A METHOD FOR THE IDENTIFICATION OF HEAT TRANSFERS ON THE SURFACE OF A MATERIAL : APPLICATION TO A PLASMA ASSISTED CHEMICAL VAPOUR DEPOSITION PROCESS

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Abstract: In this paper, a method for the identification of heat transfers on the surface of a material is investigated. From an experimental situation (the elaboration of amorphous hydrogenated silicon carbide coatings with a Plasma Assisted Chemical Vapour Deposition PACVD process), a thermal system described by a set of non linear partial differential equations is considered. From temperature measurements, the thermal effect of a the plasma surrounding is identified. Several numerical aspects of the resolution of a such an inverse problem are presented.

Keywords: Inverse dynamic problem, Identification Algorithm, Partial differential equation, Sensitivity analysis, Non-linear systems

1. Introduction

This paper is dedicated to the problem of determining the heat flux on the surface of a steel sample by means of the time history of measured temperatures. The problem of determining the input signal of a dynamic system when only the output is known, is known as inverse problem. An inverse problem is well-posed if it satisfies the three Hadamard conditions of existence, uniqueness and stability. If any of the previous conditions are not satisfied, then the problem is ill-posed. For continuous diffusive systems such the thermal process studied in this paper, the inverse operator is ill-conditioned and the presence of measurement noise in the actual data makes the problem instable, the inverse problem is then ill posed. The regularisation is one way of handling the ill-posedness of the inverse problem.

In the context of inverse heat conduction problems, a regularisation procedure has been introduced by Alifanov, *et al.* [1] : an iterative regularisation scheme with the *gradient* methods. In this paper, the conjugate gradient method is proposed for the resolution of the inverse problem and leads to iterative resolution of three well conditioned problem: direct problem, sensitivity problem, adjoint problem.

In the following paragraph, the experimental situation is exposed and modeled. Numerical results are exposed. Then, the inverse problem is stated and the conjugate gradient method is presented. Sensitivity analysis leads to the determination of a strategy for sensor location. Numerical results for a simulated experiment are briefly exposed and the interest of the algorithm is discussed.

2. The Plasma Assisted Chemical Vapour Deposition (PACVD) process

Elaboration of amorphous hydrogenated silicon carbide coatings are interesting for their mechanical properties such as hardness, low friction coefficient. Such coatings can be elaborated with PACVD techniques which are developed since the early eighties in many high technological domains such as microelectronics, optics or aeronautics. Amorphous Hydrogenated Silicon Carbide coatings for tribological applications are realised at the IMP-CNRS with a PACVD reactor (see figure 1). The experimental setup is detailed elsewhere[2].

The PACVD process allows to elaborate coatings at low temperature range (300 K ⇔ 1100 K)[3]. In order to ensure a better reactivity of the surface of the substrate to the reactive species, an heating element embedded in a ceramic material heats the back of the substrate. The heating element is connected to a PID controller. For technological reasons, the temperature measurements can not be performed during elaboration of coatings. This is an important limitation of the PID efficiency.

Figure 1 : *PACVD reactor in the IMP CNRS*

Reproducibility of specific coatings is very difficult to ensure due to the large number of experimental parameters. It is well known that the temperature of the substrate (where the coating occurs) is a key-parameter for the mechanical properties of Silicon Carbide films[4]. However measurements are difficult to obtain because of experimental conditions. Then, an efficient predictive tool is essential in order to determine the evolution of the substrate temperature. A three dimensional thermal modelling has been proposed in a previous work in order to estimate the thermal effect of the heating element on the substrate (without the plasma surroundings)[5]. Several geometries of the substrate holder are shown on figure 2 ; due to the shape of the sample, 3D configuration is needed.

Figure 2: *Ceramic, Substrate holder and substrates*

A method to estimate the heat flux due to the plasma surroundings is presented in this communication. In the following paragraph, the model is briefly exposed.

3. *Thermal modelling*

3.1 Notations and equations

Let us denote by:

- $x=(x_1,x_2,x_3)\in\Omega\subset\mathbb{R}^3$, the space variable. The surface of Ω is : $\Gamma=\underset{j}{\cup}\Gamma_j$.
- $t\in T=\left[0,t_f\right]$ is the time variable.
- $\theta(x,t)$ is the temperature and the initial temperature is θ_{ini} (assumed to be uniform).
- The thermophysical properties of the steel substrate are denoted by : $\rho(\theta)$ the mass density, $C_p(\theta)$ the specific heat, $\varepsilon(\theta)$ the emissivity and $\lambda(\theta)$ the thermal conductivity.
- $\phi_p(x,t)$ the heat flux due to the plasma surroundings which depends on time and space because of electrons collisions on boundary Γ_1.
- ϕ_{thc} the heat flux corresponding to heat losses due to thermal contact between the ceramic and the substrate holder.

