A DESIGN METHOD OF 2-DEGREE-OF-FREEDOM REPETITIVE CONTROL SYSTEMS

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Abstract

In the present paper, we examine a design method of 2-degree-of-freedom repetitive control systems. The 2-degree-of-freedom control has good characteristics, such as the input-output characteristics for the reference input and feed-back control characteristics can be independently settled. 2-degree-of-freedom controllers consist of the feed-forward controller and the feed-back controller. Design methods of feed-back controller for the repetitive control systems have been considered sufficiently, but design methods of feed-forward controllers for repetitive control systems have not been considered yet. In order to design feed-forward controller for the repetitive control system, we must solve the interpolation problem. In addition, the interpolation problem for feed-forward controller for the repetitive control system has many interpolation points. No method has been proposed to solve the interpolation problem for feed-forward controller for the repetitive control system. The purpose of this paper is to propose a simple design method for feed-forward controllers and to present a design method of 2-degree-of-freedom repetitive control systems.

1 Introduction

In this paper, we examine a design method of 2-degree-of-freedom repetitive control systems. The repetitive control system is a type of servo-mechanism for repetitive reference input. That is, the repetitive control system follows the periodic reference input without steady state error even if a periodic disturbance or uncertainty exists in the plant [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. The repetitive control system was initially proposed for 'high accuracy control magnet power supply of proton synchrotron' [1]. Subsequently, several papers on the theory and application of repetitive control systems have been published [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. Since a repetitive control system follows any periodic reference input without steady state error is a neutral type of time-delay control system, it is difficult to design stabilizing repetitive controllers for the plant [8]. In order to design a repetitive control system that follows any periodic reference input without steady state error, the plant needs to be biproper [3, 4, 5, 6, 7, 8]. Ikeda and Takano [9] pointed out that it is physically difficult for the output to follow any periodic reference input without steady state error. In addition they showed that the repetitive control system is L_2 stable for periodic input that do not include infinite frequency input if the relative degree of the controller is one. However, since the actual control system is strictly proper and has any relative degree, many design methods of repetitive controller for the strictly proper plant are given in [3, 4, 5, 6, 7, 8]. These studies are divided into two types. One uses a low pass filter [3, 4, 5, 6, 7] and the other uses an attenuator [8]. Since the former type of repetitive control system has a simple structure, and is easy to design, this design method is used in many applications [3, 4, 5, 6, 7]. The method of later can make steady state error small. But it is difficult to design because the later uses a state variable time-delay in repetitive controller. Therefore the former, called modified repetitive control systems, have been widely applied [1, 2, 3, 4, 5, 6, 7]. However, all designs of repetitive control systems announced so far are examining the method of designing feed-back controller, and no method of feed-forward controller for repetitive control systems has been examined.

The purpose of this paper is to give a design method for feed-forward controllers and to propose a design method of 2-degree-of-freedom repetitive control system. First, the structure of 2-degree-of-freedom repetitive control system is presented. Next, the problem considered in this paper is summarized. We describe the basic idea to solve the problem of design method of feed-forward repetitive controller, which is the fusion of the stable filtered inverse system [13, 14] and the idea of predictive control [15]. A design procedure of the feed-forward controllers for single-input/single-output systems is proposed. Next, we present a design method of 2-degree-of-freedom repetitive control systems. We expand this result and propose a design procedure of the feed-forward controllers for multiple-input/multiple-output systems. A numerical example is illustrated to show the effectiveness of the proposed method.

Notations

$R(s)$	the set of real rational function with s.
RH_∞	the set of stable proper real rational functions.
H_∞	a set of stable causal functions.

