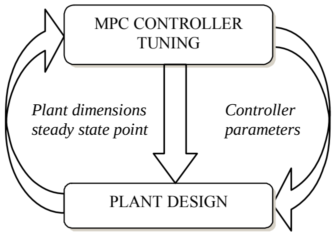

Figure 8: Iterative loop for Integrated Design

5. Integrated Design results

In table I can be seen Integrated Design results for two different weights *Wp*:

$$Wp_1 = \frac{26.6s+32}{s+0.0001};\ Wp_2 = \frac{33.3s+40}{s+0.0001} \tag{20}$$

Weight Wp_2 is more restrictive for *S'* magnitude, so the Integrated Design performed with this weight produces a plant with better disturbance rejection, with only a small increase of reactor dimensions. In figure 9 substrate responses are presented.

TABLE I
INTEGRATED DESIGN RESULTS

	Wp_1	Wp_2
W_u	0.0027	0.00042
H_p	10	10
H_c	2	4
V_1	7262	7307
A	1857	1857
S_{1r}	70	70
$Max(s_1)$	78.66	76.49
$Max(qr_1)$	870.69	1021.3
Plant cost= $f_1(x)$	1.5127	1.5177

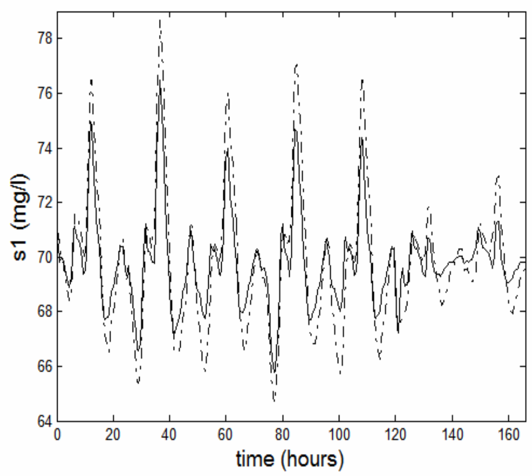

Figure 9: Comparison of substrate responses for Integrated Design with two weights Wp_1 and Wp_2

Then a comparison of results obtained with different goals for objective function f_1 is presented in Table II, keeping constant the goals for f_{22} and f_{24}. Although goals are not reached in any of the three cases, they have an important effect in plant costs (dimensions) and in the controller parameters. When costs are forced to be small with a low goal, disturbance rejection is worse than when costs are less restricted by considering a higher goal (Fig. 10).

TABLE II
INTEGRATED DESIGN RESULTS

Goal for f_1	0.3	0.7	1.6
W_u	0.00066	0.0027	0.0021
H_p	7	10	10
H_c	4	2	2
V_1	5268	7262	9305
A	1936	1857	1873
$Max(s_1)$	84.37	78.66	78.95
$Max(qr_1)$	205.31	870.69	1116
Plant cost= $f_1(x)$	1.3682	1.5127	1.7548

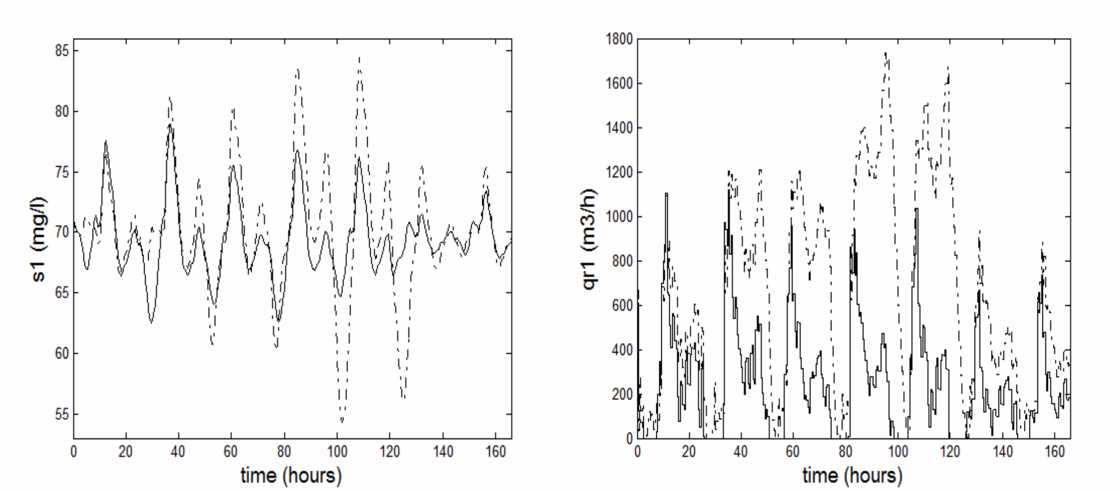

Figure 10: Comparison of substrate responses (s_1) and control actions (qr_1) for Integrated Design with goals 0.3 for f_1 (dashed dotted line) and 1.6 (solid line)

