A STATE SPACE INTERPRETATION OF SIMULTANEOUS OBSERVATION FOR LINEAR SYSTEMS

Jaime A. Moreno

Automatización, Instituto de Ingeniería, UNAM, Ap. Postal 70-472, 04510 México D.F., MEXICO,
JMorenoP@iingen.unam.mx, Tel: ++(52)5556228127, Fax:++(52)5556228130

Keywords: Linear observers, simultaneous observation, robust observation, Indistinguishability, Unknown Input Observers

Abstract

The problem of simultaneously observing two or more LTI systems is considered. Obvious necessary conditions are derived using distinguishability concepts. In this way the existence of a common observer for two systems is related to the existence of an unknown input observer for the parallel system, formed by the difference of the systems. These conditions are also shown to be suf£cient for constructing a common observer, and a structure for such observers is proposed. The use of a special normal form allows to easily check if the conditions are satis£ed and to effectively construct a family of common observers. Similar conditions are derived for the case of more than two plants. In this case it is shown that the solution of the simultaneous observation problem for N plants can be reduced to the simultaneous observation problem of $N-1$ plants, but with *unknown inputs*. This latter problem is of a higher level of dif£culty but in a lower number systems, and has not been solved yet (for $N > 2$).

1 Introduction

The problem of simultaneous observation of two or several linear plants is of interest when, for example, a robust observer has to be designed that converges despite of failures in some components in a system. Or when for a nonlinear system, that has several operating points, an (linear) observer has to be designed. In contrast to the (dual) simultaneous control problem [7], that has received a lot of attention in the literature since its appearance around 1980, the simultaneous observation problem has only recently been studied [8, 4]. In these references the problem has been characterized using a MFD.

The objective of this note is to give an state space characterization of the solvability conditions for the problem. In doing so we obtain more general results than those in [8], since our results are valid for systems that are not necessarily detectable and/or stabilizable, for systems without inputs, and the conditions obtained are shown to be necessary and suf£cient for the existence of any kind of common observers, and not only of linear ones. Moreover, an algorithm and a structure for simultaneous observers are proposed for the solution of the problem. Using an special normal form the solution of the problem can be completely and easily obtained for the case of two systems.

A second objective is also to obtain an ”intuitive” characterization of the conditions for simultaneous observability in system theoretic terms, and not only as algebraic conditions. We show that simultaneous observation conditions are equivalent to an easy to grasp detectability condition, that generalizes detectability for a single system. Moreover, it is shown that the simultaneous observation problem is intimately related to the problem of unknown input observation, a classical observation problem. In fact the simultaneous observation of N systems is reduced to the problem of constructing a simultaneous observer for a family of $N-1$ systems, but with *unknown inputs*. For $N > 2$ there is no a general solution for such a problem.

Let $\lambda(A)$ be the set of eigenvalues of A, and $\mathbb{C}^-$, and $\mathbb{C}_0^+$ the open left and the closed right halfplanes of the complex plane, respectively.

2 Preliminaries

The classical theory of observers is concerned with the reconstruction of the state from the input and output of the system. For LTI systems it is a well-known fact that the necessary and suf£cient condition for the existence of such a classical observer is detectability. When the input is not completely available for measurement the existence conditions for an unknown input observer (UIO) are more restrictive than detectability: a necessary and suf£cient condition has been termed strong* detectability by Hautus [2].

Consider the LTI continuous time system

$$\Sigma : \begin{cases} \dot{x} = Ax + Bu + Dw\ , \quad x(0) = x_0 \\ y = Cx \\ z = Ex \end{cases} , \qquad (1)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^p$, $w \in \mathbb{R}^q$, $y \in \mathbb{R}^m$, and $z \in \mathbb{R}^r$ are the state, the (known) input, the unknown input, the output vectors, and the functional of the system, respectively.

The typical observer design problem is to construct a *Functional Observer (FO)* for system (1) when there are only known inputs (u), i.e $D = 0$. If unknown inputs (w) exist, then an important problem is the design of a functional observer that is robust against the unknown signals, i.e. a so called *unknown input functional observer (UIFO)*. A *(UI)FO* is a dynamic system that, with the information of the input $u(t)$, and of the output $y(t)$ (without information of the derivatives, and of the unknown inputs if they exist) can reconstruct asymptotically the functional $z(t)$. We will consider in particular linear (UI)FO.