2 2-degree-of-freedom repetitive control

Let us consider a 2-degree-of-freedom control system shown in Fig. 1 . Here $G(s) \in R(s)$ is the plant, $C(s) \in R(s)$

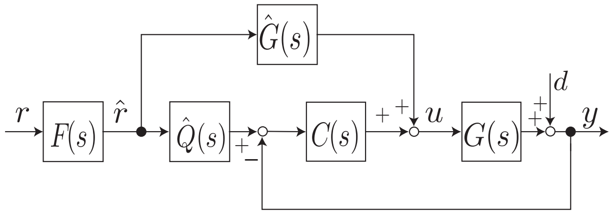

Figure 1: 2-degree-of-freedom control system

is the feed-back controller, $\hat{Q}(s) \in R(s)$, $\hat{G}(s) \in R(s)$ and $F(s) \in R(s)$ are the feed-forward controllers and satisfying

$$G(s)\hat{G}(s) = \hat{Q}(s). \tag{1}$$

$G(s)$ is assumed to be controllable and observable. y is the output, u is the control input, r is the periodic reference input with period T satisfying

$$r(t+T) = r(t) \quad (\forall t \geq 0) \tag{2}$$

and has settled beforehand, d is the periodic disturbance with period T.

The transfer function from r to y in Fig. 1 and that from d to y in Fig. 1 are given by

$$\begin{aligned} y &= G(s)\hat{G}(s)F(s)r \\ &= \hat{Q}(s)F(s)r \end{aligned} \tag{3}$$

and

$$y = \frac{1}{1+G(s)C(s)}d, \tag{4}$$

respectively. Therefore 2-degree-of-freedom control system in Fig. 1 can settle independently the input-output characteristics for the reference input and the feed-back control characteristics.

If the plant $G(s)$ has a periodic disturbance d with period T and uncertainty, and the output y follows the periodic reference input r with period T and the frequency component

$$\omega_i = \frac{2\pi}{T} i \quad (i = 0, \cdots, N) \tag{5}$$

without steady state error, then the controller $C(s)$ must be described by

$$C(s) = C_r(s)\hat{C}(s) \tag{6}$$

[4], where $C_r(s)$ is an internal model for the periodic reference input r with period T written as

$$C_r(s) = \frac{1}{1-q(s)e^{-sT}} \tag{7}$$

and $q(s)$ is strictly proper, asymptotically stable low pass filter satisfying

$$1 - q(j\omega_i) = 0 \quad (i = 0, \cdots, N). \tag{8}$$

For practically, in many cases, $q(s)$ is settled by

$$q(s) = \frac{1}{(1+s\tau_q)^{\alpha_q}} \tag{9}$$

satisfying

$$1 - q(j\omega_i) \simeq 0 \quad (i = 0, \cdots, N), \tag{10}$$

where $\tau_q > 0$ is sufficiently small positive real number and α_q is an arbitrary positive integer [3, 4, 5, 6, 7, 8].

In addition, in order the output y to follow the reference input r with the frequency components in Equation (5), the feed-forward controller $\hat{G}(s)$, $\hat{Q}(s)$ and $F(s)$ must satisfy

$$\begin{aligned} 1 - G(s)\hat{G}(s)F(s)\Big|_{s=j\omega_i} &= 1 - \hat{Q}(s)F(s)\Big|_{s=j\omega_i} \\ &= 0 \quad (i = 0, \cdots, N). \end{aligned} \tag{11}$$

In oder the output y to follow the reference input r in Equation (2) without steady state error, it needs that control system in Fig. 1 is stable. According to [12], the necessary and sufficient stability condition of the control system in Fig. 1 is satisfying following expressions:

1. $C(s)$ stabilizes $G(s)$.
2. $\hat{G}(s)$ is stable.
3. $\hat{Q}(s)$ is stable.
4. $F(s)$ is stable.

In this paper, we examines a design method of feed-forward controller $\hat{G}(s)$, $\hat{Q}(s)$ and $F(s)$ for 2-degree-of-freedom control systems in Fig. 1 to make 2-degree-of-freedom repetitive control systems in Fig. 1 stable.

We adapt a design method of filtered inverse system in [14] for designing the feed-forward controller. Because the design method for feed-forward controllers in [14] has following advantages:

1. the output y follows the step reference input r without steady state error.
2. even if the plant is multiple-input/multiple-output, the transfer function from r to y is decoupled. This implies that the input-output characteristics is reduced to single-input/single-output control problem.