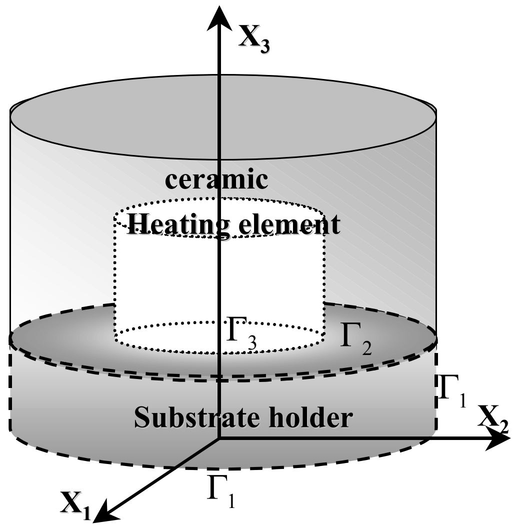

Figure 3: *Domain of interest, full substrate holder and ceramic.*

The figure 3 is the domain of study, the top is the ceramic and the bottom the full substrate holder. Instead of using the real substrate holder which is complicate to grid (figure 2), a full substrate holder is used to realise the numerical study. The boundary conditions are presented on the figure 4 and wrote in this paragraph.

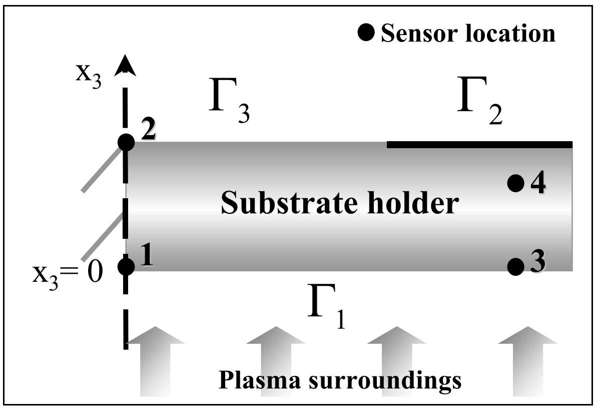

Figure 4 : *Domain of investigation : substrate holder (and sensors locations).*

The thermal evolution of the material during the ignition of the plasma surroundings is described by the following equations:

- the state equation: $\forall(x,t)\in\Omega\times T$

$$\rho(\theta)C_p(\theta)\frac{\partial\theta}{\partial t}-\nabla(\lambda(\theta)\nabla\theta)=0 \qquad \text{<1>}$$

- the initial condition: $\forall x\in\Omega$

$$\theta(x,t=0s)=\theta_{ini} \qquad \text{<2>}$$

- the heat transfer on Γ_1, see figure 4, due to the plasma surroundings :

$$\forall(x,t)\in\Gamma_1\times T$$

$$-\lambda(\theta)\frac{\partial\theta}{\partial\vec{n}_1}=\phi_p(x,t) \qquad \text{<3>}$$

where $\vec{n}_i$ is the normal exterior vector to the surface Γ_i.

- the heat transfer on Γ_2, see figure 4, due to the contact between the substrate and the ceramic: $\forall(x,t)\in\Gamma_2\times T$

$$-\lambda(\theta)\frac{\partial\theta}{\partial\vec{n}_2}=\phi_{thc} \qquad \text{<4>}$$

- the heat transfer on Γ_3 corresponding to the radiative exchange between the back of the full substrate holder and the exterior environment at temperature θ_{env} :

$$\forall(x,t)\in\Gamma_3\times T$$

$$-\lambda(\theta)\frac{\partial\theta}{\partial\vec{n}_3}=\varepsilon(\theta)\sigma(\theta^4-\theta_{env}^4) \qquad \text{<5>}$$

The direct problem can be formulated as follows:

Problem: P_{dir}
Find the temperature $\theta(x,t)$ solution of the system of non linear partial differential equations:
$(P_{dir})\begin{cases}\text{the state equation <1>}\\ \text{the initial condition <2>}\\ \text{the boundaries conditions <3,4,5>}\end{cases}$