I.e. a *linear (UI)FO* for system (1) is a LTI £nite dimensional system

$$\Omega : \begin{cases} \dot{\xi} = F\xi + Gu + Hy\ , \quad \xi(0) = \xi_0 \\ \hat{z} = J\xi + Ky\ , \end{cases} \tag{2}$$

where $\xi \in \mathbb{R}^s$ is the state vector; F, G, J, H and K, are constant matrices of appropriate dimensions.

De£nition 1 *System Ω (2) is a* linear FO *for system (1) with $D = 0$ if $\forall u$, $\forall x_0$ and $\forall \xi_0$ it is satis£ed that*

$$\lim_{t\to\infty} (\hat{z}(t) - z(t)) = 0\ . \tag{3}$$

System Ω is a linear UIFO *for Σ (1) if $\forall u$, $\forall w$, $\forall x_0$ and $\forall \xi_0$ (3) is satis£ed.*

Existence conditions for such observers can be given in system theoretic terms (see [2]):

De£nition 2 *System (1) is* z-detectable[1] *if*

$$\lim_{t\to\infty} y(t) = 0 \Longrightarrow \lim_{t\to\infty} z(t) = 0 \tag{4}$$

for any initial state x_0, and for zero known and unknown inputs, i.e. $u(t) = 0$, and $w(t) = 0$.
System (1) is strong* z-detectable *if (4) is satis£ed for zero known input, i.e. $u(t) = 0$, and irrespective of the unknown input w and of the initial state x_0.*

The relationship between (strong*) z-detectability and the existence of a (UI)FO for system Σ is clari£ed by the following result:

Lemma 3 *[2, 5] System Σ has an* (unknown input) functional observer ((UI)FO) *if and only if it is* (strong*) z-detectable. *Moreover, if a (UI)FO exist, then there is a linear one.*

If Σ is brought to a special form, then necessary and suf£cient conditions for existence of Observers can be very easily stated, and their construction is very transparent.

Theorem 4 *[6, 3, 5]. For system Σ (1) suppose that* rk $(D) = q$, rk $(C) = m$. *There exist state $x \to Px$, output $y \to Qy$ and input $w \to Rw$ transformations such that, in the new coordinates, Σ takes the form (Σ_{fn})*

$$\begin{aligned}
\dot{x} &= \begin{bmatrix} A_{11} & 0 & 0 & 0 & A_{15}C_2 \\ A_{21}C_1 & A_{22} & 0 & 0 & A_{25}C_2 \\ A_{31}C_1 & 0 & A_{33} & 0 & A_{35}C_2 \\ A_{41}C_1 & D_2A_{42} & D_2A_{43} & A_{44} & A_{45}C_2 \\ D_1A_{51} & D_1A_{52} & D_1A_{53} & D_1A_{54} & A_{55} \end{bmatrix} x + \\
&+ \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \end{bmatrix} u + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & D_2 \\ D_1 & 0 \end{bmatrix} w\ , \\
y &= \begin{bmatrix} C_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & C_2 \end{bmatrix} x\ , \\
z &= \begin{bmatrix} E_1 & E_2 & E_3 & E_4 & E_5 \end{bmatrix} x\ ,
\end{aligned} \tag{5}$$

and satis£es properties (i)-(vi). Denote as $n_i = \dim(A_{ii})$, $i = 1, \cdots, 5$, $q_j =$ rk (D_j), $m_j =$ rk (C_j), $j = 1, 2$.

(i) Matrices C_1, C_2, D_1 and D_2 are full rank.

(ii) Transmission zeros of Σ are $\lambda(A_{22}) \cup \lambda(A_{33})$.

(iii) $\lambda(A_{22}) \subset \mathbb{C}^-$ and $\lambda(A_{33}) \subset \mathbb{C}_0^+$.

(iv) The couples (C_1, A_{11}) and (C_2, A_{55}) are observable. The couples (A_{44}, D_2) and (A_{55}, D_1) are controllable.

(v) $q_1 =$ rk $(D_1) =$ rk $(C_2) = m_2$.