Therefore, we consider the design problem that the output y follows the general reference input r written in Equation (2) without steady state error maintaining the advantage of the method in [14]. In order to maintain the advantage of the method in [14], if $\hat{G}(s)$ in Fig. 1 is designed using the method in [14], a design problem of feed-forward controller for multiple-input/multiple-output is reduced to single-input/single-output control problem. $F(s)$ is designed so that the output y in Fig. 1 may follow the periodic reference input r without steady state error. Note that if $\hat{r}$ in Fig. 1 is calculable even if $F(s)$ is not necessarily causal in Fig. 1 the control system in Fig. 1 can be built. That is, in the repetitive control system, the periodic reference input r is given in advance as in Equation (2). A design method of $F(s)$ using the characteristics of the repetitive control system such that the reference input is also acquired in advance is proposed.

3 Basic idea of designing $F(s)$

In this section, we describe a basic idea to design method of feed-forward controller in Fig. 1 .

For easy explanation, it is assumed that the reference input r is written by

$$r = \frac{\bar{\omega}}{s^2 + \bar{\omega}^2}, \tag{12}$$

where $\bar{\omega} > 0$.

According to [14], $\hat{G}(s)$ in Fig. 1 is settled by

$$\begin{aligned} G(s)\hat{G}(s) &= \hat{Q}(s) \\ &= Q(s)G_K(s), \end{aligned} \tag{13}$$

where $G_K(s) \in RH_\infty$ is the inner function of $G(s)$ satisfying

$$G_K(-s)G_K(s) = 1 \tag{14}$$

and

$$G_K(0) = 1. \tag{15}$$

$Q(s)$ is

$$Q(s) = \frac{1}{(1 + s\tau)^\alpha} \tag{16}$$

and α is arbitrary positive integer which makes $\hat{G}(s)$ be proper and τ is sufficiently small positive real number.

Next we describe a design method of $F(s)$ to satisfy Equation (11). For easy explanation, τ is assumed to be settled to be sufficiently small positive real number and to satisfy

$$Q(j\bar{\omega}) = 1. \tag{17}$$

From the assumption of Equation (17) and Equation (13), we have

$$\begin{aligned} 1 - G(s)\hat{G}(s)F(s)\Big|_{s=j\bar{\omega}} &= 1 - Q(s)G_K(s)F(s)|_{s=j\bar{\omega}} \\ &= 1 - G_K(s)F(s)|_{s=j\bar{\omega}} . \end{aligned} \tag{18}$$

We define $F(s)$ by

$$F(s) = e^{sW}, \tag{19}$$

where $W \in R$ is settled by

$$W = -\frac{\angle G_K(j\bar{\omega})}{\bar{\omega}}. \tag{20}$$

From the assumption of Equation (14) and simple manipulation, using $F(s)$ written by Equation (19), it is confirmed that

$$1 - G_K(s)F(s)|_{s=j\bar{\omega}} = 0. \tag{21}$$

In this way, we can design $F(s)$ satisfying Equation (11).

Since $G_K(s)$ is an inner function satisfying Equation (14) and Equation (15), $\angle G_K(j\bar{\omega}) < 0$ is confirmed. Therefore, since $W > 0$ in Equation (20), $F(s)$ in Equation (19) is not causal. However, from the assumption that r is obtained beforehand, $\hat{r}$ can be calculated and the control system in Fig. 1 is evaluated.

$F(s)$ in Equation (19) works as the predictor, the proposed method is a method using predictor.

4 Design method for feed-forward controller

In this section, we expand the idea described in 3 and propose a design procedure of $F(s)$ such that the output y follows the reference input r in Equation (2) without steady state error.

In the same manner as the method in 3, $\hat{G}(s)$ in Fig. 1 is designed using the method in [14]. That is, $\hat{G}(s)$ satisfies Equation (11).

From Equation (13), the problem of designing $F(s)$ satisfying Equation (11) is equivalent to hold

$$1 - Q(s)G_K(s)F(s)|_{s=j\omega_i} = 0 \ \ (i = 0, \cdots, N). \tag{22}$$

The rest of designing problem is to find the design procedure of $F(s)$ satisfying Equation (22). Let $F(s)$

$$F(s) = \sum_{i=0}^{N} h_i(s)e^{sW_i}, \tag{23}$$

where $h_i > 0 \in R(i = 0, \cdots, N)$ and $W_i \in R(i = 0, \cdots, N)$. For simplicity, we rewrite Equation (23) to