The value of the thermophysical properties of the 304L steel are given in the following table:

mass density $\rho(\theta)$ $[kg.m^{-3}]$	$8158.6-0.672\theta$
Specific heat $C_p(\theta)$ $[J.kg^{-1}.K^{-1}]$	$436+0.1465\theta$ $t<573K$ $391+0.225\theta$ $573K\le t<773K$ $275.1+0.375\theta$ $773K\le t<973K$
Thermal conductivity $\lambda(\theta)$ $[W.m^{-1}.K^{-1}]$	$10.76+0.0128\theta$
Emissivity $\varepsilon(\theta)$	$0.156+0.00008\theta$

Table 1: *thermophysical properties of 304 L steel*

If both the boundary conditions and the thermophysical properties of the material are known, the resolution of the direct problem leads to the determination of the temperature evolution on the substrate surface. Numerical resolution is performed by a finite element method which is efficient for such a non linear system of partial differential equations in a 3D geometry.

3.2 Resolution of the direct problem

In this paragraph, numerical results are shown while the heat flux due to the plasma surrounding is assumed to be known (an arbitrary flux is considered). According to the following formulation of $\phi_p(x,t)$, a preliminary step is considered and the axis symmetry of the electronic distribution is taken into account :

$$10^4\exp\left[-\frac{\sqrt{x_1^2+x_2^2}}{r_{\max}}\right]\sqrt{\frac{t}{600}} \qquad \text{<6>}$$

for $t\le 600s$

$$10^4\exp\left[-\frac{\sqrt{x_1^2+x_2^2}}{r_{\max}}\right] \qquad \text{<7>}$$

for $t>600s$.

The time interval is $T=[0,1800]$. On the following figures (5 and 6), the spatial distribution of the temperature at the end of the simulation is presented (Modulef finite element software is used). The known parameters as $\lambda(\theta)$, $\rho(\theta)$, $C(\theta)$ and $\varepsilon(\theta)$ are given from the literature for stainless steel 304L and ϕ_{thc} has been estimated experimentally. On the two following figures, a quadrant is considered ; the bright domain (centre and border of the full substrate holder) corresponds to hot temperature whereas the dark domain corresponds to cold temperature.

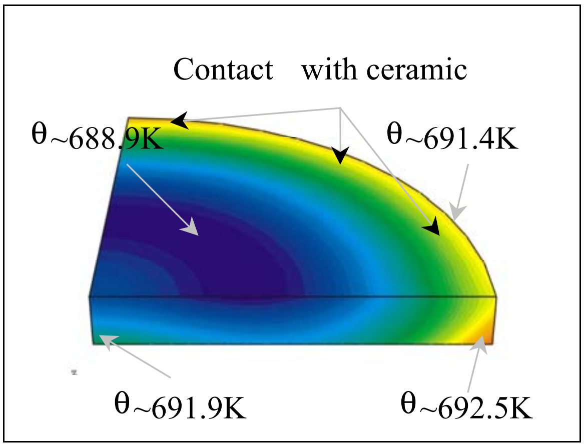

Figure 5: *back side of the full substrate holder*

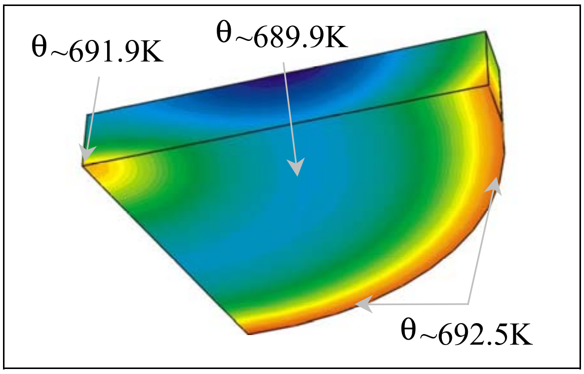

Figure 6: *front side of the full substrate holder*

On the *figure 5,* heat losses by radiative exchanges are shown. On the circular edge of the surface, heat losses are less important since the sample is in contact with the ceramic.

The *figure 6* is the front side where is applied the heating flux $\phi_p(x,t)$ (due to the plasma). For this arbitrary given flux, the temperature is practically uniform in the full substrate holder (less than 3K on the front surface where the film grows). The temperature evolution is shown on figure 7 ; temperature differences between each sensors is less than $5K$ and thermal equilibrium is not obtained after 1800s. The weak difference of temperature between the sensors is due to the little dimension of the domain (with a good thermal conductivity) which is plunged in a plasma medium whom heats quickly all the full substrate holder. The plasma medium is applicated on the boundary named Γ_1, see figure 4, which is the biggest area of the full substrate holder.