(vi) For every K the couple $(C_2, A_{55} - D_1K)$ is observable, i.e. system (C_2, A_{55}, D_1) is perfectly observable.

Note that in the classical case, when there are only known inputs ($D = 0$), the normal form (5) reduces to

$$\begin{aligned}
\dot{x} &= \begin{bmatrix} A_{11} & 0 & 0 \\ A_{21}C_1 & A_{22} & 0 \\ A_{31}C_1 & 0 & A_{33} \end{bmatrix} x + \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} u\ , \\
y &= \begin{bmatrix} C_1 & 0 & 0 \end{bmatrix} x\ , \\
z &= \begin{bmatrix} E_1 & E_2 & E_3 \end{bmatrix} x\ .
\end{aligned} \tag{6}$$

In this case x_2 and x_3 correspond to the unobservable states, and the system is detectable[2] iff x_3 has dimension zero.

For systems in the normal form (5) (or (6) if there are no unknown inputs), the detectability properties can be easily checked, and observers can be easily constructed:

Lemma 5 *[5] System (1) is* z-detectable *if and only if in the special normal form[3] (6) $E_3 = 0$. A (reduced order) observer is given by (7) (setting $y_2 \equiv 0$, and $y_1 = y$).*
System (1) is strong* z-detectable *if and only if in the special normal form (5) $E_3 = 0$, $E_4 = 0$, and $E_5 = MC_2$, for some*

[1] This name is not standard.

[2] Or x-detectable in the terminology of De£nition 2.

[3] Here condition $G = 0$ is also satis£ed if G has one dimension zero.

matrix M. A (reduced order) observer with the stated properties is given by

$$\begin{array}{l}\dot{\xi}_1 = A_{11}\xi_1 + A_{15}y_2 + B_1 u + H_{11}\left(C_1\xi_1 - y_1\right) ,\\ \dot{\xi}_2 = A_{22}\xi_2 + A_{25}y_2 + A_{21}y_1 + B_2 u ,\\ \hat{z} = E_1\xi_1 + E_2\xi_2 + My_2 .\end{array} \tag{7}$$

where H_{11} is such that $(A_{11} + H_{11}C_1)$ is Hurwitz.

Note that (x-)detectability is not necessary to construct a FO for z.

Remark 6 *For a z-detectable system in the normal form (6) a full order Luenberger observer can be constructed as*

$$\begin{array}{l}\dot{\xi} = \begin{bmatrix} A_{11}+H_1C_1 & 0 & 0 \\ (A_{21}+H_2)\,C_1 & A_{22} & 0 \\ (A_{31}+H_3)\,C_1 & 0 & A_{33}\end{bmatrix}\xi + \begin{bmatrix} B_1 \\ B_2 \\ B_3\end{bmatrix} u + \\ \quad - \begin{bmatrix} H_1 \\ H_2 \\ H_3 \end{bmatrix} y ,\\ \hat{z} = \begin{bmatrix} E_1 & E_2 & E_3 \end{bmatrix}\xi ,\end{array} \tag{8}$$

where H_1 is such that $(A_{11} + H_1C_1)$ is Hurwitz. Note that this observer is internally stable only if Σ is detectable. However, it can always be reduced to an internally stable one.

3 Problem formulation

Consider a family of linear time invariant systems

$$\Sigma_i : \begin{cases} \dot{x}_i = A_i x_i + B_i u_i , \quad x_i(0) = x_{i0} \\ y_i = C_i x_i , \\ z_i = E_i x_i , \; i = 1, \cdots , N \end{cases} \tag{9}$$

where $x_i \in \mathbb{R}^{n_i}$ is the state, $u_i \in \mathbb{R}^p$ the (known) input, $y_i \in \mathbb{R}^m$ the output vectors, and $z_i \in \mathbb{R}^r$ is the vector of signals to be estimated. A_i, B_i, C_i, and E_i are constant matrices of adequate dimensions. Note that the dimensions of the state variables can be different. The dimensions of the input and output vectors will be considered constant, although they can be different for different systems. In that case (dummy) inputs and/or outputs can be added to some of the systems.