$$\begin{aligned} F(s) &= \sum_{i=0}^{N} h_i(s)e^{sW_i} \\ &= \bar{h}_0(s)e^{s\bar{W}_0}\left(1 + \bar{h}_1(s)e^{s\bar{W}_1}\right. \\ &\quad \left.\left(1 + \bar{h}_2(s)e^{s\bar{W}_2}\left(1 + \cdots\left(1 + \bar{h}_N(s)e^{s\bar{W}_N}\right)\right)\right)\right), \end{aligned} \tag{24}$$

where

$$h_i(s) = \prod_{k=0}^{i} \bar{h}_k(s) \ \ (i = 0, \cdots, N) \tag{25}$$

and

$$W_i = \sum_{k=0}^{i} \bar{W}_k \ \ (i = 0, \cdots, N). \tag{26}$$

The design procedure for $F(s)$ satisfying Equation (22) is summarized following procedure.

<u>The design procedure</u>

1. Let $i = 0$.
   (a) Let $h_i(s)$ is designed so that $|Q(j\omega_i)G_K(j\omega_i)h_i(j\omega_i)| = 1$ are satisfied. $h_i(s)$ is established by
   $$h_i(s) = 1. \tag{27}$$
   (b) A design of W_i
   W_i is established to hold
   $$\left\{\angle\left(Q(s)G_K(s)\right) + \angle e^{sW_i}\right\}\Big|_{s=j\omega_i} = 0. \tag{28}$$
   That is, W_i is established by
   $$W_i = 0. \tag{29}$$
   (c) Add 1 to i and go to next step.

2. From Equation (24), $h_i(s)$ is settled as
$$h_i(s) = \prod_{k=0}^{i} \bar{h}_k(s), \tag{30}$$
where
$$\bar{h}_i(s) = l_i \hat{h}_i(s), \tag{31}$$
$l_i \in R$, $\hat{h}_i(s) \in RH_\infty$. When $i = 1$, $\hat{h}_i(s)$ is settled by
$$\hat{h}_i(s) = \frac{s}{b_i(s)} \tag{32}$$
to hold $\hat{h}_i(\omega_0) = \hat{h}_i(0) = 0$. When $i > 1$, $\hat{h}_i(s)$ is settled by
$$\hat{h}_i(s) = \frac{s^2 + \omega_{i-1}^2}{b_i(s)} \tag{33}$$
to hold $\hat{h}_i(j\omega_{i-1}) = 0$. Here, $b_i(s)$ is any stable polynomial expression that makes $\hat{h}_i(s)$ in Equation (32) or Equation (33) proper. Let l_i
$$l_i = \frac{\left|1 - Q(j\omega_i)G_K(j\omega_i)\displaystyle\sum_{k=0}^{i-1} h_k(j\omega_i)e^{j\omega_i W_k}\right|}{\left|Q(j\omega_i)G_K(j\omega_i)\hat{h}_i(j\omega_i)\displaystyle\prod_{k=0}^{i-1} \bar{h}_k(j\omega_i)\right|}. \tag{34}$$
Next, we present a design method for W_i. W_i is established so that the phase angle of $Q(j\omega_i)G_K(j\omega_i)h_i(j\omega_i)e^{j\omega_i W_i}$ is equal to that of $1 - Q(j\omega_i)G_K(j\omega_i)\displaystyle\sum_{k=0}^{i-1} h_k(j\omega_i)e^{j\omega_i W_k}$. That is, W_i is designed to hold
$$\angle\left(Q(j\omega_i)G_K(j\omega_i)h_i(j\omega_i)e^{j\omega_i W_i}\right) = \angle\left(1 - Q(j\omega_i)G_K(j\omega_i)\sum_{k=0}^{i-1} h_k(j\omega_i)e^{j\omega_i W_k}\right). \tag{35}$$
W_i to hold Equation (35) is given by
$$W_i = \frac{1}{\omega_i}\left\{-\angle\left(Q(j\omega_i)G_K(j\omega_i)h_i(j\omega_i)\right) + \angle\left(1 - Q(j\omega_i)G_K(j\omega_i)\sum_{k=0}^{i-1} h_k(j\omega_i)e^{j\omega_i W_k}\right)\right\}. \tag{36}$$

3. If $i + 1 > N$, end up. In other cases, add 1 to i and return preceding step.

5 A design procedure of 2-degree-of-freedom repetitive control system

In this section we describe a design procedure of 2-degree-of-freedom repetitive control system.