6. Conclusions

In this paper an Integrated Design procedure to obtain one optimal plant for the activated sludge process and its MPC tuning parameters has been developed. The design procedure shown here produces better controllable plants that the classical procedure. The responses for closed loop design with MPC show clearly a good behaviour for interest variables. When Integrated Design procedure is solved, the designed plant is able to fulfil disturbance rejection requirements with optimum cost units. This is an important result because one can obtain an optimum plant with lower construction costs and good disturbance rejection. Note also that no further MPC tuning is needed because the optimization gives also its optimum parameters. The solved problem guarantees that the dynamic non-linear model of the plant is satisfied, as well as the operating and process constraints.

Acknowledgments

The authors gratefully acknowledge the support of the Spanish Government through the MEC project DPI2006-15716-C02-01.

References

[1] Al-Ghazzawi,. A., E. Ali, A. Nouh and E. Zafiriou (2001). On-line tuning strategy for model predictive controllers. *Journal of Process Control*, **11**, pp. 265-284.

[2] Ali, E. and E. Zafiriou (1993). On the tuning of Nonlinear Model Predictive Control Algoritms. *Proceedings of the American Control Conference,* pp. 786-790.

[3] Copp, J.B. (2002). The COST Simulation Benchmark: Description and Simulator Manual. *Office for Official Publications of the European Community.* ISBN 92-894-1658-0.

[4] Fisher, W. R., M. F. Doherty and J. M. Douglas (1988). The Interface Between Design and Control. 1. Process Controllability. *Ind. Eng. Chem. Res*., **27**, pp. 597-605.

[5] Francisco, M., P. Vega, O. Pérez and M. Poch (2003). Dynamic Optimization for Activated Sludge Integrated Design. *European Control Conference (UK)*

[6] Francisco, M., P. Vega, O. Pérez (2005). Process Integrated Design within a Model Predictive Control framework. *Proceedings of IFAC World Congress (Prague)*

[7] Francisco, M., P. Vega (2006). Optimal automatic tuning of model predictive controllers for the activated sludge process. *Proceedings of Chemical Process Control CPC7 (Canadá)*

[8] Gil, A., P. Vega and M. Francisco (2001). Integrated Design of pH processes. *IASTED MIC Conference*, pp 226-229.

[9] Luyben, M. L. (1993). Analyzing the Interaction Between Process Design and Process Control. *Ph.D. Thesis, Princeton University*.

[10] Maciejowsky, J. M. (2002). Predictive Control with Constraints. *Prentice Hall.*

[11] Morari, M. and Zafiriou, E. (1989). Robust process control. *Prentice Hall.*

[12] Moreno, R. (1995). Estimación de estados y control predictivo del proceso de fangos activados. *Ph.D. Thesis, Autonomous University of Barcelona. (Spain)*

[13] Ross, R., J.D. Perkins, E. N. Pistikopoulos, G.L.M. Koot and J.M.G. van Schijndel. (2001). Optimal design and control of a high purity industrial distillation system. *Computers and Chemical Engineering,* **25**, pp. 141-150.

[14] Sakizlis, V., J. D. Perkins and E. N. Pistikopoulos (2004). Recent advances in optimization-based simultaneous process and control design. *Computers and Chemical Engineering,* **28**, pp. 2069-2086.

[15] Solis, F.J. and R. J-B. Wets (1981). Minimization by random search techniques. *Mathematics of Operations Research*, **6**, pp. 19-30.

[16] Sotomayor, O. A. Z. and C. García (2002). Model-Based Predictive Control of a pre-denitrification plant: a linear state-space model approach. *Proceedings of the IFAC World Congress, Barcelona (Spain).*

[17] Vega, P. and G. Gutiérrez (1999). Optimal Design Control and Operation of wastewater treatment plants. *European Control Conference (Germany).*