The *simultaneous functional observation problem (SFOP)* consists in designing a *single* functional observer, called a *Common Functional Observer (CFO)*, such that

$$\lim_{t\to\infty} \left(\hat{z}(t, \xi_0, u_i, y_i) - z_i(t, x_{i0}, u_i)\right) = 0 . \tag{10}$$

is satis£ed for every $\forall u_i, x_{i0}, \xi_0$, $i = 1, \cdots , N$. This implies that for any plant in the set of systems given by (9) the common functional observer Ω estimates the functional asymptotically. In particular it is of interest to construct a linear CFO as (2).

4 State space characterization of simultaneous observation of two systems

In this section the simplest case, when only two plants have to be simultaneously observed, will be analyzed. The approach will be to consider £rst very intuitive necessary conditions for the existence of a common observer, expressed in terms of indistinguishability of trajectories, and then to show that they are also suf£cient.

Fact 7 *Any observer Ω (2) that is detectable is internally stable, i.e. F is Hurwitz.*

Proof. Consider any plant with input $u = 0$, and $x_0 = 0$. Then $x(t, 0, 0) = 0$, $y(t) = 0$, and $z(t) = 0$. Therefore the observer output $\hat{z}(t) \to 0$ for every observer initial state. From detectability and the observation that under these conditions the observer is an autonomous system, then F has to be Hurwitz. ■

Remark 8 *The condition for the observer to be detectable can always be satis£ed, without loss of generality.*

Now consider two systems (9) Σ_1, and Σ_2. Suppose that they have a common observer Ω, and that applying some input function $u(t)$ to both of them they produce the same output $y(t)$. Under such conditions the observer has the same inputs independently of the system it is connected to, and it produces the same output signal $\hat{z}(t)$ for the same initial condition. Since Ω is a common observer it must happen that $\hat{z}(t) \to z_1(t)$, and $\hat{z}(t) \to z_2(t)$, as $t \to \infty$. Therefore $z_2(t) \to z_1(t)$. Moreover, because of the linearity and the internal stability of the observer this has to be also true if with the same input $u(t)$ the outputs of systems Σ_1, and Σ_2 are convergent, i.e. $y_1(t) \to y_2(t)$. We have then an obviously necessary condition for systems Σ_1, and Σ_2 to have a common functional observer:

Condition 1 *Whenever for the same inputs $u_1(t) = u_2(t)$ and some initial conditions x_{10}, x_{20} the outputs of systems Σ_1, and Σ_2 converge, i.e. $y_1(t) \to y_2(t)$, then the functional outputs also converge, i.e. $z_1(t) \to z_2(t)$.*

Remark 9 *Note that with the same arguments as above an (apparently) stronger necessary condition can be obtained: $u_1(t) \to u_2(t)$ and $y_1(t) \to y_2(t)$ imply $z_1(t) \to z_2(t)$. However, it is easy to show that both conditions are equivalent.*

There is another (obvious) necessary condition for simultaneous observation

Condition 2 *Σ_i is z_i-detectable, for $i = 1, 2$.*

It will be shown (in three steps) that these two (obvious) conditions are not only necessary but also suf£cient for the existence of a common observer, and a method to construct a common (linear) observer will be derived.

4.1 First step

This step consists in reinterpreting condition 1 as a condition on Σ_p, the parallel system formed by Σ_1, and Σ_2: its input is $u(t) = u_1(t) = u_2(t)$, its state is $x = \left[x_1^T, x_2^T\right]^T$, and its output and functional are the difference of the outputs $y(t) = y_1(t) - y_2(t)$, and of the functionals $z(t) = z_1(t) - z_2(t)$, respectively, i.e.

$$\Sigma_p : \begin{cases} \dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} x + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u \,, \\ y = y_1 - y_2 = \begin{bmatrix} C_1 & -C_2 \end{bmatrix} x \,, \\ z = z_1 - z_2 = \begin{bmatrix} E_1 & -E_2 \end{bmatrix} x \,, \end{cases} \tag{11}$$

Note that Σ_p is the difference system: if the transfer matrix of Σ_i is $G_i(s)$, $i = 1, 2$, then the transfer matrix of Σ_p is $G_1(s) - G_2(s)$. It is easy to see that condition 1 is equivalent to the following property of Σ_p: Whenever for some $u(t)$, and some initial state x_0, $y(t) \to 0$ then $z(t) \to 0$. From the results in section 2 one has immediately