A design procedure of 2-degree-of-freedom repetitive control system using design method of feed-forward controller in 4. is as follows:

1. A frequency component Equation (5) of the periodic reference input r with period T which the output y should follow is decided. That is, N in Equation (5) is settled.
2. design of feed-back controller
   (a) Low pass filter $q(s)$ is given by Equation (9), $\tau_q > 0$, α_q is settled satisfying $1 - q(j\omega_i) \simeq 0 \quad (i = 0, \cdots, N)$, $C_r(s)$ is give by Equation (7).
   (b) $\hat{C}(s)$ in Equation (6) is settled satisfying
   $$\left\|\frac{q(s)}{1 + G(s)\hat{C}(s)}\right\|_\infty < 1. \tag{37}$$
   According to [10], if $\hat{C}(s)$ in Equation (6) is settled satisfying Equation (37), the feed-back controller in Equation (6) stabilities the plant $G(s)$.
3. design of feed-forward controller
   (a) The feed-forward controller $\hat{G}(s) \in RH_\infty$, $\hat{Q}(s) \in RH_\infty$ are designed satisfying Equation (13).
   (b) $F(s)$ is designed by the design procedure in 4.
4. The 2-degree-of-freedom repetitive control system in Fig. 1 is designed by using $C(s)$, $\hat{G}(s)$, $\hat{Q}(s)$ and $F(s)$.

6 Feed-forward controller for multiple-input/multiple-output systems

In this section, we expand the result in 4 and propose a design method of feed-forward controller for multiple-input/multiple-output systems.

Let us consider the 2-degree-of-freedom repetitive control systems in Fig. 1 . Here $G(s) \in R^{p\times p}(s)$ is the plant. $G(s)$ is assumed to be controllable and observable, and has no polo on the imaginary axis, and satisfies

$$\mathrm{rank}\ G(s) = p. \tag{38}$$

$C(s) \in R^{p\times p}(s)$ is the feed-back controller, $\hat{Q}(s) \in R^{p\times p}(s)$, $\hat{G}(s) \in R^{p\times p}(s)$ and $F(s) \in R^{p\times p}(s)$ are the feed-forward controllers and satisfying Equation (1). $y \in R^p$ is the output, $u \in R^p$ is the control input, $r \in R^p$ is the periodic reference input with period T written as Equation (2).

In the same manner as 4, the problem considered in this section is to find the design procedure of $F(s)$ satisfying

$$I - G(s)\hat{G}(s)F(s)\Big|_{s=j\omega_i} = 0 \ \ (i = 0,\cdots,N). \tag{39}$$

$\hat{G}(s)$ in Fig. 1 is settled by

$$\begin{aligned} G(s)\hat{G}(s) &= \hat{Q}(s) \\ &= Q(s)G_K(s), \end{aligned} \tag{40}$$

where $G_K(s) \in RH_\infty^{p\times p}$ is the diagonal inner function of $G(s)$ satisfying

$$G_K(s) = \mathrm{diag}\ \{\ G_{K1}(s),\cdots,G_{Kp}(s)\ \} \tag{41}$$

$$G_{Ki}(-s)G_{Ki}(s) = 1 \ \ (i = 1,\cdots,p), \tag{42}$$

and

$$G_K(0) = I, \tag{43}$$

$Q(s)$ is

$$\begin{aligned} Q(s) &= \mathrm{diag}\ \left\{\ \frac{1}{(1+s\tau_1)^{\alpha_1}},\cdots,\frac{1}{(1+s\tau_p)^{\alpha_p}}\ \right\} \\ &= \mathrm{diag}\ \{\ q_1(s),\cdots,q_p(s)\ \} \end{aligned} \tag{44}$$

$\alpha_i(i = 1,\cdots,p)$ is arbitrary positive integer which makes $\hat{G}(s)$ be proper and $\tau_i(i = 1,\cdots,p)$ is sufficiently small positive real number.