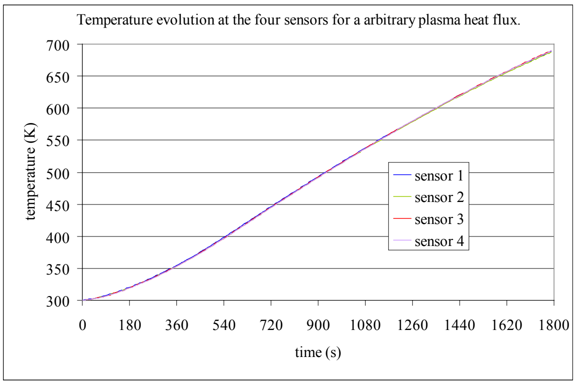

Figure 7: *temperature evolution at the four sensors*

In order to estimate the temperature of the substrate during the PACVD process, the heat flux due to the plasma ignition has to be identified. An inverse problem is formulated in order to identify the unknown boundary condition.

4. *Inverse Problem*

The unknown heat flux is assumed to be space and time dependent. In fact, in a plasma medium, the heat is created by collisions between electrons and atoms or molecules. It is usual to consider that the electronic distribution in a plasma is a gaussian distribution (probability density) where the most part of electrons are located in the center of the plasma and are decreased versus the distance. This assumptions are comforted by bibliography where temperature of substrate are measured during microwave plasma treatment[6,7].

The boundary condition on Γ_1 is written in equation <3> and $\phi_p(x,t)$ is formulated as a product of continuous piecewise linear functions see [1] (such a formulation is quite general) :

$$\phi_p(x,t) = \left(\sum_{i=1}^{Nx} \varphi_i \xi_i(x)\right)\left(\sum_{j=1}^{Nt} \varphi_j \xi_j(t)\right)$$

According to this notation $\phi_p(x,t)$ is known when all the parameters :

$$\varphi_i, i = 1,\dots,Nx, Nx+1,\dots,Nx+Nt$$

are identified. The identification is realised, when the vector $\overline{\phi}_p = (\varphi_i)_{i=1,\dots,Nx+Nt}$ is found. The functions $\xi_i(x)$ and $\xi_j(t)$ are piecewise linear function defined as follow:

$\xi_i(x_j) = \{1 \text{ if i=j else 0;i,j=1,N}_x\}$ where N_x is the number of space interval and $\xi_i(t_j) = \{1 \text{ if i=j else 0;i,j=1,N}_t\}$, N_t the number of time interval.

The inverse problem consists in the minimization of a quadratic criterion and is defined as follows:

Find $\overline{\phi}_p \in R^{Nx+Nt}$ which minimizes the cost function:

$$J(\theta,\overline{\phi}_p) = \frac{1}{2}\int_T \sum_{k=1}^{Ns}\left(\theta(x^k,t,\overline{\phi}_p) - \hat{\theta}^k\right)^2 dt$$

where Ns is the number of sensor, $\hat{\theta}_k$ the measured temperature at the sensor k at the location x_k at instant t and with the constraint: the temperature $\theta(x_k,t,\overline{\phi}_p)$ is solution of the problem P_{dir} .

The previous inverse problem is ill- posed. The regularisation method is the conjugate gradient method which is useful to solve non linear system of PDE with a large number of unknown parameters. The algorithm of minimisation is :

General conjugate gradient algorithm :

1. initialisation $n=0$; let us denote by $\overline{\phi}_p^0$ the given initial approximation of $\overline{\phi}_p$ and

$$\overline{d}^0 = -\nabla J\left(\overline{\phi}_p^0\right) = -\left(\left(\frac{\partial J}{\partial \varphi_i^0}\right)_{i=1,\cdots,Nx+Nt}\right)\left(\overline{\varphi}_p^0\right)_i$$

the initial descent direction,

2. at iteration n, from point $\overline{\phi}_p^{n-1}$, the next point is obtained : $\overline{\phi}_p^n = \overline{\phi}_p^{n-1} + \gamma^n \overline{d}^n$ where γ^n is computed by estimating the value of γ which minimize the following function:

$$\gamma = \arg\min_{\gamma \in R}\left(J\left(\overline{\phi}_p^n + \gamma \overline{d}^n\right)\right)$$

3. the next direction is defined by : $\overline{d}^n = -\nabla J\left(\overline{\phi}_p^n\right) + \beta^n \overline{d}^{n-1}$ with :

$$\beta^n = \left\|\nabla J\left(\overline{\phi}_p^n\right)\right\|^2 / \left\|\nabla J\left(\overline{\phi}_p^{n-1}\right)\right\|^2$$

4. stopping of the iterative process if $J\left(\overline{\phi}_p^n\right) \le J_{stop}$ or : $n \leftarrow n+1$ and go to 2 .