Lemma 10 *Consider systems Σ_1, Σ_2, and Σ_p. Consider for Σ_p the input u as an unknown one. Then the following conditions are equivalent:*

1. *Condition 1.*
2. *Σ_p is strong* z-detectable.*
3. *Σ_p has an* unknown input functional observer (UIFO) *for z.*

4.2 Second step

The following result is a generalization for (UI)FO of well known results for state observers, and is therefore interesting in itself. For a system Σ (1) consider the family of systems

$$\Sigma_{KF} : \begin{cases} \dot{x} = (A + DK + FC)\, x + Bu + Dw \\ y = Cx \\ z = Ex \end{cases} ,$$

obtained from Σ by arbitrary output injection, and/or state feedback (through the unknown input).

Lemma 11 *System Σ has an (UI)FO for z if and only if system Σ_{KF} has also an (UI)FO for z for arbitrary K, and F.*

Proof. Consider £rst the case with unknown inputs (UIFO). *Suf£ciency:* First suppose that Σ is given in its normal form (5), then, using the same transformations, Σ_{KF} can be written exactly as (5), but with matrix A replaced by $\tilde{A} = A + DK + FC$,

$$\tilde{A} = \begin{bmatrix} \tilde{A}_{11} & 0 & 0 & 0 & \tilde{A}_{15}C_2 \\ \tilde{A}_{21}C_1 & A_{22} & 0 & 0 & \tilde{A}_{25}C_2 \\ \tilde{A}_{31}C_1 & 0 & A_{33} & 0 & \tilde{A}_{35}C_2 \\ \tilde{A}_{41} & D_2\tilde{A}_{42} & D_2\tilde{A}_{43} & \tilde{A}_{44} & \tilde{A}_{45} \\ \tilde{A}_{51} & D_1\tilde{A}_{52} & D_1\tilde{A}_{53} & D_1\tilde{A}_{54} & \tilde{A}_{55} \end{bmatrix} ,$$

with $\tilde{A}_{11} = A_{11} + F_{11}C_1$, $\tilde{A}_{21} = A_{21} + F_{21}$, $\tilde{A}_{31} = A_{31} + F_{31}$, $\tilde{A}_{41} = (A_{41} + F_{41})\, C_1 + D_2K_{21}$, $\tilde{A}_{51} = D_1(A_{51} + K_{11}) + F_{51}C_1$, $\tilde{A}_{42} = A_{42} + K_{22}$, $\tilde{A}_{52} = A_{52} + K_{12}$, $\tilde{A}_{43} = A_{43} + K_{23}$, $\tilde{A}_{44} = A_{44} + D_2K_{24}$, $\tilde{A}_{53} = A_{53} + K_{13}$, $\tilde{A}_{54} = A_{54} + K_{14}$, $\tilde{A}_{45} = (A_{45} + F_{42})\, C_2 + D_2K_{25}$, $\tilde{A}_{35} = A_{35} + F_{32}$, $\tilde{A}_{25} = A_{25} + F_{22}$, $\tilde{A}_{15} = A_{15} + F_{12}$, $\tilde{A}_{55} = A_{55} + F_{52}C_2 + D_1K_{15}$, and where F_{ij}, and K_{ij} are block matrices of F and K, respectively. Note that if Σ has an UIFO for z then, because of lemma 5, $E_3 = 0$, $E_4 = 0$, and $E_5 = MC_2$. And so these are also satis£ed for Σ_{KF}. Note that since (A_{11}, C_1) is observable, then so is $\left(\tilde{A}_{11}, C_1\right)$. Therefore, system

$$\begin{aligned} \dot{\xi}_1 &= \tilde{A}_{11}\xi_1 + \tilde{A}_{15}y_2 + B_1u + H_{11}(C_1\xi_1 - y_1) \,, \\ \dot{\xi}_2 &= A_{22}\xi_2 + \tilde{A}_{25}y_2 + \tilde{A}_{21}y_1 + B_2u \,, \\ \hat{z} &= E_1\xi_1 + E_2\xi_2 + My_2 \,, \end{aligned}$$

where H_{11} is such that $\left(\tilde{A}_{11} + H_{11}C_1\right)$ is Hurwitz, is an UIFO for Σ_{KF}. *Necessity:* Note that Σ can be obtained from Σ_{KF} by output injection, and/or state feedback. So if the previous arguments are applied initiating with Σ_{KF}, then the necessity part follows.