From Equation (40), the problem of designing $F(s)$ satisfying Equation (39) is equivalent to hold

$$I - Q(s)G_K(s)F(s)|_{s=j\omega_i} = 0 \ \ (i = 0,\cdots,N). \tag{45}$$

We settle $F(s)$ by

$$F(s) = \mathrm{diag}\ \{F_1(s),\cdots,F_p(s)\}. \tag{46}$$

From Equation (41), Equation (44) and Equation (46), Equation (45) is equivalent to

$$\begin{aligned} &1 - q_m(s)G_{Km}(s)F_m(s)|_{s=j\omega_i} = 0 \\ &\qquad (i = 0,\cdots,N : m = 1,\cdots,p). \end{aligned} \tag{47}$$

Equation (47) is a designing problem of feed-forward controller for single-input/single-output systems. Therefore, for multiple-input/multiple-output systems, we can design feed-forward controller $F(s)$ using the same procedure in 4.

7 Numerical example

In this section, a numerical example is illustrated to demonstrate the effectiveness of the proposed method.

Let the unstable plant $G(s)$ be

$$G(s) = \frac{-s+250}{s^2+17s-200}. \tag{48}$$

Let us consider to design repetitive control system for the plant $G(s)$ in Equation (48). The periodic reference r with period $T = 1$[sec] is written by

$$r = \sin(2\pi t) + \sin(4\pi t). \tag{49}$$

The feed-back controller $\hat{C}(s)$ is given by Equation (6). Where $C_r(s)$ and $q(s)$ are settled by Equation (7) and by

$$q(s) = \frac{1}{1+0.01s}, \tag{50}$$

respectively.

If $\hat{C}(s)$ in Equation (6) selected by

$$\begin{aligned} &\hat{C}(s) \\ &= \frac{100s^4+7000s^3+170000s^2+1570700s+2779000}{s^4+500s^3+24800s^2+99341s}, \end{aligned} \tag{51}$$

satisfies stability condition in Equation (37). It is confirmed that feed-back controller $C(s)$ in Equation (6) stabilizes the plant $G(s)$.

Next we design feed-forward controller. Since the reference input r is given by Equation (49), $N = 2$. $G_K(s)$ satisfying Equation (41) and Equation (42) is given by

$$G_K(s) = \frac{-s+250}{s+250}. \tag{52}$$

The low pass filter $Q(s)$ is settled by

$$Q(s) = \frac{1}{1+0.01s}. \tag{53}$$

From Equation (13), the feed-forward controller $\hat{G}(s)$ and $\hat{Q}(s)$ are written by

$$\hat{G}(s) = \frac{100s^2+1700s-20000}{s^2+350s+25000} \tag{54}$$

and

$$\hat{Q}(s) = \frac{-100s+25000}{s^2+350s+25000}, \tag{55}$$

respectively.

When we design feed-forward controller $F(s)$ method in 4, $h_i(s)(i = 0,1,2)$, $W_i(i = 0,1,2)$ in Equation (23) are calculated by

$$h_0(s) = \bar{h}_0(s) = 1, \tag{56}$$

$$h_1(s) = \prod_{k=0}^{1} \bar{h}_k(s), \tag{57}$$

$$h_2(s) = \prod_{k=0}^{2} \bar{h}_k(s), \tag{58}$$

$$\bar{h}_1(s) = \frac{0.018s}{0.01s+1}, \tag{59}$$

$$\bar{h}_2(s) = \frac{10^{-3}\left(0.045s^2+1.80\right)}{\left(0.01s+1\right)^2}, \tag{60}$$

$$W_0 = 0, \tag{61}$$

$$W_1 = 0.0162, \tag{62}$$

$$W_2 = 0.2781. \tag{63}$$

Next we show the difference of the response for the reference input between $F(s)=1$ and the proposed method ($F(s)$ is setled by Equation (23)). The error $e = r - y$ for the reference input r is shown in Fig. 2 . Here, the solid line shows time