J_{stop} is a positive scalar which can be chosen according to the variance of the temperature measurement errors in order to avoid unstable solutions ; see the iterative regularizing principle[4].

The descent method needs to calculate the gradient of the quadratic criterion defined as follows:

$$\nabla J\left(\theta, \overline{\phi}_p^n\right) = \left(\frac{\partial J\left(\theta, \overline{\phi}_p^n\right)}{\partial \varphi_i}\right)_{i=1,\ldots,Nx+Nt}$$

The calculation of the gradient is performed by introducing the following functional corresponding to the minimisation of $J\left(\theta, \overline{\phi}_p\right)$ under the constraint : temperatures $\theta\left(x_k, t, \overline{\phi}_p\right)$ satisfy the state equation .

$$\mathcal{L}\left(\theta, \overline{\phi}_p, \psi\right) = J\left(\theta, \overline{\phi}_p\right) + \ldots$$

$$.. + \int_0^{t_f}\int_\Omega \left(\rho C_p \frac{\partial \theta}{\partial t} - \nabla\left(\lambda \nabla \theta\right)\right)\psi dt d\Omega$$

where $\psi(x,t) \in \Omega \times T$ is the lagrangian function.

When $\theta(x,t)$ is solution of P_{dir}, we have $\mathcal{L}\left(\theta, \overline{\phi}_p, \psi\right) = J\left(\theta, \overline{\phi}_p\right)$ and the lagrangian differential is equal to : $\delta\mathcal{L}\left(\theta, \overline{\phi}_p, \psi\right) = \delta J\left(\theta, \overline{\phi}_p\right)$. To estimate the gradient of $J\left(\theta, \overline{\phi}_p\right)$, we write that the lagrangian derivative is related to $\nabla J\left(\theta, \overline{\phi}_p\right)$ by the following expression :

$$\delta\mathcal{L}\left(\theta, \overline{\phi}_p, \psi\right) = \left\langle \nabla J\left(\theta, \overline{\phi}_p\right), \delta\overline{\phi}_p \right\rangle \qquad \text{<8>}$$

Where $\langle .,. \rangle$ is the inner product in $L_2\left(\Gamma_1 \times T\right)$ ($L_2(.)$ is the space of integrable square functions). $\psi(x,t)$ is chosen in order to have :

$$\frac{\partial \mathcal{L}}{\partial \theta}\delta\theta = 0 \;\forall\; \delta\theta \qquad \text{<9>}$$

in the lagrangian differential :

$$\partial\mathcal{L}\left(\theta, \phi_p, \psi\right) = \frac{\partial \mathcal{L}}{\partial \theta}\delta\theta + \frac{\partial \mathcal{L}}{\partial \phi_p}\delta\phi_p$$

$\delta\theta$ is a temperature variation and $\delta\phi_p$ a variation of the unknown heat flux. From equation <9>, adjoint equations are established by developing the lagrangian differential :

- ***Adjoin equations***

$$\left(P_{adj}\right)\begin{cases} -\rho(\theta)Cp(\theta)\dfrac{\partial \psi}{\partial t} - \lambda(\theta)\Delta\psi = -E_k \text{ in } \Omega \times \mathrm{T} \\ \psi\left(x, t_f\right) = 0 \text{ in } \Omega \\ -\lambda(\theta)\dfrac{\partial \psi}{\partial \vec{n}_1} = 0 \text{ on } \Gamma_1 \times T \\ -\lambda(\theta)\dfrac{\partial \psi}{\partial \vec{n}_2} = 0 \text{ on } \Gamma_2 \times T \\ -\lambda(\theta)\dfrac{\partial \psi}{\partial \vec{n}_3} = \sigma\left[4\varepsilon(\theta)\theta^3 + \dfrac{\partial \varepsilon(\theta)}{\partial \theta}\left(\theta^4 - \theta_{env}^4\right)\right]\psi \text{ on } \Gamma_3 \times T \end{cases}$$

where the function $E_k(t) = \int_\Omega \sum_{k=1}^{Ns}\left(\theta\left(x^k, t, \overline{\phi}_p\right) - \hat{\theta}^k\right)\delta\left(x - x^k\right)d\Omega$ with $\delta(.)$ the Dirac function. For more explanation on the formulation of the adjoin problem, the reader can see [1] for the method.