The case for known inputs follows easily from the previous result by noting that in that case the blocks 4 and 5 in the normal form do not exist. ■

Note, however, that systems Σ, and Σ_{KF} do not necessarily share the same (UI)FO. It is easy to see, from the previous arguments, that suf£cient conditions on F and K for systems Σ, and Σ_{KF} to share an UIFO is that (when Σ is in the normal form): $F_{22} = 0$, $F_{21} = 0$, $F_{12} = 0$, and $A_{11} + (F_{11} + H_{11})\, C_1$ is Hurwitz.

4.3 Third step

Now we are in position to construct a simultaneous observer for systems Σ_1, and Σ_2 when conditions 1, and 2 are satis£ed.

If condition 2 is satis£ed, then, by Lemma 3 a functional observer can be constructed for each Σ_1 and Σ_2. Note that this condition is symmetric with respect to Σ_1 and Σ_2, and so it is irrelevant if in the coming arguments one takes either one. Let us take Σ_1, and construct a full order observer Ω_{1f} of the form (cfr. remark 6)

$$\Omega_{1f} : \begin{cases} \dot{\hat{x}}_1 = A_1\hat{x}_1 + B_1u_1 + F_0(\hat{y}_1 - y_1) \,, \\ \hat{y}_1 = C_1\hat{x}_1 \,, \\ \hat{z}_1 = E_1\hat{x}_1 \,, \end{cases} \tag{12}$$

for some matrix F_0.

Now assume that condition 1 is satis£ed. Then an UIFO exists for Σ_p and for Σ_{pKF}, for arbitrary K, and F. Consider an UIF observer designed for system Σ_{p0}, i.e. system Σ_{pKF}, when $K = 0$, and $F = \left[F_0^T \;\; 0\right]^T$, the UIFO Ω_{20}, that is internally stable. Now connect both observers, Ω_{1f}, and Ω_{20} as shown in £gure 1, and denote it as Ω_c.

Now it will be shown that if the plant is either Σ_1, or Σ_2 the observer Ω_c is a common one. Suppose £rst that the

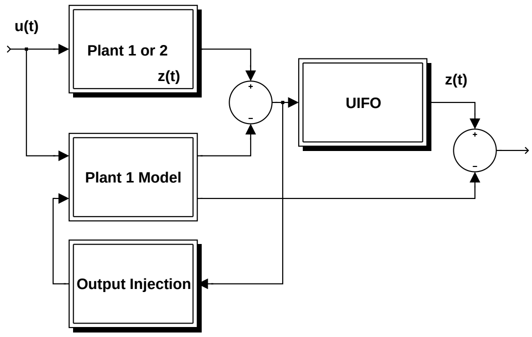

Figure 1: Block diagram of the structure of a common observer

plant is system Σ_1. Due to the properties of Ω_{1f} both $\lim_{t\to\infty}(\hat{z}_1(t) - z_1(t)) = 0$, and $\lim_{t\to\infty}(\hat{y}_1(t) - y_1(t)) = 0$. And so the input to Ω_{20}, i.e. $(\hat{y}_1(t) - y_1(t))$, converges to zero. Since Ω_{20} is internally asymptotically stable, its output converges to zero. Therefore, the estimation of $z(t)$ converges to $z_1(t)$, showing that Ω_c works properly. Now suppose that the plant is Σ_2. Observer Ω_{1f} will not (usually) converge, and its estimation of the functional is given by $\check{z}_1(t)$. However, the system composed by plant and the observer Ω_{1f} is exactly system Σ_{p0}. Since Ω_{20} is an UIFO for Σ_{p0} the output of this last system converges to $\check{z}_1(t) - z_2(t)$. It follows that $z(t)$ converges to $z_2(t)$, showing that in this case Ω_c also estimates correctly the functional.