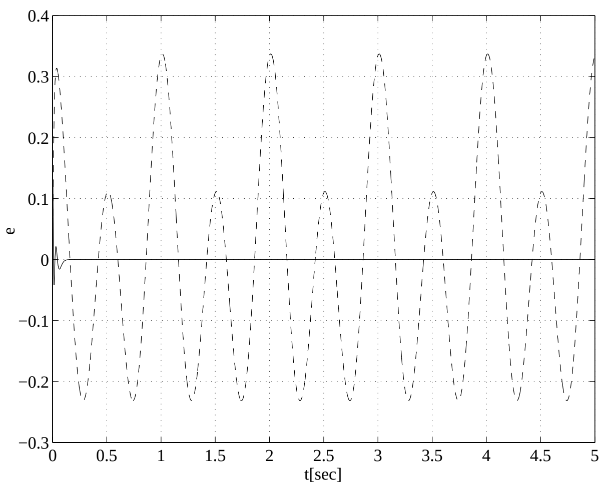

Figure 2: Error for the reference input $r = \sin(2\pi t) + \sin(4\pi t)$

response in the case that $F(s)$ is setlled by Equation (23). The dotted line show time response in the case of $F(s)=1$. From Fig. 2 , we find that the proposed feed-forward controller $F(s)$ in Equation (23) has better characteristics than $F(s)=1$ such as the output follows without steady state error and the convergence speed is high.

8 Conclusion

In this paper, we gave a design method for feed-forward controllers using the stable filtered inverse systems in [13, 14] and the predictive control, and proposed a design method of 2-degree-of-freedom repetitive control system.

References

[1] Inoue, et al. High Accuracy Control Magnet Power Supply of Proton Synchrotron in Recurrent Operation. The Trans. of The Institute of Electrical Engineers of Japan, 100:234–240, 1980.

[2] Inoue, Iwai and Nakano. High Accuracy Control of Play-Back Servo System. The Trans. of The Institute of Electrical Engineers of Japan, 101-4:89–96, 1981.

[3] Hara, Omata, and Nakano. Stability Condition and Synthesis Methods for Repetitive Control System. Trans. of the Society of Instrument and Control Engineers, 22-1:36–42, 1986.

[4] Yamamoto and Hara. The Internal Model Principle and Stabilizability of Repetitive Control System. Trans. of the Society of Instrument and Control Engineers, 22-8:830–834, 1987.

[5] Hara and Yamamoto. Stability of Multivariable Repetitive Control Systems - Stability Condition and Class of Stabilizing Controllers. 22-12:1256–1261, 1986.

[6] Hara, Yamamoto, Omata and Nakano. Repetitive Control System: A New Type Servo System for Periodic Exogenous Signals. IEEE Trans. on Automatic Control, AC-33-7:659–668, 1988.

[7] Omata, Hara and Nakano. Nonlinear Repetitive Control with Application to Trajectory Control of Manipulators. J. of Robotic Systems, 4-5:631–652, 1987.

[8] Watanabe and Yamatari. Stabilization of Repetitive Control System-Spectral Decomposition Approach. Trans. of the Society of Instrument and Control Engineers, 22-5:535–541, 1986.

[9] Ikeda and Takano. Repetitive Control for Systems with Nonzero Relative Degree. Trans. of the Society of Instrument and Control Engineers, 24-6:575–582, 1988.

[10] Nakano, et al. Repetitive Control. Trans. of the Society of Instrument and Control Engineers,1989.

[11] Gotou, et al. A Robust System with an Iterative Learning Compensator and a Proposal of Multi-Period Learning Compensator. J. of The Society of Instrument and Controll Engneers, No. 5:367–374, 1987.

[12] Sugie, T., and Yoshikawa, T., General Solution of Robust Tracking Problem in Two-Degree-Of-Freedom Control Systems, IEEE Trans., AC-31, 552-554(1986)

[13] Yamada, K., Watanabe, K., Shu, Z.B., A State Space Design Method of Stable Filtered Inverse Systems and Its Application to H_2 Suboptimal Internal Model Control, International Federation of Automatic Control World Congress'96, pp.379-382(1996)

[14] Yamada, K., and Kinoshita, W., New design method of stable filtered inverse systems, American Control Conference 2002, 4738-4743(2002)

[15] Tsuchiya, T., Egami, T., Digital preview & predictive control, Sangyotosho(1992)(in Japanese)