The gradient expression is calculated from the equation <8> and the second member of the lagrangian differential [1]:

$$\nabla J\left(\theta, \overline{\phi}_p\right) = \left(\frac{\partial J\left(\theta, \overline{\phi}_p\right)}{\partial \varphi_i}\right)_{i=1,Nx+Nt} = \int_T\int_{\Gamma_1} \psi \xi_i dt d\Gamma_1$$

The unknown heat flux is assumed to be a product of two functions : one depending on space and the other on time : $\phi_p(x,t) = f(x).g(t)$. Then the differential of is: $\delta\phi_p(x,t) = f(x).\delta g(t) + g(t).\delta f(x)$. The gradient is written as follows :

$$\nabla J_{f(x)}\left(\theta, \overline{\phi}_p\right) = \int_T\int_{\Gamma_1} g(t)\psi(x,t)\xi_i(x) dt d\Gamma_1$$

in order to identify the spatial distribution and

$$\nabla J_{g(t)}\left(\theta, \overline{\phi}_p\right) = \int_T\int_{\Gamma_1} f(x)\psi(x,t)\xi_i(t) dt d\Gamma_1$$

in order to identify the time distribution.

The calculation of the descent depth is realised by minimising the expression :

$$\min_{\gamma\in\mathbb{R}}\left(\frac{1}{2}\int_T\left(\sum_{j=1}^{N_s}\left(\theta\left(x_j,t;\bar{\varphi}_p^n+\gamma\bar{d}^n\right)-\hat{\theta}_j(t)\right)^2\right)dt\right) \quad <10>$$

we obtain the following expression[8] for the descent depth:

$$\gamma_k=\frac{\int_T\left(\sum_{j=1}^{N_s}\delta\theta\left(x_j,t;\bar{\phi}_p^n\right)\left(\theta\left(x_j,t;\bar{\phi}_p^n\right)-\hat{\theta}_j(t)\right)\right)dt}{\int_T\left(\sum_{j=1}^{N_s}\left(\delta\theta\left(x_j,t;\bar{\phi}_p^n\right)\right)^2\right)dt}$$

where $\delta\theta(x,t)$ is the sensitivity function computed for the descent direction : d^n.

In order to determine $\delta\theta(x,t)$, the effect of a variation of the heat flux $\phi_p+\eta\delta\phi_p$ on the temperature evolution has to be estimated. The resolution of the following system of equations, called sensitivity equations, leads to the determination of $\delta\theta(x,t)$.

- ***Sensitivity equations***

$$(P_{sen})\begin{cases}\dfrac{\partial\left(\rho(\theta)Cp(\theta)\delta\theta\right)}{\partial t}-\Delta\left(\lambda(\theta)\delta\theta\right)=0 \text{ in } \Omega\times\mathrm{T}\\ \delta\theta(x,0)=0 \text{ in } \Omega\\ -\dfrac{\partial\left(\lambda(\theta)\delta\theta\right)}{\partial\vec{n}_1}=\delta\phi_p(\mathrm{x,t}) \text{ on } \Gamma_1\times T\\ -\dfrac{\partial\left(\lambda(\theta)\delta\theta\right)}{\partial\vec{n}_2}=0 \text{ on } \Gamma_2\times T\\ -\dfrac{\partial\left(\lambda(\theta)\delta\theta\right)}{\partial\vec{n}_3}=\sigma\left[4\varepsilon(\theta)\theta^3+\dfrac{\partial\varepsilon(\theta)}{\partial\theta}\left(\theta^4-\theta_{env}^4\right)\right]\delta\theta \text{ on } \Gamma_3\times T\end{cases}$$

Remarks :

- Direct problem, adjoint problem and sensitivity problem are established from non linear partial differential equations. The same numerical method (finite element) and software can be implemented. Moreover, these problems are well conditionned.
- Whatever the numerical approach is, identification can lead to erroneous estimates due to sensor errors[9]. Methods for optimum sensor locations are generally based upon sensitivity analysis[10]. The problem under interest is to locate one or several sensors such that the estimation of $\varphi(\theta)$ is not adversely affected by errors in the measured state θ. In order to determine spatial location where sensitivity function is maximum, sensitivity equations can be considered[11].