When Σ_1 is detectable Ω_{1f} is internally stable, and so is Ω_c. If Σ_1 is not detectable, then Ω_c is not internally stable, but can be reduced to an internally stable observer. Therefore, conditions 1 and 2 are suf£cient for the existence of a common observer. Moreover, they give a very intuitively appealing structure and design method for such an observer. One has then the

Theorem 12 *The following statements are equivalent:*

1. *Systems Σ_1 and Σ_2 have a (internally stable) common (functional) observer*

   (a) *there exists a (functional) observer for one of the plants, and*

   (b) *the parallel system Σ_p has an UIFO.*

2. *Systems Σ_1 and Σ_2 have a linear (internally stable) common (functional) observer*

Proof. The equivalence of (2) and (3) has been proved in the previous paragraphs, except for the fact that the existence of a FO has to be checked for one system and not for both of them. This last result follows immediately from the construction of the common observer, since if Σ_1 has a FO and Σ_p has an UIFO, then Σ_2 has a FO. (3) $\Rightarrow$ (1) is trivial. That (1) $\Rightarrow$ (2) follows from the observation that Condition 1 has to be satis£ed no matter how the observer is constructed, i.e. if this condition is not satis£ed, then there is no common observer of any kind. ∎

Algorithm 13 *If systems Σ_1 and Σ_2 have a common (functional) observer, then a linear one can be designed in the following manner:*

1. *Design for Σ_1 a full order functional observer Ω_{1f} (this is possibly not internally stable).*
2. *Design for system Σ_{p0} an UIFO Ω_{20}. This can be done using Lemma 5.*
3. *Connect both systems as shown in £gure 1.*
4. *If Ω_{1f} is not internally stable, reduce it by eliminating its not detectable part.*

Remark 14 *Note that theorem 12, besides of being intuitive and natural, generalizes the known results (see [4, 8]) in different directions: (i) Previous results require the systems to be detectable and stabilizable. This is not necessary in theorem 12. This shows some lack of duality between the simultaneous observation and stabilization problems, since for the latter both detectability and stabilizability are necessary conditions! (ii) Previous results are only valid for systems with inputs. Theorem 12 is valid for both systems with and without inputs. (iii) Condition 2 above is automatically satis£ed if the systems are detectable, and therefore it does not appear in the previous results. Condition 1 was only known under the additional restriction that one of the systems is stable [8, Cor. 2. Note that its proof is incomplete, since only suf£ciency is demonstrated.]. (iv) The design of a common observer is reduced to the solution of two known problems: the design of a FO, and the design of an UIFO, for which there are algorithms to construct a solution. Moreover, the degrees of freedom in the design are clear. Note furthermore, that the structure of the proposed common observer is compatible with the general form of a linear observer for a plant given in [1] for the observation of the state. (v) It is shown that the existence of a CFO is equivalent to the existence of a linear CFO.*

Remark 15 *If systems Σ_1 and Σ_2 have an internally stable linear CFO Ω then Lemma 10 can be given an interesting interpretation: if system Σ_i is connected to Ω, that has as inputs (y_i, u_i), then its output $\hat{z}$ converges to z_i as $t \to \infty$. This is true for $i = 1, 2$, and any initial conditions x_{0i}, and ξ_0. Because of the linearity and stability of the CFO if Ω has as inputs $(y_1 - y_2, u_1 - u_2)$, then its output $\hat{z}$ converges to $z_1 - z_2$ as $t \to \infty$. Now if the inputs to systems Σ_1 and Σ_2 are selected identical, i.e. $u_1 = u_2$, then Ω with inputs $(y_1 - y_2, 0)$, estimates asymptotically $z_1 - z_2$. But this corresponds exactly to the fact that system Σ_p has an UIFO, and Ω with input $(y_1 - y_2, 0)$, is an UIFO for the parallel system Σ_p.*

5 Simultaneous observation of a (£nite) set of systems

Consider now the set of N plants (9) Σ_i, $i = 1, \cdots, N$. Corresponding to condition 1 and 2 we have then obviously necessary conditions for these systems to posses a common functional observer.

Condition 3 *Whenever for the same inputs* $u_j(t) = u_k(t)$ *and some initial conditions* x_{j0}, x_{k0} *the outputs of systems* Σ_j, *and* Σ_k *converge, i.e.* $y_j(t) \to y_k(t)$, *then the functional outputs also converge, i.e.* $z_j(t) \to z_k(t), \forall j, k \in \{1, 2, \cdots, N\}$, $j \neq k$.