4. *About Sensor Location*

The sensitivity problem P_{sen} is defined according to the set of equations established in the previous section. In the following, numerical results are presented for a variation of the flux on boundary Γ_1. A variation of 1% of the heating flux $\phi_p(x,t)$ defined in section 3 was chosen to simulate the sensitivity problem. To resolve the sensitivity problem, calculated temperature from the direct problem are necessary. The thermal parameters $\lambda(\theta)$, $\rho(\theta)$, $C_p(\theta)$ and $\varepsilon(\theta)$ are present in the sensitivity problem that is why the sensitivity problem is coupled to the direct problem. To know their value at each step time of the simulation, the temperature evolution in the full substrate holder is necessary and obtained by solving first the direct problem. The direct problem was the purpose of the 3rd paragraph.

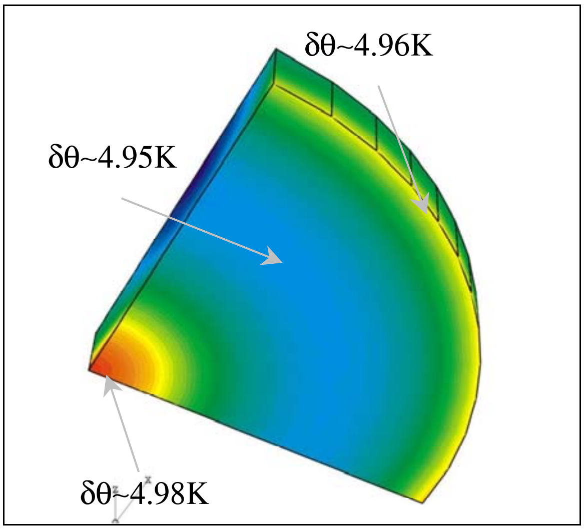

Figure 8: *side submitted to the heating flux*

The domain who are the more sensitive are bright coloured while the dark coloured are the less sensitive. Center and circular edge of the substrate seem to be more sensitive to a variation of the plasma flux. Sensor located in this area are more able to identify the unknown plasma flux.

On the back side, almost all the surface is less sensitive to the plasma heat than the front side. Sensors located on this surface are less sensitive to a variation of plasma flux. However, their observations are useful because the sensor 2 and 4 are located in the centre and in the suburb of the domain (respectively). These locations are the more sensitive of the back side.

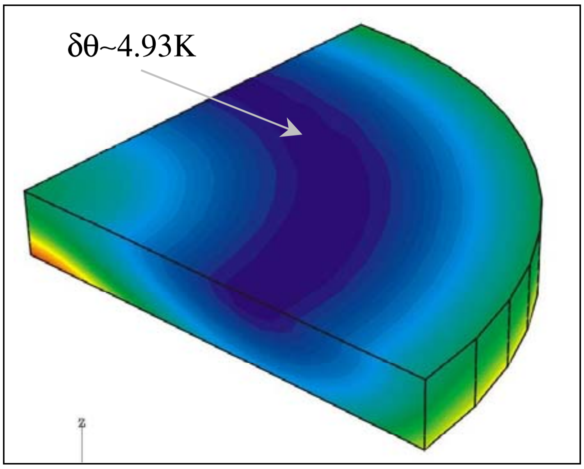

Figure 9: *side not submitted to the plasma medium*

As seen for the temperature evolution, the sensitivity evolution is pratically the same inside the full substrate holder. In fact, the boundary where is applicated the variation of the plasma heat flux is the biggest exchange area that's why the domain has the same sensitivity evolution. Moreover, the plasma heat flux used is not very different from the centre to the border of the full substrate holder.

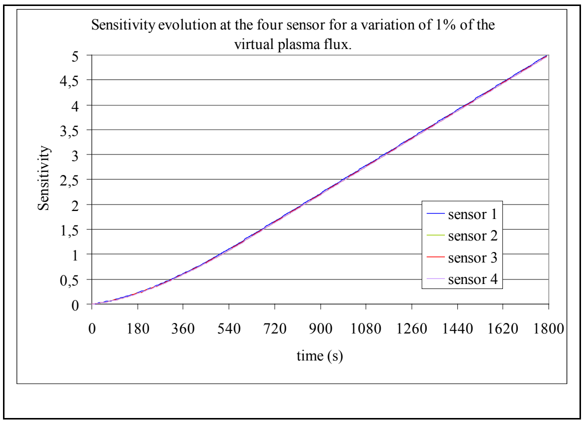

Figure 10: *time evolution of the sensitivity of the four sensors.*

The evolution of the sensitive function is shown on figure 10 (for a variation of 1% of $\phi_p(x,t)$). From this figure, it is difficult to define an optimal strategy of observation since sensor sensitivities are quite similar at each instant. Thus, measurements given by the four sensors are taken into account at each time step.

According to this numerical simulation, the identification of the unkown heating flux can be investigated since the sensors location seem to be efficient enough.

5. *Numerical resolution of the inverse problem*

In this section, a numerical identification of the arbitrary heat flux proposed in the section 3 is performed for a time interval of $1200s$ with a time step of $30s$. The criterion was arbitrary stopped at 79.45 (corresponding to iteration number 157) and the criterion for the initial plasma thermal flux was $J\left(\theta,\bar{\phi}_p^0\right)=988607$.

(the initial value is $f^\circ(x)=-100W/m^2$ for the spatial distribution and $g^\circ(t)=0.1$ for the time distribution).

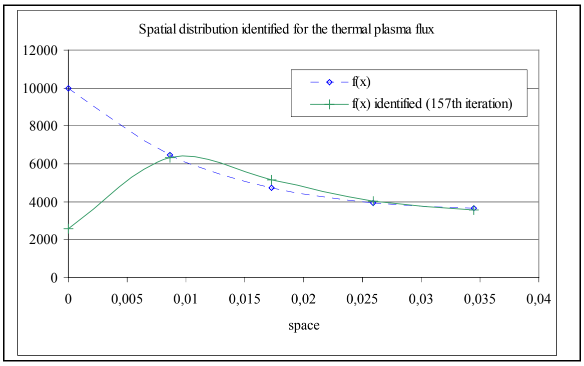

Figure 11 : *Results for the identification of the spatial distribution of the heating flux.*

The heat exchanges at the center of the sample is bad estimated due to the low number of nodes at this position. The other parameters of the spatial function are well identified.

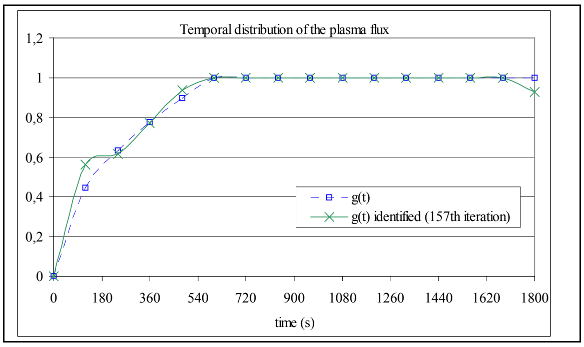

Figure 12 : *Results for the identification of the time distribution of the heating flux.*

The time distribution is well identified. Differences between simulated measurements (obtain with an arbitrary given $\phi_p(x,t)$) and calculated temperatures are shown on figure 13. The difference does not exceed $1.5K$ while the temperature range is about 550K.

Thus, the resolution of the inverse problem (in the previous simulated numerical situation) gives good results. The identification of heat transfers on the surface of the substrate by means of the conjugate gradient method seems to be efficient.

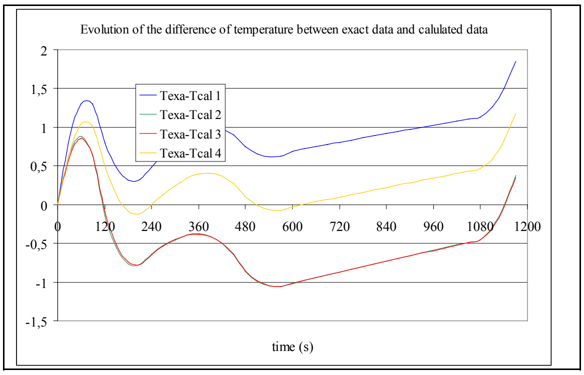

Figure 13: Differences between simulated measurements and calculated temperatures

6. Conclusions

In this communication, the problem of identification of heat transfers on the surface of a substrate is developed. The thermal model taking into account the thermal transfer between plasma medium and the substrate holder is presented. As the heating flux due to the plasma is unknown, an inverse problem is stated. The conjugate gradient method is used to minimise the quadratic criterion. This method leads to the resolution of three well-conditioned problems : the direct problem to calculate the quadratic criterion, the adjoint problem to compute the gradient and the sensitivity problem to calculate the descent depth. Sensitivity analysis is briefly presented in order to verify the strategy of sensors locations. Numerical simulations are presented and the identification of a heat flux depending on space and time variable is performed. Since the method is efficient and attractive in a numerical situation, experimental temperature measurements will be consider : in order to measure the temperature, gained thermocouple K are used[12]. From these experimental measurements, identification results obtained with the previous regularization method will be analysed.

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