De£ning the parallel system Σ_{pij} as the one formed by parallel connection of systems i and j, one has from Condition 3 and Lemma 10 that $\forall j, k \in \{1, 2, \cdots, N\}$, $j \neq k$, Σ_{pjk} has an UIFO. Under the assumption that the family of plants have a linear and internally stable CFO Ω an stronger necessary condition can be deduced. For this take systems Σ_i and Σ_j, for any $i, j \in \{1, 2, \cdots, N\}$, $i \neq j$. By the arguments in remark 15 it follows that Ω with input $(y_i - y_j, 0)$, is an UIFO for the parallel system Σ_{pij}. This shows the necessity of the following condition, when a linear CFO exists:

Condition 4 *The parallel systems* Σ_{pij} *have a* common (linear) unknown input functional observer $\forall i, j \in \{1, 2, \cdots, N\}$, $i \neq j$.

Corresponding to condition 2 there is the following necessary condition:

Condition 5 Σ_i *is* z_i*-detectable, for* $i = 1, 2, \cdots, N$.

Theorem 16 *Systems* Σ_i *for* $i = 1 \cdots N$ *(9) have a common linear (functional) observer if and only if (1) there exists a (functional) observer for one of the plants, and (2) the parallel systems* Σ_{p1j}, $j = 2, \cdots, N$ *have a common UIFO.*

Proof. *Suf£ciency:* By the same procedure as in the two systems case (see Step 3) one common observer for all N systems can be constructed, if the hypothesis of the theorem are satis£ed. Note that the common observer has the same structure given in Figure 1.
Necessity: Use the same arguments as the ones used to show the necessity of Condition 4. ■

Note that the main issue in this case is the construction of a *common* UIFO for $N - 1$ parallel systems.There is no yet an effective method to design such a common UIFO. Once a common UIFO is known, then an procedure similar to Algorithm 13 can be used to construct a CFO for the family of plants. And so the problem of the construction of a CFO for a family of N plants is reduced to the construction of a *common Unknown Input* functional observer for a family of $N - 1$ plants, a problem of a higher level of dif£culty but in a lower number of systems.

6 Conclusions

In this paper a state space characterization for simultaneous observability of several linear plants has been obtained. Generalization of previous results is obtained and an algorithm and a structure for simultaneous observers are proposed. Moreover, an ”intuitive” characterization of these conditions is given in terms of a detectability condition. It is also shown that the simultaneous observation problem is intimately related to the problem of unknown input observation, a classical observation problem. For the simultaneous observation of several plants the presentation is very brief, and generalizes the previous results.

Acknowledgements: This work has been done with the £nancial aid of Conacyt under project 34934A.

References

[1] Goodwin, G. C.; Middleton, R. H. (1989). The class of all stable unbiased state estimators. *Systems & Control Letters*, 13, pp. 161–163.

[2] Hautus, M. L. J. (1983) Strong detectability and observers. *Linear Algebra and its Applications*, 50, pp. 353-368.

[3] Knobloch, H.W. and Kwakernaak, H. (1985). *Lineare Kontrolltheorie*. Berlin, Springer-Verlag.

[4] Kovacevic, R.; Yao, Y. X.; Zhang, Y. M. (1996) Observer parametrization for simultaneous observation. *IEEE Trans. Autom. Control*, 41, pp. 255–259.

[5] Moreno, J. (2001). Quasi-unknown input observers for linear systems. *Proceedings of the 2001 IEEE International Conference on Control Applications* pp. 732–737.

[6] Sannuti, P. and Saberi, A. (1987) Special coordinate basis for multivariable linear systems - £nite and in£nite zero structure, squaring down and decoupling. *Int. J. Con.* 45, pp. 1655-1704.

[7] Vidyasagar, M. (1988) A state-space interpretation of simultaneous stabilization. *IEEE Trans. Autom. Control*, 33, pp. 506–508.

[8] Yao, Y. X.; Darouach, M.; Schaefers, J. (1995) Simultaneous observation of linear systems. *IEEE Trans. Autom. Control*, 40, pp. 696–699.